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AN

INTRODUCTION

TO

NATURAL PHILOSOPHY;

DESIGNED AS A

TEXT-BOOK

FOR THE USE OF

STUDENTS IN COLLEGE.

BY DENISON OLMSTED, LL.D.,
PROFESSOR OF NATURAL PHILOSOPHY AND ASTRONOMY IN YALE COLLEGE.

REVISED
BY E. S. SNELL, LL.D.,
PROFESSOR OF MATHEMATICS AND NATURAL PHILOSOPHY IN
AMHERST COLLEGE.

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PREFACE TO THE REVISED EDITION.

In the autumn of 1858, Professor Olmsted addressed a letter to the author of this revision, stating it to be his purpose to revise his "College Philosophy," and requesting corrections and suggestions, which might have been noted by one who had used the work as a text-book for several years. In reply to this application, such material was prepared and furnished as was deemed suitable for additions and amendments, with the understanding that the author of the treatise would make use of it so far as it should suit his purpose. But he had not entered upon the intended work, when sickness and death brought his labors to a close.

After the decease of Professor Olmsted, the manuscript which I had prepared was returned to my hands, with a request that I would make a revision of the book myself. The present volume is the result. It has been my aim to preserve the general character of the work unchanged, for reasons which are fully stated in the original Preface. Additions have been made to all the subjects treated of, but most largely to "Acoustics" and "Optics." Part VIII., on "Electro-Magnetism," is now introduced for the first time, in which is presented a brief statement of the main principles of that branch of science, sufficient to enable the pupil to understand the explanation of the present system of telegraphy, and the application of the electro-magnetic force to the mechanic arts. In three instances, there is placed at the end of a chapter a discussion of its principal subject by means of the differential and integral calculus. These processes are presented in their appropriate connection, and yet are not so interwoven with the general discussions of the work, but that they can be used or omitted without inconvenience, as the teacher, in view of the attainments of his classes, shall judge expedient. For the benefit of those who wish to exercise themselves in the practical applications of mechanical principles, several miscellaneous problems are introduced at the end of Part I.

Notwithstanding these additions, the volume, instead of being enlarged, is considerably diminished. This results from several circumstances. The more simple wood-cuts (about one-fourth of the whole number) are much reduced in size, thus giving more room for printed matter. The statements of facts and principles, and the demonstrations of propositions, have been expressed more concisely, wherever it seemed possible to make such changes without a sacrifice of perspicuity. A few articles, containing statements purely historical, which, though important and interesting, did not seem to harmonize with the general plan of the work, have on that account been omitted. A considerable number of the practical questions, especially those whose processes of solution were briefly indicated, have also been dropped; as it is believed that most teachers prefer to give such numerical examples as may occur to them, rather than to be confined wholly to the use of a stereotyped series. In these several ways room has been found for many important additions, which the present state of physical science requires that a College text-book should contain.

The practice which now considerably prevails in the preparation of text-books, of placing at the beginning of each paragraph the leading idea which it presents or discusses, has been adopted to such an extent, that one may readily find the particular topics treated of under the general subjects of the chapters.

I wish to acknowledge my indebtedness to Professor NEWTON, of Yale College, for valuable suggestions relating to the mathematical part of "Mechanics." A large proportion of the miscellaneous problems at the end of the same Part were also furnished by him.

E. S. SNELL.

AMHERST COLLEGE, March, 1860.

AUTHOR'S PREFACE.

The compiler of this work has had two objects constantly in view: first, to make the student thoroughly and familiarly acquainted with the *leading principles* of Natural Philosophy; and, secondly, to furnish him with as much *useful information* as possible, within so limited a compass. The foundation of all accurate attainments in Natural Philosophy and Astronomy being laid in the science of Mechanics, a large proportion of the work is devoted to this subject. The Mathematical Elements of Mechanics are moreover first considered, separately, that the student may have nothing to divert his mind from the contemplation of these universal and fundamental truths. Still further to render the knowledge of these truths familiar, as well as to supply a most useful intellectual exercise to the student, a great variety of Problems are annexed to each chapter, the utility of which must be obvious to every experienced instructor. Indeed, Problems hold so important a place in the estimation of the writer, that he has introduced them into various parts of the work, wherever the subject appeared to be susceptible of deriving aid from them. Problems put the student on his own resources; they compel him to think for himself; they lead him to a just understanding of the principles demonstrated; and they teach him how to reduce his knowledge to practice. These truths are so obvious, that it is difficult to account for the singular fact, that treatises on Natural Philosophy have, in general, contained few or no problems, although they occupy so large a space in most of the branches of the pure mathematics.

The first part of the following treatise on mechanics, comprising the "Mathematical Elements," is taken chiefly from *Bridge's Mechanics*. This work was peculiarly adapted to our purpose, partly because it is written in a style well suited to the average capacities and attainments of college classes, and partly because it is enriched with a finer collection of problems than any similar work with which we are acquainted. We have aimed to select such parts as promised the most practical utility; and in order to adapt the treatise to the purposes of recitation, the propositions are more distinctly enunciated than in the original work, and various alterations and occasional additions are introduced, and explanations added by note or otherwise, with the view of suiting it better to the course of instruction adopted in Yale College.

In Part II., the *Practical Applications of the Principles of Mechanics to the Arts and to the Phenomena of Nature*, are pursued as far as our limits would permit, and further, perhaps, than some instructors will deem necessary; for we are aware that some maintain the expediency of occupying the attention of the student almost exclusively with *general principles*, leaving him to make the application for himself. According to our experience, however, the student who is furnished with the knowledge of abstract principles merely, seldom acquires the necessary readiness in reducing them to practice. It has appeared to us no less necessary to initiate the learner in the habit of philosophizing, than in the doctrines of philosophy. In this manner, he will indeed acquire the knowledge of fewer principles; but he will know much better how to use his acquisitions.

We can not, however, agree with those instructors who have yielded to the spirit of the age (which is still hunting after a "royal road" to knowledge) so far as to forsake demonstration altogether, and substitute for the mathematical elements of Natural Philosophy, text-books in which the principles of the science rest on no better basis than mere popular illustration, although works of this kind may, indeed, furnish us with very useful materials for exhibiting the *applications* of these principles. In philosophy, as in morals, the most important principles are usually characterized by great simplicity: in this respect the "golden rule" and the law of gravitation resemble each other.

The subsequent parts of the work are compiled from various authors. In the belief that the truths most important to be inserted in a text-book on Natural Philosophy, are, in general, such as have long been known, no effort has been made to conform the style, either of the propositions or the demonstrations, to a modern dress; but authors, old and new, English and French, have been consulted and used indiscriminately. Aiming solely at preparing such an elementary work as would be most useful to the academic student, we have not aspired to the praise of originality, nor felt at liberty to consult the pride of authorship. Among the truths, however, found in the wide field of Natural Philosophy, a vast difference exists in regard to their value; and no small acquaintance and familiarity with the subject is requisite in the writer of a text-book, in order that he may be able to cull the choicest truths and present them to the young learner in their native beauty and simplicity. For this purpose, no powers of original investigation, or gifts of genius, can compensate for the want of the experience of the instructor.

Since the publication of the first edition of this work, the compiler has been favored with the opinions of a number of his brethren of other Colleges, who have used it as a text-book. A few would prefer to have a more strictly mathematical complexion preserved throughout, in the place of those parts which are written in a more popular style, since the method of expressing philosophical truths by mathematical formulæ is more concise and comprehensive, and better suited to mental discipline, and the cultivation of a mathematical taste. A greater number, however, of those who have been so good as to communicate their views to the writer, have approved of the method adopted, namely,—to establish the *general principles* of the science by rigorous mathematical demonstration, or by precise experiments, but to rehearse the *applications* of those principles to the arts, and to the phenomena of nature, in a style divested, as far as possible, of technical phraseology. They believe that such a method renders the study of Natural Philosophy peculiarly attractive to the young learner, and conducive to the formation of habits of philosophical observation, while they rely more on other parts of the scientific course, particularly on the pure mathematics, to fulfill the purposes of mental discipline, and to inspire a mathematical taste. We have desired to accomplish, as far as possible, these several objects of philosophical education,—to improve the faculties of the mind itself, to imbue it with a love of rigorous demonstration, and to commence the formation of habits of philosophical observation, which shall be carried forward beyond the pale of academic study, to be confirmed and strengthened throughout the period of after-life. The variety of subjects comprehended under Natural Philosophy, some admitting of strict geometrical and analytical reasoning, and others conducted wholly by experimental research, is well adapted to the attainment of these important objects; and it is the prerogative of this science, at once to enlarge the mind by the most profound inquiries, and to conduct it through the most delightful and varied fields of experiment and observation.

In a work necessarily so limited as this (when compared with the vast extent of the subject), many topics must be treated with extreme conciseness, and many others, essential to a complete philosophical education, must be omitted altogether. While we aim to furnish the student with a knowledge of the *great laws of nature*, and to exemplify and illustrate them by numerous applications, we can claim nothing more than an "Introduction to Natural Philosophy," suited to beginners.

It is recommended to the student to make a free use of the *Analysis*, especially in reviewing. Let him submit each of the particulars indicated in this outline to deliberate and repeated reflection, and he will not only fully possess himself of the contents of the work, but will lay up in the mind a system of heads, or "common-places," under which he can conveniently and useful ly arrange all his future acquisitions on similar subjects.

The course of instruction in Natural Philosophy pursued in Yale College, to which the treatise is adapted, proceeds as follows. The *mathematical* part of mechanics is first recited in the same manner as a branch of the pure mathematics. With the *practical* part commences a series of familiar Lectures,* designed to amplify the text and to illustrate it by numerous experiments. These are continued during the perusal of the remainder of the work. To the same class is afterward delivered a select course of Lectures, which are chiefly devoted to the discussion of the great principles of Philosophy and Astronomy, and especially to such subjects as require a fuller attention than they can receive in the elementary course.

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*Should any teacher who may use this work, think it better to connect the Lectures and Practical Applications immediately with the theoretical part, it will be easy to do so, by giving occasional lessons in Part II., while the student is reading Part I.

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NATURAL PHILOSOPHY.

PART I.—MATHEMATICAL ELEMENTS OF MECHANICS.

PRELIMINARY DEFINITIONS.

ART. 1. *Natural Philosophy* is the science which treats of the laws of the material world. This is the primitive signification of the term; but the vast extension given to inquiries into the laws of nature, rendered a division of them necessary. Hence, those laws of nature which relate to *masses* of matter were retained by Natural Philosophy (which has been farther divided into Mechanical Philosophy and Astronomy), while those which relate to *particles* of matter, and to the changes of constitution produced by their action on each other, were assigned to Chemistry.

The term *law*, as here used, signifies the mode in which the powers of nature act. Laws are *general truths*, comprehending a great number of subordinate truths.

Natural or Mechanical Philosophy is divided into several branches, which usually receive the names, Mechanics, Hydrostatics, Pneumatics, Acoustics, Electricity, Magnetism, and Optics.

Mechanics, taken in the widest sense of the term, is that branch of Natural Philosophy which treats of the equilibrium and motion of bodies. As the changes which occur between masses of matter involve the idea of motion, hence, the causes which produce motion, or which prevent it, and the manner in which it takes place (its *laws*), constitute the great object of inquiry in mechanical philosophy.

Body is any collection of matter existing in a separate form.

Force is any cause which moves or tends to move a body, or which changes or tends to change its motion. The force which acts on a body may not produce motion, because counteracted by one or more other forces: in such a case, the body is said to be in a state of *equilibrium*.

That part of Mechanics which relates to the action of forces producing *equilibrium* or *rest* in bodies, is called *Statics;* that which relates to the action of forces producing *motion*, is called

Dynamics. But in the present treatise, the statical and dynamical propositions relating to any subject will be given in connection with each other.

There are three forms of matter differing widely from each other, *solid*, *liquid*, and *gaseous;* hence, for convenience, the science of Mechanics is subdivided into three branches:

1st. The mechanics of solids, also called *Mechanics;*
2d. The mechanics of liquids, called *Hydrostatics;*
3d. The mechanics of gases, called *Pneumatics*.

2. The two *essential* properties of matter, both of which are inseparable from it, and even from our conceptions of it, are *extension* and *impenetrability*. Extension, in the three dimensions of length, breadth, and thickness, belongs to matter under all circumstances; and impenetrability, or the property of excluding all other matter from the space which it occupies, appertains alike to the largest body and the smallest particle. In Natural Philosophy, we mean by *particles* the smallest parts into which a body may be supposed to be divided by mechanical means, without any reference to the different elements of which such particles may be composed. The smallest parts into which matter can be divided by any means whatever, are called *atoms*, or *molecules*. Inquiries respecting these belong to Chemistry.

The quantity of matter which a body contains is called its *mass;* the space it occupies, its *volume;* its relative quantity of matter under a given volume, its *density*. All bodies have empty spaces denominated *pores*. In solids, we may often see the pores with the naked eye, and almost always by the microscope; in fluids, their existence can be proved by experiment. The ratio of the space occupied by the pores of a body to that occupied by the atoms, is not known; but there are reasons for believing that even in the densest bodies, the space actually occupied by matter is small compared with the empty spaces.* Hence it is inferred that the particles of matter touch each other only in a few points.†

3. *Attraction* is the tendency which one portion of matter has to another. So far as can be known, it is a *universal* property, existing between all bodies, and all the atoms which compose them. It receives different names, according to the circumstances of its action. The attraction which binds together the atoms of different kinds, is called *affinity;* that which unites particles, whether simple or compound, so as to form a body, is called *cohesion;* and the tendency of masses of matter to each other, is called *gravity*. In Mechanics, we are concerned with gravity, principally as it exists between the earth and the bodies on its surface.

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* See Newton's Optics, Lib. II, iii, Pr. 8.

† Playfair.

Inertia is another *universal* property of matter, and signifies its tendency to remain in its present condition of motion or rest. It is a negative term, expressing the observed facts, that matter can not move itself, nor stop itself, nor change either its velocity, or the direction of its motion.

In *Geometry*, we conceive figures to possess extension only, without solidity; and they may have *one* dimension alone, or *two*, or *three:* but in *Mechanics*, we have to deal not with space itself, but with that which occupies space; and every portion of matter has three dimensions, called its length, breadth, and thickness.

CHAPTER I.

MOTION AND FORCE.

4. *Classification of motions.*—Motion is change of place, and is distinguished into *uniform*, in which equal spaces are passed over in equal times; *accelerated*, in which the space is greater in each successive unit of time; and *retarded*, in which the spaces passed over become continually less. Motion is said to be *uniformly accelerated*, if the increments of space in equal times are equal; and *uniformly retarded*, if the decrements are equal.

Velocity is the space described in the *unit* of time. For many purposes of Mechanics, one second is the unit of time, and one foot the unit of space; hence, velocity is the number of feet described in one second.

5. *Uniform motion.*—When motion is uniform, the number of feet described in one second, multiplied by the number of seconds, obviously gives the whole space; that is, if $S =$ space, $T =$ time, and $V =$ velocity, $S = TV$; $\therefore\ T = \frac{S}{V}$, and $V = \frac{S}{T}$.

If this space is compared with another $= s$, described in the time $= t$, with the velocity $= v$, then

$S : s :: TV : tv$; or briefly, in the form of a variation, $S \propto TV$. In like manner, $T \propto \frac{S}{V}$, and $V \propto \frac{S}{T}$.

If the spaces in two uniform motions are equal, $S = s$; $\therefore$ $TV = tv$; $\therefore\ T : t :: v :: V$. That is, in order that equal spaces may be described, the velocities must vary inversely as the times.

In the next chapter, examples of accelerated and retarded motion will be discussed.

6. *Questions on uniform motion.*—

1. A ball was rolled on the ice with a velocity of 30 feet per second, and moved uniformly 45 seconds; what *space* did it describe? *Ans.* 1350 feet.

2. A steamboat moved steadily across a lake 53 miles wide, at the rate of 16 miles per hour; what *time* was occupied in crossing? *Ans.* $3\frac{5}{16}$ hours.

3. On the supposition that the earth describes an orbit of 600 millions of miles in $365\frac{1}{4}$ days, with what *velocity* does it move per second? *Ans.* 19 miles, nearly.

4. Three planets describe orbits which are to each other as 15, 19, and 12, in times which are as 7, 3, and 5; what are their *comparative velocities?* *Ans.* 225, 665, and 252.

7. *Momentum.*—The momentum of a body signifies its quantity of motion, and is reckoned according to the number of particles moving, and the velocity with which they move. The momentum, therefore, varies as the quantity of matter multiplied by the velocity.

Let the momentum of a body $= M$, its quantity of matter $= Q$, and its velocity $= V$; then $M = QV$; $Q = \frac{M}{V}$, $V = \frac{M}{Q}$. In order to compare the momentum of two bodies, $M : m :: QV : qv$; that is, $M \propto QV$; hence $Q \propto \frac{M}{V}$; and $V \propto \frac{M}{Q}$.

If the momentum of one body equals that of another, then, since $M = m$, $QV = qv$, and $Q : q :: v : V$. Hence, the velocities of two bodies must vary inversely as their quantities, in order that their momenta may be equal.

Since there are two elements entering into the momentum of a body,—namely, its *quantity of matter*, usually expressed in pounds, and its *velocity*, expressed in feet per second,—therefore momentum can not be measured either in pounds or in feet, being in its nature unlike either. The compound word *foot-pound* is employed for the unit of momentum whenever the unit of quantity of matter is a pound, and the unit of velocity is a foot per second.

8. *Questions on momentum.*—

1. A ship weighing 336,000 lbs. is dashed against the rocks in a storm, with a velocity of 16 miles per hour; with what momentum did she strike? *Ans.* 7,884,800 foot-pounds.

2. A ball weighing 1 oz. is fired into a log weighing 53 lbs.,

suspended so as to move freely, and imparts a velocity of 2 ft. per second. Assuming that the log and ball have a momentum equal to the previous momentum of the ball alone, required the velocity of the ball. *Ans.* 1,698 ft. per sec.

3. If a comet moving at the rate of 1,000,000 miles per hour, were to meet the earth moving 19 miles per second; what ratio will the mass of the comet bear to that of the earth, supposing that they mutually destroy each other's motions?

Ans. 1 : 14.6; or the comet must have nearly $\frac{1}{15}$ as much matter as the earth.

4. Two railway cars have their quantities of matter as 7 to 3, and their momenta as 8 to 5; what are their respective velocities? *Ans.* As 24 to 35, or nearly 5 to 7.

5. The momentum of a cannon-ball was 434 foot-pounds; what must be the velocity of a half-ounce bullet, in order to have the same momentum? *Ans.* 13,888 feet.

9. *Classification of forces.*—The forces employed in Mechanics are mostly reducible to three classes, *gravity*, *elasticity*, and *muscular strength.* Thus, not only the action of heavy weights, but the power of water, and even that of wind, are due to gravity; and the energy of steam and of springs to the force of elasticity.

If a force has no sensible continuance, it is called an *impulsive* force; but if its action is continued, it is an *accelerating* force. Accelerating forces are subdivided into *constant* and *variable:* a force is called *constant*, if its intensity is always the same; *variable*, if its intensity changes.

10. In view of the property of inertia, it is easy to associate these several forces with the kinds of motion which they respectively produce.

1. An *impulsive* force causes *uniform* motion;
2. A *continued* (i. e. *accelerating*) force, *accelerated* motion;
3. A *constant* force, *uniformly accelerated* motion;
4. A *variable* force, *unequally accelerated* motion.

If the force *opposes* the previous motion of the body, the connection is as follows:

5. An *impulsive* force causes *uniform* motion or *rest;*
6. An *accelerating* force, *retarded* motion;
7. A *constant* force, *uniformly retarded* motion;
8. A *variable* force, *unequally retarded* motion.

Nos. 1 and 5 are obvious, because, the impulse being given, the body is left to itself, and can not change the state of motion or rest impressed on it.

In 2, 3, and 4 we must consider that the force at each instant adds a new *increment* to the uniform motion which the body would have had, if the force had ceased; and if the force is constant, those increments are equal; if variable, they are unequal.

In 6, 7, and 8 the same statements may be made in regard to *decrements*. It is also clear, that in these three last cases, if the force continues to act indefinitely, the body will come to a state of momentary rest, and then be accelerated in the direction of the force.

11. *Measure of force.*—The intensity of an *impulsive* force is measured by the *momentum* which it will produce or destroy; or $M \propto F$. If the weight of the body is given, then, since its momentum varies as its velocity, the *velocity*, which a force produces, becomes a measure of the force; or $V \propto F$.

But, in the case of a *constant* force, the momentum produced depends not only on the intensity of the force, but also on the *time* during which it is applied. The same force, acting during *two* or *five* seconds, causes twice or five times the momentum as during *one* second. Hence, $M \propto FT$; and $F \propto \frac{M}{T}$. If the weight is constant, then $V \propto FT$; and $F \propto \frac{V}{T}$.

Example.—A given train was set in motion by an engine of 50 horse-power, operating during 2 minutes; and at another time by an engine of 32 horse-power, operating during 2m. 35s. Compare the velocities produced. *Ans.* As 75 : 62.

12. *The three laws of motion.*—All the phenomena of motion in Mechanics and Astronomy are found to be in accordance with three first principles, which Newton announced in his Principia, and which are to be regarded as the *axioms* of mechanical science. They may be named and defined as follows:

1. The law of *inertia.*—A body at rest tends to remain at rest; and a body in motion tends to move forever, in a straight line, and uniformly.

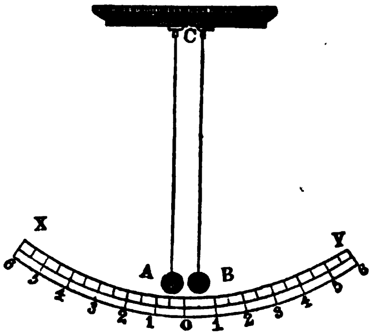
2. The law of the *coexistence of motions.*—If several motions are communicated to a body, it will ultimately be in the same position, whether those motions are simultaneous or successive.

3. The law of *action and reaction.*—If any kind of action takes place between two bodies, it produces equal momenta in opposite directions.

The truth of these laws can not be established, except approximately, by direct experiments, because gravity, friction, and the resistance of air, interfere more or less with every possible experiment. They are to be learned rather by a careful study of the phenomena of motion in general. We see an approximation to the *first* law, in rolling a ball on a horizontal surface;

first, on the earth, then, on a floor, and again, on smooth ice, the motion approaching toward uniformity as obstructions are diminished, and gravity producing no direct effect, because acting at right angles to the line of motion. The discussion of the *second* law is reserved for Chapter III. The *third* law is illustrated by a variety of cases in collision, attraction, and repulsion. In Fig. 1, if A in motion strikes B at rest, B is found to acquire a certain momentum, and A, to *lose* (that is, *acquire* in an opposite direction) an equal amount; and this is equally true, when B is in motion. In the accidental collision of railroad trains, it is immaterial as to the effects which they will respectively suffer, whether both are moving, or one is at rest, provided that, in the latter case, the moving train has a momentum equal to the sum of the momenta of the two trains in the former case. When a magnet attracts a piece of iron, each moves toward the other with an equal momentum. A spring between two bodies, A and B, drives A from B with as much momentum as B from A; and the sudden expansion of burning gunpowder, which propels the balls, when a broadside is fired, causes an equal amount of motion of the ship in the opposite direction.

Fig. 1.



13. *Force of gravity.*—Every mass of matter near the earth, when free to move, pursues a straight line toward its center. The force by which this motion is produced is called *gravity;* either the gravity of the body, or the gravity of the earth: for the attraction is mutual and equal, in accordance with the third law of motion. It is easy to understand why a small mass should attract a large one, as much as the large one attracts the small one. Let A consist of *one* atom of matter, and B, at any distance from it, consist of *ten* atoms. If it is admitted that A attracts *one* atom of B as much as *that one* atom attracts A, then the above conclusion follows. For A attracts *each* of the ten atoms of B as much as *each* of the same ten attracts A; so that A exerts ten units of attraction on B, while B exerts ten units of attraction on A. The same reasoning obviously applies to the earth and the smallest body on its surface.

14. *Relation of gravity and quantity of matter.*—At the same distance from the center of the earth, *gravity varies as the quantity of matter.* This is because it operates equally on every atom of a body; hence, the greater the number of atoms in a body, the greater, in the same ratio, is the attraction exerted upon it. That gravity does operate equally on every atom, is proved from the observed fact, that in a vacuum it gives the same velocity, in the same time, to every mass, however great or small, and of whatever species of matter. For an accelerating force is measured by the *momentum* which it produces in a given time; and that momentum, if the velocity is the same, varies as the quantity of matter: therefore the force also varies as the quantity of matter to which it imparts the given velocity.

If a body is not free to move, its tendency toward the earth causes *pressure;* and the measure of this pressure is called the *weight* of the body. Weight is usually employed as a measure of the quantity of matter in bodies. The foregoing relations are embodied in the following expressions: $G \propto Q$; and $W \propto Q$.

15. *Relation of gravity and distance.*—At different distances from the earth, *gravity varies inversely as the square of the distance from the center.* The demonstration of this proposition is reserved for astronomy, where it is shown by the movements of the bodies in the solar system that this law applies to them all.

The moon is 60 times as far from the earth's center as the distance from that center to the surface: therefore the attraction of the earth upon the particles of the moon is 3600 times less than upon particles near the earth. At the height of 4000 miles from the earth's surface, gravity is four times less than at the surface. But the heights at which experiments are commonly made upon the weights of bodies bear so small a ratio to the radius of the earth, that this variation is commonly imperceptible. At the height of *half a mile*, the diminution does not amount to more than about $\frac{1}{4000}$th part of the weight at the surface. For, let r = the radius of the earth = 4000 miles, nearly; and let x be the height of the body, W its weight at the earth's surface, and W' its weight at the height x. Then,

$$W : W' :: (r+x)^2 : r^2 :: r^2+2rx+x^2 : r^2.$$

$$W : W-W' :: r^2+2rx+x^2 : 2rx+x^2 \therefore W-W' = \frac{W\,(2rx+x^2)}{r^2+2rx+x^2} \quad \text{(A)}$$

But when x is a small fraction of r, x^2 may be neglected, and the formula becomes $W-W' = \dfrac{W \times 2x}{r+2x}$ . . . . . . (B)

Let x be *half a mile;* then $\dfrac{W \times 1}{4000+1} = \frac{1}{4001}$th part of the whole weight; or, a body would weigh so much less at the height of half a mile than at the surface of the earth. But if the height were 100 miles above the earth, then $\frac{100}{4000}=\frac{1}{40}$; and the square of this $=\frac{1}{1600}$ of the radius of the earth, a quantity too large to be neglected; and the difference of weights at the surface and at the height of 100 miles, will be found by formula (A).

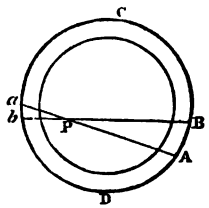
What loss of weight would a body sustain, by being elevated 500 miles above the earth?

Ans. $\frac{17}{81}$. Were x^2 neglected, then the loss by formula (B) would be $\frac{1}{5}$, which differs from $\frac{17}{81}$ by only $\frac{4}{405}$.

The relation of gravity to distance is expressed by the formula, $G \propto \frac{1}{D^2}$; and since $G \propto Q$ (Art. 14), $\therefore G \propto \frac{Q}{D^2}$.

16. *Gravity within a hollow sphere.*—A particle situated *within a spherical shell* of uniform density, is equally attracted in all directions, and *remains at rest.* This is true, because, in every direction from the body, the quantity of matter varies at the same rate as the square of the distance, so that attraction increases for one reason, as much as it diminishes for the other; which is proved as follows:

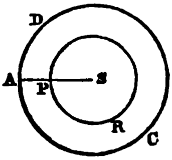
Fig. 2.



Let the particle P (Fig. 2) be at any point within the spherical shell ABCD. Let two opposite cones of revolution, of very small angle, have their vertices at P, and suppose the figure to be a section through the center of the sphere and the axis of the cones. Then AB and *ab* will be the major axes of the indefinitely small ellipses, which form the bases of the cones, and which may be considered as plane figures. Since by geometry $AP \times Pa = BP \times Pb$, $\therefore AP : PB :: Pb : Pa$; and the angles between them being equal, the triangles are similar; hence the angles B and *a* are equal. Therefore, the bases of the cones are similar ellipses, being sections of similar cones, and equally inclined to the sides. By similar triangles, $AP : Pb :: AB : ab$; and $AP^2 : Pb^2 :: AB^2 : ab^2$. But the bases of the cones are as $AB^2 : ab^2$, and therefore as $AP^2 : Pb^2$. Let Q and *q* represent the quantity of matter in the thin laminæ which form the bases, and D and *d*, the distances AP and P*b*; and we have $Q : q :: AP^2 : Pb^2$, or $Q \propto D^2$. Now (Art. 15), $G \propto \frac{Q}{D^2}$; $\therefore G \propto \frac{D^2}{D^2} \propto 1$; that is, G is constant; and the particle is equally attracted by all the opposite parts of the same thin lamina. This reasoning applies to every lamina of the shell, and therefore to the sum of them all.

17. *Gravity within a solid sphere.*—Within a *solid sphere* of uniform density, gravity varies directly *as the distance from the center.*

Fig. 3.



Let a particle P (Fig. 3) be within the solid sphere ADC; and call its distance from the center D. Now, by the preceding article the shell exterior to it, ADR, exerts no influence upon it, and it is attracted only by the sphere PRS. Let Q represent the quantity of this sphere; then gravity varies as $\frac{Q}{D^2}$. But $Q \propto D^3$; $\therefore$ $G \propto \frac{D^3}{D^2} \propto D$. Hence, in the earth (if uniformly dense), a body, at the depth of 1000 miles, weighs *three-fourths* as much as at the surface, and at 2000 miles it weighs half as much, while at the center it weighs nothing.

Comparing this proposition with Art. 15, we learn that just at the surface of the earth a body weighs more that at any other place without or within. Within, gravity diminishes as the distance diminishes; without, it diminishes as the square of the distance increases. The same remark is applicable, of course, to any other planet.

At the surface of spheres, of the same density, gravity varies as the radius of the sphere.

Let R be the radius of a sphere, and Q its quantity; then, since $G \propto \frac{Q}{D^2}$, in this case, it varies as $\frac{R^3}{R^2} \propto R$.

Required the ratio of gravity on the planets Jupiter and Uranus, whose densities are equal, and their diameters respectively 89,000 and 35,000 miles. *Ans.* As $2\frac{1}{2}$: 1, nearly.

Suppose a particle lying on the surface of a ball two feet in diameter, and of the same density as the earth; how much less is it attracted by the ball than by the earth?

Ans. More than 20 millions of times less.

18. *Questions for practice.*—

1. A bird of passage was observed to fly with a uniform velocity of 19 feet per second; over what space would she pass in 24 hours? *Ans.* 310.9 miles.

2. A lame man set out to travel round the world. He could walk but two miles an hour for seven hours out of the twenty-four. Provided he could go forward, without impediment, on the circumference of a great circle of the globe (25,000 miles), what time would he require to complete the journey?

Ans. 4 years and $324\frac{5}{7}$ days.

3. A wind blows uniformly from the equator to the pole (say 6000 miles), in 12 days; what is its velocity per hour?

Ans. $20\frac{5}{6}$ miles.

4. How much weight would a rock that weighs ten tons (22,400 lbs.) at the level of the sea, lose if elevated to the top of a mountain five miles high? *Ans.* 55.8952 lbs.[*]

5. If the earth were a hollow sphere, and if, through a hole bored through the center, a man were let down by a rope, would the force required to support him be increased or diminished as he descended through the solid crust, and where would it become equal to nothing?

6. How much would a 44-pound shot weigh at the center of the earth; how much at a point half way from the center to the surface; and how much 100 miles below the surface?

7. If a hole were bored through the center of the earth, and a stone were dropped into it, in what manner would the stone move in its way to the center and after it reached the center?

8. Suppose the battering-ram of Vespasian weighed 5,760 lbs., and was found sufficient, when propelled with a certain velocity, to demolish the walls of Jerusalem; and suppose that a 32-pound cannon-ball, fired with the velocity of 2,000 feet per second, is found capable of doing the same execution; what was the velocity of the battering-ram? *Ans.* 11.11 per sec.

9. Suppose light to have weight, and one grain of it moving at the rate of 192,000 miles per second, to impinge directly against a mass of ice moving at the rate of 1.45 feet per second; what weight of ice would the light stop?

Ans. 99877.832 lbs.,[†] or nearly 44½ tons.

10. If a ball of the same density with the earth, $\frac{1}{10}$th of a mile in diameter, were placed at the distance of $\frac{1}{10}$th of a mile above the earth; what space would the earth move through to meet it, the diameter of the earth being taken at 8,000 miles?

$\frac{1}{10}$: 8000 :: – $\frac{1}{10}$ – 4 *Ans.* $\frac{1}{80000000000}$ inch, nearly.

11. The quantity of matter in the sun being 354,000 times that of the earth, and their distances from each other being 96,000,000 miles; if the two bodies were abandoned to their mutual attraction, where would they meet?

Ans. 271.186 miles from the center of the sun; or the sun would advance so far to meet the earth.[‡]

12. Two men are pulling a boat ashore by a rope, one at each end, A being in the boat and B on the shore; how will the time of bringing the boat ashore compare with the time in which A would pull it ashore alone, were the other end of the rope fixed to an immovable post?[§]

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[*] The weight would be ascertained, in this case, by the effect on a spring, and not by scales, since a counterpoise would sustain a loss of weight in the same degree with the body in question.

[†] 1 lb. av. = 7,000 grains.

[‡] In this problem it is supposed that all the matter of each body is collected in its center, and that their own semi-diameters present no obstacle to the approach of their centers.

[§] This problem is intended merely to illustrate the doctrine of action and reaction. No numerical answer is required.

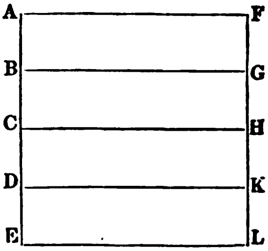
13. Suppose the rope to pass from A in one boat, to B in another equal boat; how fast will B's boat move? will A's boat have the same velocity as when B was on the shore?

CHAPTER II.

ACCELERATED MOTION.—FALLING BODIES.

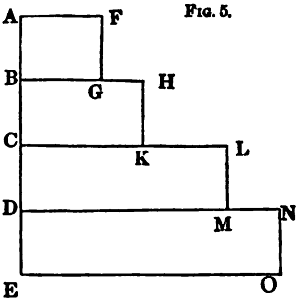
19. *Uniform and accelerated motion represented geometrically.*—Since, when a body moves with a uniform velocity for a given time, the space described is in proportion to that time and velocity conjointly (Art. 5), therefore if one side of a rectangle represents the *time* of a body's motion, and the other the uniform *velocity* with which it moves, the rectangle itself (whose area is equal to the product of the two sides) will represent the *space* described in that time. Thus, let the line AE be divided into any number of equal parts in the points B, C, D, &c., and from those points draw the equal straight lines AF, BG, CH, &c., at right angles to AE, and complete the rectangle AFLE; then if AB, BC, CD, &c., represent equal successive portions of time, and AF, BG, CH, &c., represent the uniform velocity with which a body moves, then will the rectangles AG, BH, CK, &c., represent the spaces described in those equal portions of time, and the rectangle AFLE the whole space described in the time represented by AE.*

FIG. 4.



20. Suppose now that a body moves uniformly as before, during the equal successive portions of time represented by AB, BC, CD, &c., but at the end of each portion of time receives an *increase* of velocity; for instance, during the time AB let it move with a velocity represented by AF, during the time BC with a velocity represented by BH, &c.; complete the rectangles AG, BK, CM, and DO, then the space described

FIG. 5.



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* The space described by a moving body is represented by a rectangle, not because of any resemblance, but because a rectangle has the same relation to the sides, as the space has to the time in which, and the velocity with which, it is described.

in the time AB will be represented by the rectangle AG, in the time BC by the rectangle BK, &c., and the whole space described in the time AE by the irregular figure AFOE.

21. Let us next suppose that a body receives *equal* increments of velocity at the end of each successive portion of time, so that during the second interval of time it moves with twice the velocity, during the third interval with three times the velocity, &c., then will $CK = 2BG$, $DM = 3BG$, &c. But $AC = 2AB$, $AD = 3AB$, &c.

Therefore, $AB : BG :: AC : CK :: AD : DM$, &c. Hence, the figures ABG, ACK, ADM, &c., are similar triangles; and if AG, GK, KM, &c., be joined, AO will be a straight line, and the figure AFOE, which represents the space described in the time AE, will differ from the triangle AOE only by the sum of the triangles AFG, GHK, KLM, MNO, which are all equal to each other.

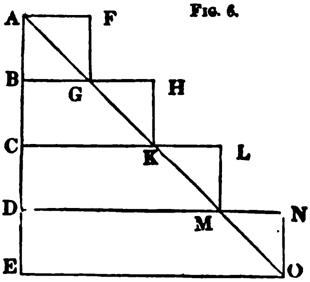


FIG. 6.

22. Now let the intervals of time and the corresponding increments of velocity be only half what they were in the former instance. Bisect AB, BC, CD, &c. (Fig. 7), in *b*, *c*, *d*, &c., and complete the rectangles as before, then the figure which represents the space described in the time AE will differ from the triangle AOE by the sum of the small triangles A*fg*, *g*FG, G*hk*, &c., which is only *half* the sum of the triangles AFG, GHK, KLM, &c., in the preceding figure. By continually halving these intervals of time and the corresponding increments of velocity, the figure AFOE will approach to the form of the triangle AOE; and when they are diminished *ad infinitum*, A*f* (which represents the velocity with which the body begins to move) will be equal to 0; as will also the sum of the triangles A*fg*, *g*FG, &c.; the space therefore described by a body beginning to move from rest by the continued action of a force which generates equal increments of velocity in equal times, will be accurately represented by the right-angled triangle AOE, one of whose sides AE represents the time of the body's motion, and the other, OE, the last acquired velocity.

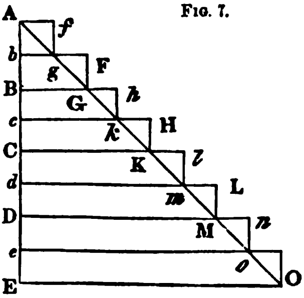


FIG. 7.

Thus we have a rectangle for the geometrical representation of *uniform* motion, and a right-angled triangle, of *uniformly accelerated* motion. An example of the latter is the unobstructed fall of a body by gravity through small distances. For gravity acts *incessantly*, and within the range of our observations, it acts with *equal intensity*. The properties of the triangle enable us to deduce very readily the laws of the fall of a body.

23. *Laws of the fall of bodies.*—When bodies fall from rest by the force of gravity, and unobstructed by the air, the following relations exist between the space, time, and velocity.

1. *The spaces vary as the squares of the times.*
2. *The spaces vary as the squares of the acquired velocities.*
3. *The times vary as the acquired velocities.*

For let S be the space described, V the velocity acquired by a body falling from rest for the time T, s the space described, v the velocity acquired at any other period t, of its fall; then, from what has already been demonstrated, if the ratio of $T : t$ be represented by the lines AB, Ab (Fig. 8), and the ratio of $V : v$ by the lines BC, bc drawn at right angles to them, the ratio of $S : s$ will be represented by the triangles, ABC, Abc. Now,

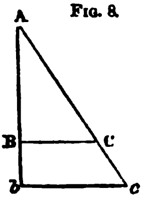


FIG. 8.

$$ABC : Abc :: AB^2 : Ab^2;\text{ or, as } BC^2 : bc^2;$$
$$\text{hence, } S : s :: T^2 : t^2,\text{ or as } V^2 : v^2.$$

As equal increments of velocity are generated in equal times, it is farther evident that the velocity acquired varies as the time; the same conclusion may also be deduced from the similar triangles ABC, Abc; for $BC : bc :: AB : Ab$, i. e. $V : v :: T : t$.

Since the spaces described are as the squares of the times; if a body falls from rest for times which are represented by the numbers 1, 2, 3, 4, 5, &c., the spaces described in those times will be as the square numbers, 1, 4, 9, 16, 25, &c.; and the spaces described in equal successive portions of time will be as the odd numbers 1, 3, 5, 7, 9, &c., as exhibited in the following table:

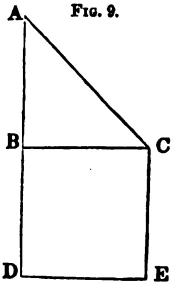
Times.	Spaces described.	Spaces described in equal successive portions of time.
1	1	In 1st portion of time............ 1
2	4	2d " " ... 4— 1=3
3	9	3d " " ... 9— 4=5
4	16	4th " " ... 16— 9=7
5	25	5th " " ... 25—16=9
&c.	&c.	&c................. &c.=&c.

24. *Uniformly retarded motion.*—If a body be projected perpendicularly upward, with the velocity which it has acquired in falling from any height, it will rise to the point from which it fell, before it begins to descend again, and the motion will be uniformly retarded. As, in the descent of a body, the force of gravity generates equal increments in equal times, so, in its ascent, equal portions of velocity will be destroyed in equal times. The spaces described in equal successive parts of time, by a body thus ascending, reckoning from the beginning of its motion, will be the same as those stated in the foregoing table, but in an inverted order: thus, if the time be divided into four equal parts, then the spaces described in the descent of the body during these equal times are as the numbers 1, 3, 5, 7, but in its ascent they will be as, 7, 5, 3, 1; that is, the space described in the first portion of time, in its ascent, will be the same as that described in the last portion of time, in its descent, and so on till the body arrives at its highest point. Equal decrements in equal times are the characteristic of uniformly retarded motion.

25. *Acquired velocity.*—If a body moves uniformly with the acquired velocity, it will pass over twice as great a space, in the same time, as it falls through to acquire it.

Let the triangle ABC (Fig. 9) represent the space described by gravity in the time AB, and BC the last acquired velocity; produce AB to D, making BD equal to AB, and complete the rectangle BCDE; then, if a body moves for the time BD with the uniform velocity represented by BC, the space described in that time will be represented by the rectangle BCDE; but the triangle ABC is half BCDE; hence the space described with the velocity BC continued uniformly is twice that which would be described in the same time AB, falling from rest.

FIG. 9.



Since the space described by a body falling from rest, is half that which it would describe in the same time with its greatest velocity continued uniformly, and since a body projected perpendicularly upward rises to the same height as that from which it must fall to acquire the velocity of projection, the whole space described by a body projected perpendicularly upward, is *half* that which it would describe in the same time with its first velocity continued uniformly.

26. *Projection downward.*—The space described in any time by a body *projected downward* with a given velocity, is equal to the space which would be described with that velocity continued uniformly for that time, together with the space through which a body would fall from rest by the action of gravity in the same time.

Let AD (Fig. 10) represent the given velocity of projection, and AB the given time, and complete the rectangle ABED;

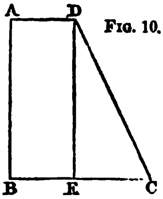


FIG. 10.

produce BE to C, and let EC represent the velocity generated by gravity in the time AB or DE, and join DC. Then, according to what has been said in Art. 19, a body moving under the influence of projection alone, with a uniform velocity represented by AD, would describe the rectangle ABED in the time AB; and by Art. 22, a body falling from a state of rest during the same time, so as to acquire the velocity represented by EC, would describe the triangle DEC; hence the figure ABCD truly represents the joint effects of both forces, or the whole space described.

27. *Projection upward.*—The space described by a body ascending for a given time, is equal to the difference between the space which would be described by the body moving uniformly for that time with the velocity of projection, and the space through which a body would fall from rest by the action of gravity in the same time.

Let BC (Fig. 11) represent the given velocity of projection, and AB the time in which it must fall from rest to acquire that velocity; draw BC at right angles to AB, and join AC, then the triangle ABC will represent the space through which the body must ascend to lose all its velocity. (Art. 24.) In AB take any point *b*, and complete the rectangle BCD*b*; then will *bc* represent the velocity of the body at the end of the time B*b* of its ascent, and *c*D will represent the velocity destroyed by gravity in the same time. But the velocity *destroyed* by gravity in any time is equal to the velocity *generated* by gravity in the same time (Art. 24); hence the triangle CD*c* will represent the space through which a body would fall from rest in the time CD or B*b*. Now the figure BC*cb* represents the space through which the body would ascend in the time B*b*, and the rectangle BCD*b* represents the space through which a body would move in the time B*b* with the velocity BC continued uniformly; but the figure BC*cb* is equal to the *difference* between the rectangle BCD*b* and the triangle CD*c*.

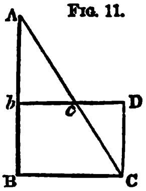


FIG. 11.

28. *Formulæ for the fall of bodies.*—The foregoing investigations show the *ratios* between the velocities, times, and spaces of falling bodies; but in estimating the actual motion of bodies descending or ascending by the force of gravity, it is necessary to have recourse to some fixed standard of measurement of space and velocity. Now it has been ascertained, by the most accurate experiments, that a body falling freely* from rest describes a space equal to $16\frac{1}{12}$ feet in the first second of its fall; and (Art. 25) a body so falling would acquire a velocity which, if continued uniformly, would carry it over $32\frac{1}{6}$ feet (that is, *twice the space*) in the same time. If, therefore, $g = 32\frac{1}{6}$ feet, g will express the *velocity* acquired in *one* second, and may be used as the measure of gravity; and $\frac{1}{2}g = 16\frac{1}{12}$ feet, the *space* fallen through in the first second. Let S be the space described in any other time T, and V the velocity acquired. Then, according to the laws (Art. 23), we have

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* All bodies descending or ascending near the surface of the earth, meet with

$$(1.)\quad S : \tfrac{1}{2}g :: T^2 : 1^2;\ \therefore \quad \dots \quad S = \tfrac{1}{2}gT^2.$$

$$(2.)\qquad \text{and} \quad \dots \quad T = \sqrt{\frac{2S}{g}}.$$

$$(3.)\quad S : \tfrac{1}{2}g :: V^2 : g^2;\ \therefore \quad \dots \quad S = \frac{V^2}{2g}.$$

$$(4.)\qquad \text{and} \quad \dots \quad V = \sqrt{2gS}.$$

$$(5.)\quad 1 : T :: g : V;\ \therefore \quad \dots \quad T = \frac{V}{g}.$$

$$(6.)\qquad \text{and} \quad \dots \quad V = gT.$$

29. *Applications of the above formulæ.*—

1. A body has been falling for 6 seconds; what *space* has it fallen through in that time, and what is the *velocity* which it has acquired?

$$S = \tfrac{1}{2}gT^2 = 16\tfrac{1}{12} \times 36 = 579 \text{ feet.}$$
$$V = gT = 32\tfrac{1}{6} \times 6 = 193 \text{ feet per second.}$$

2. *How far* must a body fall to acquire a velocity of 50 feet in a second, and *how long* will it be in falling?

$$S = 38.86 \text{ feet.} \qquad T = 1.55 \text{ seconds.}$$

3. A body fell from the top of a tower which was 150 feet high; *how long* was it in falling, and what velocity had it acquired when it got to the bottom?

$$T = 3.054 \text{ seconds.} \quad V = 98.237 \text{ feet per second.}$$

4. A body was projected perpendicularly upward with a velocity of 100 feet in a second; *how far* would it ascend before it began to return?

By Art. 24, the height to which the body would ascend is

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more or less resistance from the air; so that, strictly speaking, a body can never be said to descend *freely* but in the exhausted receiver of an air-pump. It is in a vacuum that a body describes $16\frac{1}{12}$ feet in a second; the conclusions, therefore, deduced in this section, will approximate to the truth only in those cases where the resistance of the air bears little or no proportion to the weight of the body.

equal to that through which a body must fall from rest to acquire the velocity of projection; here, therefore,

$$V = 100, \text{ and } S = \frac{V^2}{2g} = \frac{10000}{2 \times 32\frac{1}{6}} = \frac{30000}{193} = 155.44 \text{ feet.}$$

5. A body was observed to fall for 3 seconds, and afterward to move uniformly for 2 seconds along the horizon with the velocity which it had acquired by its fall; what was the *whole space* described in its perpendicular and horizontal motion?

The space described in its fall $= \frac{1}{2}gT^2 = 16\frac{1}{12} \times 9 = 144\frac{3}{4}$ feet. The velocity acquired $= gT = 32\frac{1}{6} \times 3 = 96\frac{1}{2}$ feet per second; and as it moved along the horizon for 2 seconds with this velocity, it must in that time have described 193 feet; hence the whole space described from the beginning of its fall to the end of its horizontal motion is $144\frac{3}{4} + 193$, or $337\frac{3}{4}$ feet.

6. A cannon-ball fired perpendicularly upward, was gone 10 seconds, when it returned to the same place; how *high* did it rise, and what was the *velocity* of projection?

Ans. Height $402\frac{1}{12}$ feet. Velocity of projection $160\frac{5}{6}$ feet per second.

30. Since the spaces described in equal successive parts of time (by Art. 23), are as the odd numbers 1, 3, 5, 7, 9, &c., and since the space described by a body falling from rest is in the first second $\frac{1}{2}g$ feet, the space described in successive seconds will be $\frac{1}{2}g$, $\frac{3}{2}g$, $\frac{5}{2}g$, &c., feet.

1. A body has been falling for 5 seconds; compare the spaces described in the *third* and *fifth* seconds of its fall.

Ans. The space described in the *third* second $= 80\frac{5}{12}$ feet; the space described in the *fifth* second $= 144\frac{3}{4}$ feet.

2. A body has fallen through 579 feet; what was the space described by it in the *last* second?

Ans. It will be found that the body has been falling six seconds; therefore, the space described in the last second is $176\frac{11}{12}$ feet.

31. The method adopted in the last example for finding the space described by a body in the *last* second of its fall, is only applicable when the time consists of an *integral* number of seconds; but it is not difficult to investigate a *general* expression for the space described in the last n seconds, whatever be the value of T. For the space described in T seconds $= \frac{1}{2}gT^2$; and the space described in $(T - n)$ seconds $= \frac{1}{2}g\,(T - n)^2$. Subtract $(T - n)^2$ from T^2, and we have $2nT - n^2$; therefore, the space described in the last n seconds $= \frac{1}{2}g\,(2nT - n^2)$. If $n = 1$, then the space described in the *last* second $= \frac{1}{2}g\,(2T - 1)$; which expression will lead to the same results as the method practiced in Example 1, and is likewise applicable in cases where the time does not consist of any whole number of seconds. For example, let the time of falling be $6\frac{1}{2}$ seconds; then the space described in the last second, namely, from $5\frac{1}{2}$ to $6\frac{1}{2}$ seconds, will be $16\frac{1}{12} \times (13 - 1) = 193$ feet.

If it were required to find the space described in the second *immediately previous* to the last n seconds, we have,

Space described in the last n seconds $= \frac{1}{2}g\,(2\mathrm{T}n - n^2)$. . (A)

Ditto in the last $(n+1)$ seconds $= \frac{1}{2}g\,\{2\mathrm{T}(n+1) - (n+1)^2\}$. (B)

Subtract (A) from (B), then the space described in the second immediately previous to the last n seconds $= \frac{1}{2}g\,(2\mathrm{T} - 2n - 1)$.

1. What was the space described in the *last* 2 *seconds* by a body which had fallen from the top of a tower 300 feet high?

Ans. The whole time is found to be 4.32 seconds; therefore, the space fallen through in the last 2 seconds is 213.58 feet.

2. A body has been falling for $9\frac{1}{2}$ seconds: what was the space described in the *last* second but 4 of its fall?

Ans. $160\frac{5}{6}$ feet.

32. To find the space described in a given time by a body projected upward or downward with a given velocity. Let V be the given velocity with which a body is projected *downward*, and T the time of its motion; then the space described in the time T with the uniform velocity V will be equal to TV, and the space through which a body would fall by gravity in the same time is $\frac{1}{2}g\mathrm{T}^2$; hence, from what was shown in Art. 26, the space described in the time T by a body projected downward with the velocity V is equal to $\mathrm{TV} + \frac{1}{2}g\mathrm{T}^2$; and applying the same process of reasoning to Art. 27, the space through which a body would ascend in the time T, if projected upward with a given velocity V, will be equal to $\mathrm{TV} - \frac{1}{2}g\mathrm{T}^2$.

1. A body is projected downward with a velocity of 30 feet in a second; *how far* will it fall in 4 seconds?

Ans. $377\frac{1}{3}$ feet.

2. A body is projected upward with a velocity of 120 feet in a second; *how far* will it rise in 3 seconds?

Ans. $215\frac{1}{4}$ feet.

3. Suppose at the same instant that a body begins to fall from rest from the point D (Fig. 12), another body is projected upward from B with a velocity which would carry it to A; it is required to find the point where they would meet.

Let C be the point where the bodies would meet; and let $\mathrm{AB} = a$, $\mathrm{BD} = b$, $\mathrm{DC} = x$; then will $\mathrm{AD} = a - b$, $\mathrm{AC} = a - b + x$.

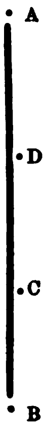

Fig. 12.

Now the time of descending through DC $= \left(\frac{2x}{g}\right)^{\frac{1}{2}}$; and the time of ascending through BC (= time down AB − time down AC) $= \left(\frac{2a}{g}\right)^{\frac{1}{2}} - \left(\frac{2(a-b+x)}{g}\right)^{\frac{1}{2}}$; but the time down DC must be equal to the time up BC; hence we have

$$\left(\frac{2x}{g}\right)^{\frac{1}{2}} = \left(\frac{2a}{g}\right)^{\frac{1}{2}} - \left(\frac{2(a-b+x)}{g}\right)^{\frac{1}{2}}, \text{or } x^{\frac{1}{2}} = a^{\frac{1}{2}} - (a-b+x)^{\frac{1}{2}};$$

$$\therefore (a-b+x)^{\frac{1}{2}} = a^{\frac{1}{2}} - x^{\frac{1}{2}}, \text{and } a-b+x = a+x-2(ax)^{\frac{1}{2}};$$

$$\therefore 2(ax)^{\frac{1}{2}} = b, \text{ or } 4ax = b^2, \text{ and } x = \frac{b^2}{4a}.$$

4. Suppose a body to have fallen from A to B (Fig. 13), when another body begins to fall from rest at D; *how far* will the latter body fall before it is overtaken by the former?

Let C be the point where one body overtakes the other, and let AB $= a$, BD $= b$, DC $= x$; then AC $= a + b + x$.

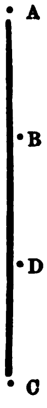

Fig. 13.

Now time down DC $= \left(\frac{2x}{g}\right)^{\frac{1}{2}}$, and time down BC = time down AC − time down AB $= \left(\frac{2(a+b+x)}{g}\right)^{\frac{1}{2}} - \left(\frac{2a}{g}\right)^{\frac{1}{2}}$; but at the moment when the lower body is overtaken, time down DC = time down BC, or

$$\left(\frac{2x}{g}\right)^{\frac{1}{2}} = \left(\frac{2(a+b+x)}{g}\right)^{\frac{1}{2}} - \left(\frac{2a}{g}\right)^{\frac{1}{2}};$$

$$\therefore x^{\frac{1}{2}} + a^{\frac{1}{2}} = (a+b+x)^{\frac{1}{2}}, \text{and } x + a + 2(ax)^{\frac{1}{2}} = a + b + x,$$

$$\text{or } 2(ax)^{\frac{1}{2}} = b, \therefore x = \frac{b^2}{4a}.$$

33. *Questions on falling bodies.*—

1. From a black cloud a flash of lightning was observed, and 15 seconds afterward it began to rain; on the supposition that the rain began to fall on the instant of the flash, what was the height of the cloud?* *Ans.* 3618.75 feet.

2. A meteoric stone fell upon a projecting stick of timber, with a momentum which, from the motion given to the stick, was estimated at 18,435 pounds. It occupied in falling, 10 sec-

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[*] No allowance is here made for the resistance of the air, which, in fact, greatly retards the descent of drops of rain.

onds; from what height did it fall, and what was the weight of the stone? *Ans.* Height, $1608\frac{1}{3}$ feet; Weight, 57.31 lbs.

3. An archer wishing to know the height of a tower, found that an arrow sent to the top of it occupied 8 seconds in going and returning; what was the height of the tower?
Ans. $257\frac{1}{3}$ feet.

4. In what time would a man fall from a balloon three miles high, and what velocity would he acquire?
Ans. $T = 31.38$ seconds, or about half a minute.
$V = 1009.39$ feet per second, or about half the maximum velocity of a cannon-ball.

5. A body having fallen for $3\frac{1}{2}$ seconds, was afterward observed to move along the horizon (with the velocity which it had acquired in its descent) for $2\frac{1}{2}$ seconds; what was the whole space described by the body from the beginning of its fall?
Ans. $478\frac{1}{2}$ feet, very nearly.

6. Through what space would the aeronaut (in Question 4) fall during the last second? *Ans.* 993.3 feet.

7. A body has fallen from the top of a tower 340 feet high; what was the space described by it in the last three seconds?
Ans. 298.957 feet.

8. Suppose a body be projected downward with a velocity of 18 feet in a second; how far will it fall in 15 seconds?
Ans. $3888\frac{3}{4}$ feet.

9. A body is projected upward with a velocity of 65 feet in a second; how far will it rise in 2 seconds? *Ans.* $65\frac{2}{3}$ feet.

10. With what velocity must a stone be projected into a well 450 feet deep, that it may arrive at the bottom in four seconds? *Ans.* $V = 48\frac{1}{6}$ feet in a second.

11. The space described in the fourth second of fall was to the space described in the last second, except four, as 1 : 3; what was the whole space described by the body?
Ans. $3618\frac{3}{4}$ feet.

12. A staging is at the height of 84 feet above the earth. A ball thrown upward from the earth, after an absence of 7 seconds, fell on the staging; what was the velocity of projection? *Ans.* 124.58 feet per second.

34. *Vis viva.*—We have seen that when a body is projected upward in a vacuum, the height to which it will ascend varies as the *square* of the velocity of projection. In the ascent, the body is constantly and uniformly opposed by gravity. If the motion of a given body were opposed by any other uniform obstruction, the distance it would proceed before coming to rest would also vary as the square of the velocity. This *penetrating* power (if it may be so termed), which varies not merely as the momentum, that is, the product of quantity and velocity, but as the product of quantity and *square* of velocity, is called the *vis viva*, or *living force*, in distinction from the *inertia*, or *dead force*. If two balls of the same *size*, but weighing 1 *lb.* and 2 *lbs.*, have velocities respectively of 2000 and 1000 feet per second, then their *momenta* are equal; and they will give the same amount of motion to another body against which they impinge. But the *living forces* are as $1 \times 2^2 : 2 \times 1^2 :: 2 : 1$; the first, therefore, has twice the living force, and will move upward or in a resisting medium, during twice as long a time as the other. In applying the principle to the *ascent* of bodies in vacuo, the quantity of matter is not to be regarded, and the distance is simply as the square of the velocity. This is because gravity itself, which opposes the motion, varies as the quantity of matter, so that all bodies, light and heavy, reach the same height, if projected with the same velocities.

THE FALL OF BODIES DISCUSSED BY MEANS OF THE CALCULUS.

35. *Differential equations.*—The differential equations relating to force and motion are the three following:

$$1. \quad v = \frac{ds}{dt}.$$

$$2. \quad f = \frac{dv}{dt} = \frac{d^2s}{dt^2}.$$

$$3. \; fds = vdv.$$

These equations are readily derived from principles already presented. In Art. 5, we have $v = \frac{s}{t}$. Reducing the numerator and denominator to infinitesimals, v remains finite, and the equation becomes $v = \frac{ds}{dt}$; which is Equation 1st. Therefore, if the space described by a body is regarded as a function of the time, the *first* differential coefficient expresses the *velocity*.

Again (Art. 11), $f = \frac{v}{t}$, where f represents a *constant* force. Making velocity and time infinitely small, we get the intensity of the momentary force, $f = \frac{dv}{dt}$. But, by Equation 1st, $v = \frac{ds}{dt}$; $\therefore f = \frac{d^2s}{dt^2}$; which is Equation 2d. Hence we learn, that the *first* differential coefficient of the *velocity* as a function of the time, or the *second* differential coefficient of the *space* as a function of the time, expresses the *force*.

Equation 3d is obtained by multiplying the 1st and 2d crosswise, and removing the common denominator.

We proceed to apply these equations to the preparation of formulæ for falling bodies.

36. *Bodies falling through small distances near the earth's surface.*

In this case, let the accelerating force, which is considered *constant*, be called g. Then, by Eq. 2, $g = \frac{dv}{dt}$, $\therefore dv = gdt$. Integrating, we have $v = gt + C$. But, since $v = 0$ when $t = 0$, $\therefore v = gt$, and $t = \frac{v}{g}$, as in Art. 28.

Again, substituting gt for v in Eq. 1, $ds = gtdt$; and by integration, $s = \frac{1}{2}gt^2 + C$; but $C = 0$, for the same reason as before; $\therefore s = \frac{1}{2}gt^2$, and $t = \sqrt{\frac{2s}{g}}$, as in Art. 28.

Once more, equating the two foregoing values of t, we have $v = \sqrt{2gs}$, and $s = \frac{v^2}{2g}$, as in Art. 28.

If, in the equation, $s = \frac{1}{2}gt^2$, v be substituted for gt, we have $s = \frac{1}{2}vt$, or $vt = 2s$; that is, the acquired velocity multiplied by the time of fall gives a space twice as great as that fallen through. (Art. 25.)

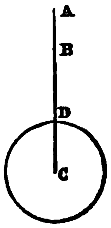
37. *Bodies falling through great distances, so that gravity is variable, according to the law in Art.* 15.

Suppose a body to fall from A to B, toward the center C. Let $AC = a$; $BC = x$; $DC = r$, the radius of the earth.

The force f at B, is found by the principle, Art. 15,

$$x^2 : r^2 :: g : f = gr^2 \frac{1}{x^2} = gr^2x^{-2}.$$

Fig. 14.



38. *To find the acquired velocity.*—Substitute gr^2x^{-2} for f, and $a - x$ for s, in Equation 3d., and we have $gr^2x^{-2} . d(a - x) = vdv$; $\therefore$ by integration, $\frac{1}{2}v^2 = \int - gr^2x^{-2}dx = gr^2x^{-1} + C$. But $v = 0$, when $x = a$; $\therefore C = -gr^2a^{-1}$; and

$$\frac{1}{2}v^2 = gr^2x^{-1} - gr^2a^{-1};$$

$$\therefore\ v^2 = \frac{2gr^2(a - x)}{ax};$$

$$\therefore\ v = \left\{\frac{2gr^2(a - x)}{ax}\right\}^{\frac{1}{2}}.$$

This is the general formula for the acquired velocity. If the body falls to the earth, $x = r$, and the formula becomes

$$v = \left\{\frac{2gr(a-r)}{a}\right\}^{\frac{1}{2}}.$$

Again, if the body falls to the earth, through so small a space, that $\frac{r}{a}$ may be regarded as a unit, the formula reduces to

$$v = \{2g(a-r)\}^{\frac{1}{2}} = (2gs)^{\frac{1}{2}};$$

the same as obtained by other methods.

If a body falls to the earth from an infinite distance, it does not acquire an infinite velocity. For then, as we may put a for $a - r$,

$$v = \left\{\frac{2gr \, . \, a}{a}\right\}^{\frac{1}{2}} = (2gr)^{\frac{1}{2}} =$$

$$(2 \, . \, 32\tfrac{1}{6} \, . \, 3956 \, . \, 5280)^{\frac{1}{2}} \text{ feet} = 6.95 \text{ miles}.$$

Therefore, the greatest possible velocity acquired in falling to the earth is less than *seven miles;* and a body projected upward with that velocity would never return.

39. *To find the time of falling.*—From equation first we obtain $dt = \frac{ds}{v}$; in this, substitute $d(a-x)$ for ds, and $\frac{\{2gr^2(a-x)\}^{\frac{1}{2}}}{(ax)^{\frac{1}{2}}}$ for v, as found in the preceding article; then

$$dt = \frac{(ax)^{\frac{1}{2}} . \, d(a-x)}{\{2gr^2(a-x)\}^{\frac{1}{2}}} = \left(\frac{a}{2gr^2}\right)^{\frac{1}{2}} . \frac{-x^{\frac{1}{2}}dx}{(a-x)^{\frac{1}{2}}};$$

$$\therefore \text{ by integration, } t = \left(\frac{a}{2gr^2}\right)^{\frac{1}{2}} . \int - x^{\frac{1}{2}}dx\,(a-x)^{-\frac{1}{2}}.$$

By the formula for reducing the index of x, we obtain

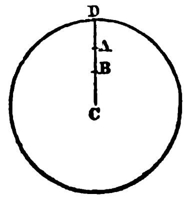
$$\int - x^{\frac{1}{2}}dx\,(a-x)^{-\frac{1}{2}} = (ax - x^2)^{\frac{1}{2}} - \frac{a}{2}\, vers^{-1}\left(\frac{2x}{a}\right) + \text{C}.$$

Now, when $t = 0$, $x = a$; $\therefore \text{C} = \frac{a\pi}{2}$;

$$\text{hence, } t = \left(\frac{a}{2gr^2}\right)^{\frac{1}{2}} \left\{(ax - x^2)^{\frac{1}{2}} - \frac{a}{2}\, vers^{-1}\left(\frac{2x}{a}\right) + \frac{a\pi}{2}\right\}.$$

40. *Bodies falling within the earth (supposed to be of uniform density), where gravity varies as the distance from the center.*

Fig. 15.



Suppose a body to fall from A to B; and let DC $= r$, AC $= a$, and BC $= x$. Then

$$r : x :: g : f = \frac{g}{r}x = \text{force at B.}$$

To find the velocity acquired.—By Eq. 3d,

$$vdv = fds;\ \therefore\ vdv = \frac{g}{r}x \,.\, d(a - x) = -\frac{gxdx}{r};$$

$$\therefore\ \tfrac{1}{2}v^2 = -\frac{gx^2}{2r} + C;\ \text{but } v = 0 \text{ when } x = a;$$

$$\therefore\ C = \frac{ga^2}{2r}, \text{ and } \tfrac{1}{2}v^2 = \frac{g(a^2 - x^2)}{2r};\ \therefore\ v = \left\{\frac{g}{r}(a^2 - x^2)\right\}^{\frac{1}{2}}.$$

If the body falls from the surface to the center, $x = 0$, and this formula becomes $v = (gr)^{\frac{1}{2}} = (32\tfrac{1}{6} \times 3956 \times 5280)^{\frac{1}{2}} = 25{,}904$ feet per second.

To find the time of falling.—By Equation 1st, and substitutions, we obtain $dt = \frac{ds}{v} = \frac{d(a-x)}{v} = -\frac{dx}{v} = \frac{-dx}{\left\{\frac{g}{r}(a^2 - x^2)\right\}^{\frac{1}{2}}}$

$$= \left(\frac{r}{g}\right)^{\frac{1}{2}} \times \frac{-dx}{(a^2 - x^2)^{\frac{1}{2}}};\ \therefore\ t = \left(\frac{r}{g}\right)^{\frac{1}{2}} \int \frac{-dx}{(a^2 - x^2)^{\frac{1}{2}}} = \left(\frac{r}{g}\right)^{\frac{1}{2}} \cos^{-1}\frac{x}{a} + C.$$

When $t = 0$, $x = a$, $\frac{x}{a} = 1$, and the arc, whose cosine is $1 = 0$;

$$\therefore C = 0.\quad \therefore t = \left(\frac{r}{g}\right)^{\frac{1}{2}} \times \cos^{-1}\frac{x}{a}.$$

If the body falls to the center, $x = 0$, and $t = \left(\frac{r}{g}\right)^{\frac{1}{2}} \times \frac{\pi}{2}$; in which a does not appear at all; so that the time of falling to the center from any point within the surface is the same; and equals $\left(\frac{3956 \times 5280}{32\frac{1}{6}}\right)^{\frac{1}{2}} \times 1.570796$ in seconds, or 21m. 7¼s.

CHAPTER III.

COMPOSITION AND RESOLUTION OF MOTION.

41. In the two preceding chapters, we have considered the motion produced in bodies by the action of only a *single* force. We now proceed to show the manner in which a body would move when acted upon at the same time by *several* forces.

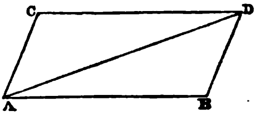
When two or more forces act simultaneously on a body, they are called *components;* and their joint effect is called the *re-*

sultant. It is customary to estimate forces by the *lines* along which they would move a body in a given time; the lines represent the forces in two particulars, the *directions* in which they act, and their *relative magnitudes.*

42. *The parallelogram of forces.*—This is the name given to the relation which exists between any two components and their resultant, and is stated as follows:

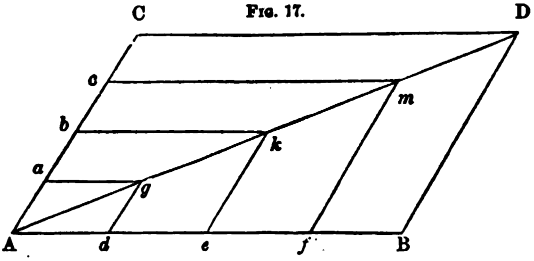
If two adjacent sides of a parallelogram represent two forces which act at the same time on a body, its diagonal represents the resultant. Suppose an impulse given to a body at A, such as acting alone would carry it over AB in a given time, and another, which would carry it over AC in the same length of time. If both impulses are given at the same instant, the body describes AD in the same time as AB by the first force, or AC by the second, and the motion in AD is uniform.

FIG. 16.



43. It will be readily seen, that this is an instance of the coexistence of motions, stated in the second law of motion (Art. 12). For the body, in passing directly from A to D, is as truly making progress in the direction expressed by AC, as though the motion AB did not exist; and, at the same time, it is uniformly advancing in the direction AB, just as fast as if AC did not exist. In Fig. 17, let AB be divided into any number of small equal parts, and AC into the same number; and let systems of parallels be drawn through the points of division, one system parallel to AB, the other to AC. Then, by similar triangles, it appears that AD meets the intersections of each pair of parallels, and is thus divided in the same ratio as the sides. At the end of the first portion of time, the body which would have been at *d* by one force, or at *a* by the other, will be at *g* by the action of both forces, having advanced $ag = Ad$, in the former direction, and $dg = Aa$, in the latter. At the end of the second portion of time, it will be at *k*, instead of *e* or *b*, having advanced two portions of the space in the direction AB, and two in the direction AC, and so on. Thus it appears that both mo-

FIG. 17.



tions fully coexist in the progress of the body along the diagonal; and it is in this sense that the motion produced by two forces is called *compound motion.* Since we may suppose AD to be described by a single force in the same time as by AB and AC jointly, such a single force is said to be *equivalent* to the other two.

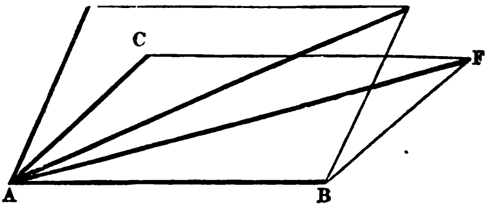
44. The lines AB and AC are described by the components separately, and the line AD by their joint action, in the same length of time; hence the *velocities* in the three lines are as the lines. In the parallelogram of forces, therefore, two adjacent sides, and the diagonal between them, represent, 1st, the *directions* and *relative magnitudes* of two component forces and their resultant; and 2d, the *velocities* with which they are respectively described.

45. *The sides of a triangle may represent two components and their resultant.*—For purposes of calculation, it is more convenient to represent two components and their resultant by the sides of a triangle, than by the sides and diagonal of a parallelogram. In Fig. 16, CD, which is equal and parallel to AB, may represent in direction and magnitude the same force which AB represents. Therefore, the components are AC and CD, while their resultant is AD; and the angle C in the triangle is the supplement of CAB, the angle between the components. Care should be taken to construct the triangle so that the sides representing the components may be taken continuously in the directions of the forces, as, AC, CD; then AD correctly represents their resultant. But, although AC and AB represent the components, the third side, CB, of the triangle ACB, does not represent their resultant, since AC and AB cannot be taken continuously in the direction of the forces. It should be observed, that though CD represents the *magnitude* and *direction* of the component, it is not in the *line* of its action, because both forces act through the same point A.

46. Since the sides of a triangle are proportional to the sines of the opposite angles, these sines may also represent two components and their resultant. Thus, the sine of CAD corresponds to the component AB (=CD); the sine of CDA (=DAB) corresponds to the component AC; and the sine of C (= sine of CAB) corresponds to the resultant, AD. Each of the three forces is therefore represented by the sine of the angle between the other two.

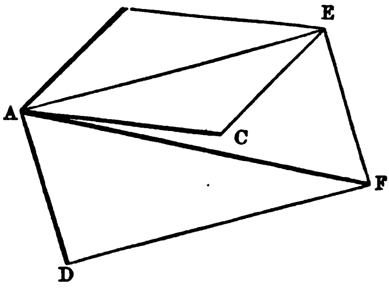
47. *Greatest and least values of the resultant.*—A change in the angle between the components alters the value of the resultant; as the angle increases from 0° to 180°, the resultant diminishes from the *sum* of the components to their *difference*. In Fig. 18, let CAB and DAB be two different angles between the same components AC (or AD) and AB. As CAB is less than DAB, its supplement ABF is greater than ABE, the supplement of DAB; therefore AF is greater than AE. When the angle CAB is diminished to 0°, the sides AB, BF, become one straight line, and AF equals their *sum*; when DAB is enlarged to 180°, E falls on AB, and AE equals the *difference* of AB and AC. Between the sum and difference of the components, the resultant may have all possible values.

FIG. 18.



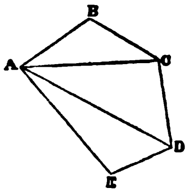
48. *The polygon of forces.*—All the sides of a polygon except one may represent so many forces acting at the same time on a body, and the remaining side will represent their resultant. In Fig. 19, suppose AB, AC, and AD, to represent three forces acting together on a body at A. The resultant of AB and AC is represented by the diagonal AE; and the resultant of AE and AD by the diagonal AF. As AF is equivalent to AE and AD, and AE is equivalent to AB and AC, therefore AF is equivalent to the three, AB, AC, and AD. But if we substitute BE for AC, and EF for AD, then the three components are AB, BE, and EF, three sides of a polygon, and the resultant AF is the fourth side of the same polygon.

FIG. 19.



So, in Fig. 20, AB, BC, CD, DE, may represent the directions and relative magnitudes of four forces, which act simultaneously on a body at A. The resultant of AB and BC is AC; the resultant of AC and CD is AD; and the resultant of AD and DE is AE, which last is therefore the resultant of all the forces, AB, BC, CD, and DE; the components being represented by four sides, and their resultant by the fifth side, of a polygon of five sides.

FIG. 20.



49. *Curvilinear motion.*—Since, according to the first law of motion, a moving body proceeds in a straight line, if no force disturbs it, whenever we find a body describing a *curve*, it is certain that some force is continually deflecting it from a straight line. Besides the original impulse, therefore, which gave it motion in one direction, it is subject to the action of an accelerating force, which operates in another direction. A familiar example occurs in the path of a projectile. Suppose a body to be thrown from P (Fig. 21), with an impulse which would alone carry it to N, in the same time in which gravity alone would carry it to V. Complete the parallelogram PNQV; then, as both motions coexist (2d law), the body at the end of the time will be found at Q. Let T be the time of describing PN or PV; and let t be the time of describing PM by the impulse, or PL by gravity. Then, at the end of the time t, the body will be at O. Now, as PN is described uniformly,

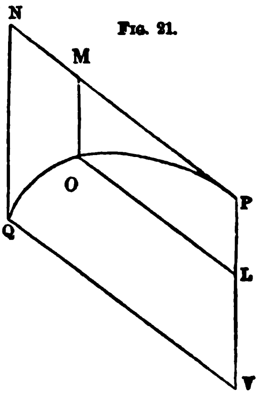


FIG. 21.

$$PN : PM :: T : t; \therefore PN^2 : PM^2 :: T^2 : t^2.$$

But (Art. 23), $PV : PL :: T^2 : t^2$;

$$\therefore PV : PL :: PN^2 : PM^2; \text{ or } QV^2 : OL^2.$$

Hence, the curve is such that $PV \propto QV^2$; that is, the abscissa varies as the square of the ordinate, which is a property of the parabola. POQ is therefore a parabola, one of whose diameters is PV, and the parameter to that diameter is $\frac{QV^2}{PV}$.

Owing to the resistance of the air, the curve actually deviates sensibly from a parabola, especially in swift motions.

50. *Calculation of the resultant.*—When two components and the angle between them are given, the resultant may be found both in direction and magnitude by trigonometry. The theorem required is that for solving a triangle, when two sides and the contained angle are given; but the contained angle is not that between the components, but its supplement (Art. 45). In Fig. 16, if AB = 54, and AC = 22, and CAB = 75°, then ACD is the triangle for solution, in which AC = 22, CD = 54, and ACD = 105°. Performing the calculation, we find the resultant AD = 63.363, and the angle DAB, which it makes with the greater force, = 19° 35′ 43″. This method will apply in all cases.

1. A foot-ball received two blows at the same instant, one directly east, at the rate of 71 feet per second, the other exactly northwest, at the rate of 48 feet per second; in what direction, and with what velocity did it move.

ngle 135° [illegible] 45° *Ans.* N. 47° 30′ 52″ E. Vel. = 50.253.

[illegible] in angle [illegible]

The process is of course abridged, if the forces act at a right angle with each other, as in the following example.

2. A balloon rises 1120 feet in one minute, and in the same time is borne by the wind 370 feet; what angle does its path make with the vertical, and what is its velocity per second? *Ans.* 18° 16′ 53″. $V = 1179.53$.

In the next example, one component and the angle which each component makes with the resultant, are given to find the resultant and the other component.

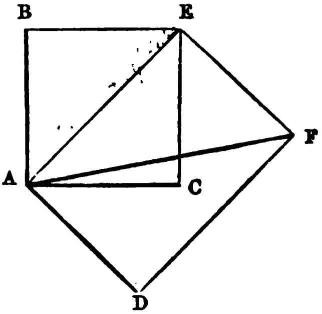
3. From an island in the Straits of Sunda, we sailed S. E. by S. (33° 45′) at the rate of 6 miles an hour; and being carried by a current, which was running toward the S. W. (making an angle with the meridian of 64° 12¼′), at the end of four hours we came to anchor on the coast of Java, and found the said island bearing due north: required *the length of the line* actually described by the ship, and *the velocity of the current?*

Ans. $S = 26.4$ miles.
$V = 3.7024$ miles per hour.

If the magnitudes and directions of any number of forces are given, the resultant of them all is obtained by a repetition of the same process as for two. In Fig. 20, first calculate AC, and the angle ACB, by means of AB, BC, and the angle B. Subtracting ACB from BCD, we have the same data in the next triangle, to calculate AD, and thus proceed to the final resultant, AE.

As it is immaterial in what order the components are introduced into the calculation, it will diminish labor, to find first the resultant of any two *equal* components, or any two which make a *right angle* with each other; since it can be done by the solution of an isosceles, or a right-angled triangle.

FIG. 22.



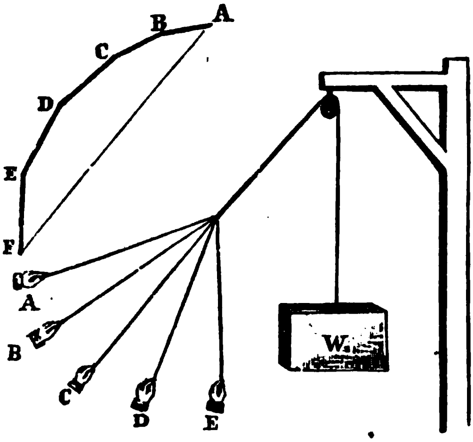
4. The particle A (Fig. 22), is urged by three equal forces AB, AC, and AD; the angle BAC = 90°, and CAD = 45°; what is the direction of the resultant, and how many times AB?

Ans. BAF = 80° 16′, and AF : AB : : $\sqrt{3} : 1$.

5. Five sailors raise a weight by means of five separate ropes, in the same plane, connected with the main rope that is fastened to the weight in the manner represented in Figure 24. B pulls at an angle with A of 20°; C with B, at 19°; D with C, at 21° 30′; and E with D, at 25°. A, B, and C, pull with equal forces, and D and E with forces one-half greater; required the magnitude and direction of the resultant?

Ans. Its angle with A is 46° 33′ 10″. Its magnitude is 5,1957 times the force of A.

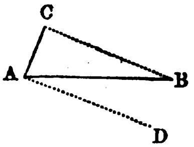
FIG. 23. FIG. 24.



If the polygon, ABCDEF (Fig. 23), be constructed for the above case, AC and DF are easily calculated in the isosceles triangles ABC and DEF, after which AD and then AF are to be obtained by the general theorem.

51. *The resultant and all components, except one, being given, to find that one component.*—If AB (Fig. 25) is the resultant to be produced, and there already exists the force AC, a second force can be found, which acting jointly with AC, will produce the motion required. Join CB, and draw AD equal and parallel to it, then AD is the force required; for AB is equivalent to AC and CB. Therefore CB has the magnitude and direction of the required force; AD is the line in which it must act.

FIG. 25.



Again, suppose that *several* forces act on A, and it is required to find the force, which in conjunction with them all shall produce the resultant AB. Let the several forces be combined into one resultant, and let AC represent that resultant. Then AD may be found as before.

The trigonometrical process for finding a component is essentially the same as for finding a resultant.

1. A ferry-boat crosses a river $\frac{3}{4}$ of a mile broad in 45 minutes, the current running all the way at the rate of 3 miles an hour; at what angle with the direct course must the boat head up the stream in order to move perpendicularly across?

Ans. 71° 34′.

2. A sloop is bound from the mainland of Africa to an island bearing W. by N. (78° 45′) distant 76 miles, a current setting N. N. W. (22° 30′) 3 miles an hour; what is the *course* to arrive at the island in the shortest time, supposing the sloop to sail at the rate of 6 knots per hour; and what *time* will she take? *Ans.* Course, S. 76° 41′ 4″ W. Time, 10h. 40m. 7 sec.

3. The resultant of two forces is 10; one of them is 8, and the direction of the other is inclined to the resultant at an angle of 36°. Find the angle between the two forces.

Ans. 47° 17′ 5″.

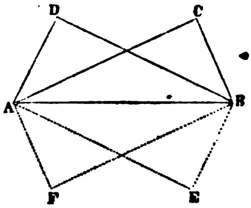
4. A ball receives two impulses: one of which would carry it N. 27 feet per second; the other, E. 30° N. with the same velocity; what third impulse must be conjoined with them, to make the ball go E. with a velocity of 21 feet?

Ans. S. 3° 22′ W. V. = 40.57.

52. *Resolution of motion.*—In the *composition* of motions or forces, we find the resultant of any given components; in the *resolution* of motion or force, we reverse the process, and find what components would produce a given resultant.

If AB (Fig. 26) represents a given motion, and it is desired to determine two forces, which acting together would produce it, we have only to construct any triangle whatever upon AB, as ADB, ACB, &c.; then the other two sides of that triangle will represent the two forces required (Art. 45). If AD is one component, then AE, equal and parallel to DB, is the other in the line of its action at the point A; or if AC is one, AF is the other; and so for any triangle whatever constructed on AB. The number of pairs is therefore infinite. It is obvious, likewise, that the *directions* of both components may be what we please, with this one condition, that the sum of the angles made with AB shall be less than 180°. Instead of determining the directions of both components, we may fix the *magnitude* and *direction* of one component, without any limitation whatever. Or, once more, we may determine the *magnitudes* of both components, provided their difference is not greater than the given resultant, AB. These several cases are obvious, from the properties of the triangle.

FIG. 26.



A motion of 153 toward the north is produced by the forces 100 and 125; how are they inclined to the meridian?

Ans. 54° 28′ and 40° 37′ 7″.

When a given force has been resolved into two others, each of those may again be resolved into two, each of those into two others still, and so on. Hence it appears that a given force may be resolved into any number of components whatever, with such limitations as to direction and magnitude as accord with the foregoing statements.

53. *Resolution into pairs, with certain conditions.*—Sometimes, by the conditions of the problem, the resolved forces are required to make a *given* angle, or to be in a certain ratio, with each other. The method of solving cases of this kind may be illustrated by a few examples.

First, let a given force AB (Fig. 27), be resolved into pairs of forces which shall always act at *right angles* to each other. Upon AB describe a semicircle, ABC, and from the extremities of the base draw straight lines to meet in any point of the circumference. The sides of the triangle thus formed, will severally contain right angles; and AC and a line drawn from A parallel and equal to CB, will represent two forces equivalent to the given force AB.

FIG. 27.

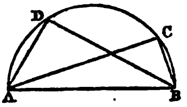
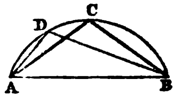


FIG. 28.



Secondly, let the resolved forces be required to make with each other *any* given angle. Upon AB (Fig. 28) describe the segment of a circle, ABC, containing an angle, the *supplement* of the given angle, and draw straight lines from the extremities of the base to any point in the circumference.

FIG. 29.

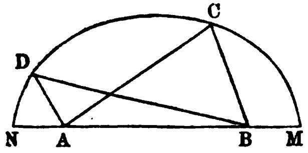
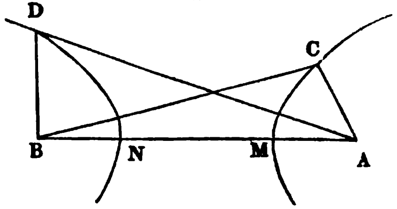


FIG. 30.



Thirdly, let the *sum* of the resolved forces be required to be equal to a given quantity. Let MN (Fig. 29) be equal to the sum of the forces required, and AB be the given force; and upon MN, as the transverse, and with the points A and B (equally distant from M, N), as foci, describe the ellipse MCN. From A and B draw straight lines to any point in the ellipse, and the sides of the several triangles will form the pairs of forces required.

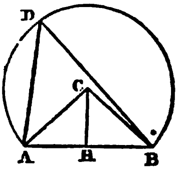
Fourthly, in like manner, pairs of forces, whose *difference* shall be always equal to the same constant quantity, may be found by making A and B the foci of an hyperbola, as in Fig. 30, and drawing straight lines from these points to the curve.

1. A given force (a) is required to be resolved into different

pairs of forces which shall act at an angle of 135° to each other. What is the radius of the circle whose segment shall contain pairs of the resolved forces? Ab : AB :: R :

Let AB = a, and upon AB describe the segment of a circle which shall contain an angle ADB of 45°; then the radius AC = $\frac{a}{\sqrt{2}}$.

FIG. 31.



sin ADB = cos ABC
b : AB :: sin ABC : 1 : cos
b : a :: cos : sin 2a
b = $\frac{a \times \cos A}{\sin 2A}$
sin 2a = $\frac{2 \cos \sin A}{R}$
$\frac{a \times \cos A}{2 \cos \sin A} = \frac{a \times \cos A}{2 \cos \sin A} = \frac{a}{2 \sin A}$
A CB = 2a

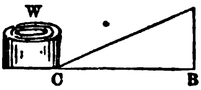
2. To determine the radius of the circle when AB (Fig. 31) is required to be resolved into pairs of forces acting at any given angle whose supplement is A. ac : ab :: R : 2

Let AB = a, ADB = A; find the center C, and join CA, CB; then BC or radius = $\frac{a \times \cos A}{\sin 2A}$ = (in a briefer form) $\frac{a}{2 \sin A}$.

Let $a = 10$, A = 40° } then radius = 7.7786.

54. *Resolution of a force, to find its efficiency in a given direction.*—By the resolution of a force into two others acting at right angles with each other, it is ascertained how much efficiency it exerts to produce motion in any given direction. For example, a weight W (Fig. 32), lying on a horizontal plane, and pulled by the oblique force CA, is prevented by gravity from moving in the line CA, and is compelled to remain on the plane. Resolve CA into CB, *in* the plane, and BA, *perpendicular* to it, then the former represents the component which is efficient to cause motion along the plane; the latter has no direct influence to aid or hinder that motion; it simply diminishes pressure upon the plane. In like manner, if AC is an oblique force, *pushing* the weight, its horizontal component, BC, is alone efficient to move it, the other, AB, merely increasing the pressure. In either case, *the whole force is to the efficient part, as radius to the cosine of the obliquity.* Also, the *whole* force is to the *component, perpendicular* to a surface, as *radius* to the *sine* of obliquity to that surface.

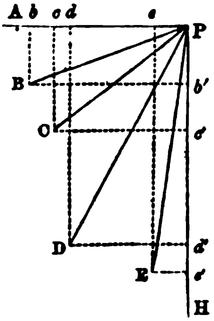
FIG. 32.



55. *Resolution upon axes, for the composition of forces.*—The preceding article suggests the most convenient method of obtaining the resultant of several forces.

Let PA, PB, PC, PD, PE (Fig. 33), represent the forces in Question 5 (Art. 50). Draw PH at right angles to PA, and resolve each force upon PA and PH, as axes. PA, lying on one axis needs no resolution; to resolve PB, say

FIG. 33.



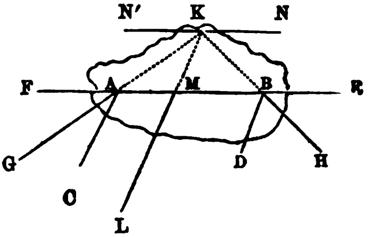
$$\begin{aligned}
& R : \cos 20° :: PB : Pb, \\
\text{and}\quad & R : \sin 20° :: PB : Pb'; \\
\text{again,}\ & R : \cos 39° :: PC : Pc, \\
\text{and}\quad & R : \sin 39° :: PC : Pc'; \ \&c. \\
\text{Let}\ & M = PA + Pb + Pc + Pd + Pe, \\
\text{and}\ & N = \qquad\ Pb' + Pc' + Pd' + Pe'.
\end{aligned}$$

Then will M and N be the components along PA and PH. If the resultant of M and N, and the angle it makes with PA, be computed, we shall have 5.1957 and 46° 33′ 10″, as before, by a shorter and more elegant process.

If any components of the resolved forces lie in an opposite direction, they must be taken negatively.

56. *Parallel Forces.*—We have thus far considered forces acting upon a single particle; or upon one point of a body, called its center of gravity. If, however, two forces, GA and HB (Fig. 34), in the same plane, act upon A and B, two different points of a rigid body, they may still have a single resultant. For, produce HB and GA to meet in K, and suppose K to be rigidly united with A and B. The two forces may then be considered as acting at K, and thus compounded into a single force at that point.

FIG. 34.



If, however, the two forces are parallel, as CA and DB, they will not meet. Still there will generally be, in this case, a single resultant; and this we now proceed to find. The effect of the forces will not be changed, if two equal forces acting in opposite directions, EB and FA, be applied along the line AB. The two forces, CA and FA, will be equivalent to a single resultant GA; and likewise the two, EB and DB, will be equivalent to HB. Let these meet in K, and draw KML parallel to AC. Consider HB and GA as acting at K, and resolve each one along KL and N′KN, parallel to AB. The latter components will be equal (being in fact the same as EB and FA), and will therefore destroy each other. The sum of the former will be equal to CA + DB. Hence, LM (= CA + DB) is the resultant sought.

Again, since the sides of the triangle AMK, will represent the three forces, FA, GA, and CA, we have

$$AM : MK :: FA : CA;$$

in like manner, $BM : MK :: EB : DB;$

therefore, since the means are equal,

$$AM : BM :: DB : CA; \text{ and, by composition,}$$
$$AM : AM + BM \ (= AB) :: DB : DB + CA = LM.$$

Hence we derive the following theorems, which contain the principles of the composition and resolution of parallel forces.

I. *The resultant of two parallel forces is in a direction parallel to them, and is equal to their sum.*

II. *If a straight line be drawn to meet the lines of these three forces (viz., the two components and their resultant), each of the three forces will be represented by that portion of the line contained between the other two.*

57. If the force CA (Fig. 35) acts in a direction opposite to that of DB, then the resultant will be found in a similar manner upon AB produced, and equal to DB – CA.

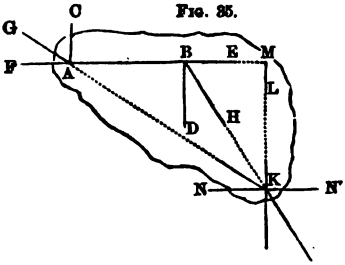


Fig. 35.

In the first case, where the forces act in the same direction, the above proportion gives

$$AM = \frac{AB \times DB}{DB + CA}.$$

In the second case we have, in like manner,

$$AM = \frac{AB \times DB}{DB - CA}.$$

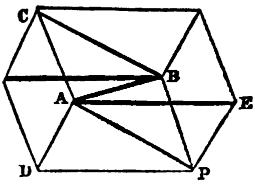
If in the last case, DB = AC, then AM will be infinite. The two forces in this case constitute what is called a *couple.* The effect of a couple is to produce rotation of the body upon which the couple acts.

Any number of parallel forces may be reduced to a single force (or else to a couple of forces), by proceeding as in Art. 56; that is, by first finding the resultant of two forces, and then a new resultant for that resultant and one of the remaining forces, and so on to the last. And any single force may be resolved into any number of parallel forces by a method the reverse of this.

58. *Forces in different planes.*—In estimating the effects produced by the composition and resolution of forces, we have hitherto considered them as acting in the same plane; we proceed now to the consideration of forces acting in different planes.

The parallelopiped.—Any three forces in different planes may be reduced to one, by making the lines which represent the forces the edges of a parallelopiped; the diagonal will then

FIG. 36.

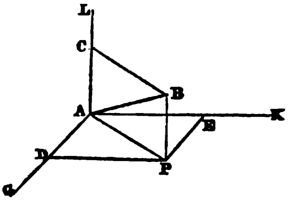


represent the resultant. Let AC, AD, and AE (Fig. 36), be three forces applied in different planes to the body at A. Construct the parallelopiped CP, and from A draw the diagonal AB. The section through the edges AC and PB is a parallelogram, and therefore AB is the resultant of AC and AP; and AP is the resultant of AD and AE. Hence AB is the resultant of AC, AD, and AE.

This process may be reversed, and a given force may be resolved into three components upon the edges of a parallelopiped, having such inclinations as we please.

59. *Rectangular axes.*—The parallelopiped generally chosen is the one whose sides are all rectangles; and its three edges, indefinitely extended, are called *rectangular axes.* All the forces which can possibly act upon a body may be resolved into equivalent forces in the direction of three such axes. And as all the forces in the direction of any line may be reduced to one by addition and subtraction, therefore all forces whatever which act through one point may be reduced to *three.* We proceed to obtain expressions for them.

FIG. 37.

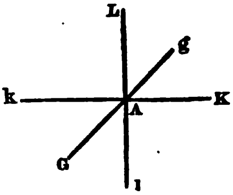


Let AK, AG (Fig. 37), be at right angles with each other, and AL perpendicular to the plane of AK, AG. Let AB represent the force acting on A; and resolve it into AC and AP; and resolve AP into AD and AE. Then AC, AD, and AE, are three rectangular forces, together equivalent to AB.

Let AK, AG, and AL (Fig. 38), be produced through A to *k*, *g*, *l*; then the planes of these axes will divide the angular space about A into eight solid right angles; and a given force may lie within any one of the eight. The eight solid angles are described by their bounding lines, as follows:

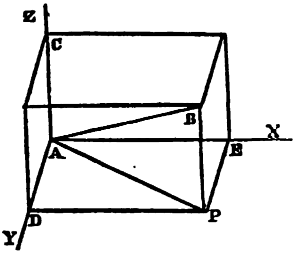
AL, AG, AK;
AL, AG, A*k*;
AL, A*g*, A*k*;
AL, A*g*, AK;
A*l*, AG, AK;
A*l*, AG, A*k*;
A*l*, A*g*, A*k*;
A*l*, A*g*, AK.

FIG. 38.



60. Let AX, AY, AZ (Fig. 39), be three rectangular axes of indefinite length, and AB a force which is resolved as before

Fig. 39.



into AC, AD, and AE. By the right-angled triangles, we see that

$$AB^2 = BP^2 + AP^2 = AC^2 + AP^2;$$

and $AP^2 = AD^2 + DP^2 = AD^2 + AE^2;$

$$\therefore AB^2 = AC^2 + AD^2 + AE^2.$$

and $AB = \sqrt{AC^2 + AD^2 + AE^2}.$

Thus we have the magnitude of any force in terms of its components on the axes.

61. In order to obtain the same in a trigonometrical form, let the angles which AB makes with the axes AX, AY, and AZ, respectively, be α, β, γ; that is, $BAE = \alpha$, $BAD = \beta$, $BAC = \gamma$. If we suppose BE to be joined, AEB is right-angled at E; and

$$AB : AE :: \text{rad} : \cos \alpha;$$

$$\therefore \quad AE = AB \,.\, \cos \alpha.$$

In like manner.

$$AD = AB \,.\, \cos \beta;$$

$$\text{and} \quad AC = AB \,.\, \cos \gamma.$$

And, since AB is equivalent to AE, AD, and AC, it is equivalent to their equals, $AB \,.\, \cos \alpha$, $AB \,.\, \cos \beta$, and $AB \,.\, \cos \gamma$. In general, the components of any force P, when resolved upon three rectangular axes, are $P \,.\, \cos \alpha$, $P \,.\, \cos \beta$, $P \,.\, \cos \gamma$.

62. Suppose the body at A to be acted on by a second force P', whose direction makes with the axes the angles α', β', γ', then, as before, P' is equivalent to $P' \,.\, \cos \alpha'$, $P' \,.\, \cos \beta'$, $P' \,.\, \cos \gamma'$; and a third force P'' may, in like manner, have for its components $P'' \,.\, \cos \alpha''$, $P'' \,.\, \cos \beta''$, $P'' \,.\, \cos \gamma''$; and so of other forces.

Therefore, the whole force acting in the axis of X

$$= P \,.\, \cos \alpha + P' \,.\, \cos \alpha' + P'' \,.\, \cos \alpha'' +, \text{\&c.};$$

the whole force on the axis of Y

$$= P \,.\, \cos \beta + P' \,.\, \cos \beta' + P'' \,.\, \cos \beta'' +, \text{\&c.};$$

and that on the axis of Z

$$= P \,.\, \cos \gamma + P' \,.\, \cos \gamma' + P'' \,.\, \cos \gamma'' +, \text{\&c.}$$

If any of the resolved forces have the negative sign, it indicates that such forces act in opposition to the others on the same axis.

It is obvious that the axes may be chosen in any directions which are most convenient.

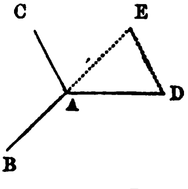
If all the components on *one* axis equal *zero*, then the body suffers no motion in the direction of that axis; *i. e.*, its motion is limited to the plane of the other two axes, and is to be treated as the example in Art. 55. If all the components on each of *two* axes are reduced to *zero*, then the whole motion of the body is in the direction of the third axis, and therefore perpendicular to the plane of the first two.

63. *Forces producing equilibrium.*—We have seen (Art. 47), that two forces produce the least resultant when they act at 180° with each other, and that the resultant then equals the difference of the forces. If the forces are equal, their difference is zero, and the resultant vanishes. Hence, *two forces produce equilibrium, when they are equal, and act in opposition to each other.*

64. From this we derive a principle for determining the circumstances of equilibrium, when more than two forces act on a body. If, for example, *three* forces are in equilibrio, each one must be *equal* to, and *opposed* to, the resultant of the other two; for that force, and only that, which neutralizes the *resultant,* would neutralize the *components,* whose action is equivalent to that resultant. But the two forces, AC and AB (Fig. 16), produce the resultant AD; therefore DA, equal and opposite to AD, will counterbalance AC and AB, or AC and CD; and the three forces which hold the body at rest are represented by AC, CD, DA, the sides of ACD, taken continuously. Hence, *three forces produce equilibrium, when they are proportional to the sides of a triangle, and are in the direction of those sides taken continuously.*

65. The law of equilibrium of three forces may also be stated in another form, thus: *when a body is in equilibrio by the action of three forces, each is as the sine of the angle between the directions of the other two.* Let BA, CA, and DA (Fig. 40) hold the particle A at rest; then the force, BA, may be represented by the sine of CAD; DA, by the sine of BAC; and CA, by the sine of BAD. For, if BA be produced until AE = BA, and ED be joined, ED will be equal and parallel to CA, since each is equal to the resultant of BA and DA. But DA, AE, and ED, are as the sines of E, D, and DAE. Now, sin D = sin CAD; sin E = sin CAE = sin BAC; and sin EAD = sin BAD. Therefore the forces BA, CA, and DA, are as the sines of CAD, BAD, and BAC, respectively.

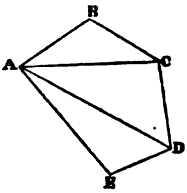
FIG. 40.



It is apparent from the law of equilibrium of three forces, that *the lines of their action must all intersect in the same point;* for otherwise, each force can not be opposed to the resultant of the other two.

66. When more than three forces acting in one plane, keep a body at rest, we may use the sides of a polygon to express their relation. In Fig. 41, if AB, BC, CD, and DE, act simultaneously on a body at A, their resultant is AE (Art. 48). Hence, EA, equal and opposite to AE, since it would counterbalance AE, will counterbalance all those forces which are equivalent to AE. Thus we have the law, that *all the forces, which together keep a body at rest, may be represented by the sides of a polygon, taken continuously in the directions in which the forces act.*

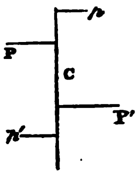
Fig. 41.



67. *Equilibrium of parallel forces.*—A force which is in equilibrio with two parallel forces, is parallel to them, equals their sum, and divides the distance between them inversely as their magnitudes. These three conditions belong to the *resultant* of two parallel forces (Art. 56), and therefore to the force which counterbalances that resultant.

A *couple* of forces can not be kept in equilibrio by *one* force, but only by another couple; and the second couple may be either equal or unequal to the first. The couple P and P′ tend to produce rotation about the middle point C (Fig. 42). If another couple Q and Q′, just equal to P and P′, should be applied to produce equilibrium, one must directly oppose P, and the other P′, and thus each *one*, and therefore the *pair* will be in equilibrio. But if the second couple is less than P and P′, then they must be applied at a distance from C, which is greater in the same ratio. Thus the couple p and p' will tend to produce rotation in one direction, and P and P′ in the opposite; and these tendencies will be equal, if the distance CP : Cp : : p : P. For, by the law of parallel forces, a force equal to $P - p$, acting toward the left at C, balances P and p, and a force equal to $P' - p'$ toward the right at the same point, balances P′ and p'; but $P - p = P' - p'$; therefore they are all in equilibrio.

Fig. 42.

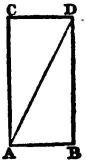


68. *Equilibrium of forces in different planes.*—Since all the forces which can operate on a body, may be reduced to three forces on rectangular axes, it is obvious that the whole system of forces can not be in equilibrio, till the components on each axis are reduced to zero. We must have, therefore, in Art. 62, as conditions of equilibrium, these three equations for the three axes, X, Y, and Z:

$$P \cdot \cos\alpha + P' \cdot \cos\alpha' + P'' \cdot \cos\alpha'' +, \&c., = 0;$$
$$P \cdot \cos\beta + P' \cdot \cos\beta' + P'' \cdot \cos\beta'' +, \&c., = 0;$$
$$P \cdot \cos\gamma + P' \cdot \cos\gamma' + P'' \cdot \cos\gamma'' +, \&c., = 0.$$

69. *The resistance of a smooth surface in equilibrio with other forces.*—Whenever any forces cause pressure upon a smooth surface, and are held in equilibrio by its resistance, the resultant of those forces must be at right angles to the surface. Suppose that DA (Fig. 43), is either a single force, or the resultant of two or more forces, and that it is held in equilibrio by the reaction of AB, a smooth surface. If DA is not perpendicular to the surface, it can be resolved into two components, one perpendicular to the surface AB, the other parallel to it. The former, DB, is neutralized by the resistance of the surface; the latter, BA, is not resisted, and produces motion parallel to the surface. Therefore DA, if held in equilibrio by the surface AB, must be perpendicular to it.

Fig. 43.



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CHAPTER IV.

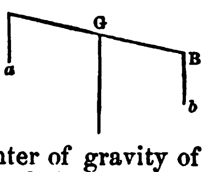
THE CENTER OF GRAVITY.

70. *The center of gravity of any body or system of bodies is that point about which, if supported, all the parts (acted on only by the force of gravity) balance each other in any position.*

The center of gravity is, therefore, the point through which, in every position, passes the resultant of all the parallel forces exerted by gravity upon the several parts of the body or system. For most purposes we may assume that all the parts are collected at that point; since, by so doing, we only substitute the resultant for all the forces to which it is equivalent. When a system of bodies is considered, it is implied, in all that follows, that the bodies are all united to each other by rigid rods, void of gravity.

71. *Equal bodies in a straight line.*—The center of gravity of two equal particles is in the middle point between them. Let A and B (Fig. 44), two equal particles, be joined by a straight line, and let A*a* and B*b* represent the forces of gravity. The resultant of these forces, since they are parallel and equal, will pass through the middle of AB (Art. 56); G is therefore the center of gravity. In like manner it is proved that the center of gravity of two equal *bodies* is in the middle point between their respective centers of gravity.

Fig. 44.



Any number of equal particles or bodies, arranged at equal distances on a straight line, have their common center of gravity in the middle; since the above reasoning applies to each pair, taken at equal distances from the extremes. Hence, the center of gravity of a material straight line (*e. g.*, a fine wire), is in its middle point.

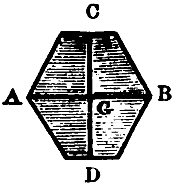
72. In the discussion of the center of gravity in relation to *form*, *bodies* are considered uniformly dense, and *surfaces* are regarded as thin laminæ of matter.

Regular figures.—In plane figures *the center of gravity coincides with the center of magnitude, when they have such a degree of regularity that two diameters can be drawn, each of which divides the figure into equal and symmetrical parts.*

The circle, the parallelogram, the regular polygon, and the ellipse, are examples.

For instance, the regular hexagon (Fig. 45) is divided symmetrically by AB, and also by CD. Conceive the figure to be composed of material lines parallel to AB. Each of these has its center of gravity in its middle point, that is, in CD (Art. 71). Hence, the center of gravity of the whole figure is in CD. For the same reason it is in AB. It is, therefore, at their intersection, which is also obviously the center of magnitude.

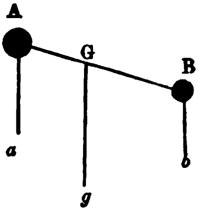
FIG. 45.



By a similar course of reasoning it is shown, that in solids of uniform density, which are so far regular that they can be divided symmetrically by three different planes, the centers of gravity and magnitude coincide; *e. g.* the sphere, the parallelopiped, the cylinder, the regular prism, and the regular polyhedron.

73. *Center of gravity between two unequal bodies.*—The center of gravity of two unequal bodies is in a straight line joining their respective centers of gravity, and at the point which divides their distance in the inverse ratio of their weights. Let Aa and Bb (Fig. 46), passing through the centers of gravity of A and B, be proportional to their weights, and therefore represent the forces of gravity exerted upon them. By the laws of parallel forces, the resultant $Gg = Aa + Bb$ (Art. 56), and $Aa : Bb :: BG : AG$. Therefore the center of gravity must be at G, through which the resultant passes (Art. 70). This obviously includes the case of *equal* weights (Art. 71).

FIG. 46.



It appears from the foregoing, that the whole pressure on a support at G is A + B, and that the system is kept in equilibrio by such support.

74. *Equal moments with respect to the center of gravity.*—If A is put for the weight of A, and B for that of B, the above proportion becomes $A : B :: BG : AG$.

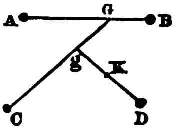
Let the proportion be changed to an equation, and we have $A \times AG = B \times BG$. Suppose now that AB is an inflexible rod, void of gravity, and free to revolve about G. Since the bodies balance each other about that point, the equal products, $A \times AG$ and $B \times BG$, may be taken to represent the equal tendencies of the bodies to turn the system about G. The tendency of the body A, expressed by $A \times AG$, is called the *moment* of A, with reference to the point G; and, similarly, $B \times BG$ is the *moment* of B, with reference to the same point. Hence the proposition, that *the moments of two bodies with reference to their center of gravity are equal.*

75. *Center of gravity between three or more bodies.*—The method of determining the center of two bodies may be extended to any number.

Let A, B, C, D, &c. (Fig. 47), be the weights of the bodies, and let the centers of gravity of A and B be connected together by the inflexible line AB.

Take $A : B :: BG : AG$, or $A + B : B :: BG + AG\,(AB) : AG$, then will the bodies A and B balance themselves about G (Art. 73), and consequently G will be their common center of gravity; and the three first terms of the above proportion being known, the distance of G from A is thus found.

FIG. 47.



Next, let the center of gravity of C be connected with G by the inflexible line CG, then, for the reason assigned in Art. 73, the pressure upon G will be the same as if a body equal to $A + B$ were placed at G; take, therefore,

$$A + B : C :: Cg : Gg, \text{ or } A + B + C : C :: Cg + Gg\,(CG) : Gg,$$

then g will be the center of gravity of $A + B$ and C, and consequently the common center of gravity of the three bodies A, B, C.

Again, let the center of gravity of D be connected with g by the inflexible line Dg, then the pressure upon g will be the same as if $A + B + C$ were placed at g. Take, therefore,

$$A + B + C : D :: DK : gK,$$

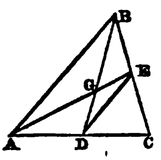
or, $$A + B + C + D : D :: DK + gK\,(Dg) : gK,$$

then K will be the center of gravity of $A + B + C$ and D, and consequently the common center of gravity of the four bodies A, B, C, D; moreover the pressure upon K will be the same as if $A + B + C + D$ were placed at K; and thus we might proceed for any number of bodies.

It is evident that the foregoing demonstration does not at all depend upon the number or weight of the bodies, or their distance from each other; it rests merely on the supposition that their centers of gravity are connected together by inflexible rods void of gravity. It may therefore be applied to any number of particles of matter situated either in the same or in different planes, and placed at all possible distances from each other. Increase the number of these particles till they become contiguous to each other, and for the imaginary line void of gravity substitute the power of cohesion; then the system of bodies, A, B, C, D, &c., may represent an irregular mass of compact matter, not unlike such as are to be met with in the works of nature or of art; and although it may be difficult to find the actual center of gravity of such a mass, yet the proposition still remains true, that if this mass be supported, its pressure downward will be the same as if the whole quantity of matter contained in it were concentrated in its center of gravity.

76. *Rectilinear plane figures.*—The center of gravity of a *triangle is one-third of the distance from the middle of a side to the opposite angle.* Bisect AC in D (Fig. 48), and BC in E; join AE, BD, and DE. BD bisects all lines parallel to AC; therefore the center of gravity of all those lines united, that is, of the triangle, is in BD. For the same reason it is in AE; and therefore at their intersection, G. Since EC$=\frac{1}{2}$BC, and DC$=\frac{1}{2}$AC, $\therefore$ ED$=\frac{1}{2}$AB. But EGD and AGB are similar;

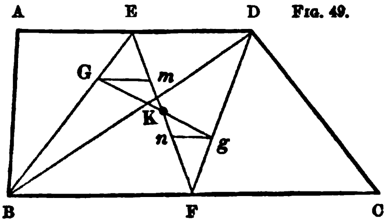
FIG. 48.



$$\therefore \text{DG} : \text{BG} :: \text{DE} : \text{AB} :: 1 : 2;\ \therefore \text{DG} = \tfrac{1}{2}\text{BG} = \tfrac{1}{3}\text{BD}.$$

77. The center of gravity of a *trapezoid* is in the line which bisects the parallel sides; and it divides that line, in the ratio of *twice the longer plus the shorter, to the longer plus twice the shorter.* In Fig. 49, bisect AD in E, and BC in F; join EF, BE, BD, and DF. The center of gravity is in EF, because it bisects all lines parallel to the bases. Make EG$=\frac{1}{3}$EB, and F$g=\frac{1}{3}$FD; then G is the center of gravity of ABD, and g, of BCD; hence the center of gravity of the trapezoid is in Gg, and therefore at K, the intersection of EF and Gg. It remains to find the ratio of EK to KF.

FIG. 49.



Draw Gm, gn, parallel to AD or BC; then since EG$=\frac{1}{3}$BE, E$m=\frac{1}{3}$EF; and for the same reason F$n=\frac{1}{3}$EF $\therefore$ E$m=mn$

$= n$F. Now K being the common center of gravity of the triangles ABD, BDC,

$$\text{GK} : \text{K}g :: \text{BDC} : \text{ABD} :: \text{BC} : \text{AD}.$$

But

$$\text{GK} : \text{K}g :: \text{K}m : \text{K}n;$$
$$\therefore \text{K}m : \text{K}n :: \text{BC} : \text{AD};$$

add and subtract,

$$\text{K}m + \text{K}n : \text{K}m - \text{K}n :: \text{BC} + \text{AD} : \text{BC} - \text{AD};$$
$$\therefore 3\text{K}m + 3\text{K}n : \text{K}m - \text{K}n :: 3\text{BC} + 3\text{AD} : \text{BC} - \text{AD};$$

add and subtract,

$$4\text{K}m + 2\text{K}n : 2\text{K}m + 4\text{K}n :: 4\text{BC} + 2\text{AD} : 2\text{BC} + 4\text{AD};$$
$$\therefore 2\text{K}m + \text{K}n : \text{K}m + 2\text{K}n :: 2\text{BC} + \text{AD} : \text{BC} + 2\text{AD}.$$

But it is apparent from the construction of the figure, that

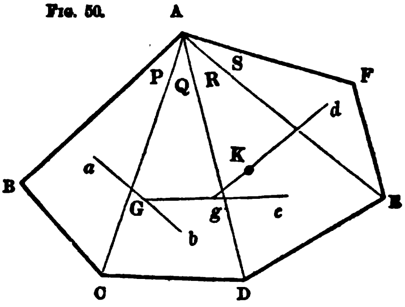
$$2\text{K}m + \text{K}n = \text{EK}, \text{ and } \text{K}m + 2\text{K}n = \text{KF};$$
$$\therefore \text{EK} : \text{KF} :: 2\text{BC} + \text{AD} : \text{BC} + 2\text{AD}.$$

When $\text{AD} = 0$, then the figure becomes a triangle, and $\text{EK} : \text{FK} :: 2\text{BC} : \text{BC}$; that is, $\text{FK} = \frac{1}{3}\text{EF}$, as was found by a different process in Art. 76.

When $\text{AD} = \text{BC}$, the figure becomes a parallelogram, and $\text{EK} : \text{FK} :: 3\text{BC} : 3\text{BC}$; consequently the center of gravity is in the center of the figure, as was shown in Art. 72.

78. To find the center of gravity of an *irregular polygon*, divide it into triangles by drawing diagonals, and then proceed according to principles already given.

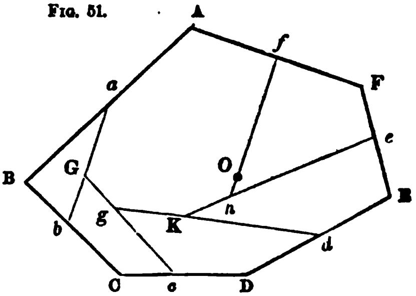
FIG. 50.



Let ABCDEF (Fig. 50), be an irregular polygon, divided into triangles, whose areas are represented by P, Q, R, S, and whose centers of gravity are respectively *a*, *b*, *c*, *d*. Conceive these triangles to be collected in the points *a*, *b*, *c*, *d*; join *ab*, and take $b\text{G} : a\text{G} :: \text{P} : \text{Q}$, then G will be the center of gravity of the figure ABCD. Join G*c*, and take $cg : \text{G}g :: \text{P} + \text{Q} : \text{R}$; then *g* will be the center of gravity of the figure ABCDE. Let *g* and *d* be joined, and make $d\text{K} : g\text{K} :: \text{P} + \text{Q} + \text{R} : \text{S}$, then K will be the center of gravity of the whole polygon; and so we might proceed, whatever be the number of sides.

79. *Perimeters of plane figures.*—The center of gravity of one side of a rectilinear figure is in its middle point (Art. 71).

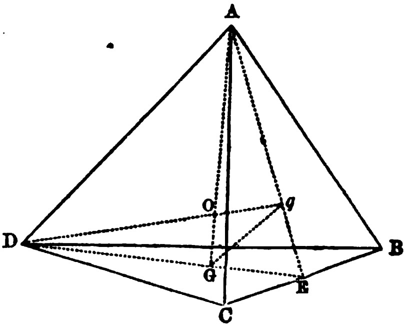
FIG. 51.



If the figure is a regular polygon, it is easily shown that the center of gravity of the whole perimeter coincides with the center of magnitude. If it is irregular, it must be found by successive proportions, as follows: In Fig. 51, having found *a*, *b*, *c*, &c., the centers of the several sides, join *ab*, and take *b*G : *a*G : : AB : BC; then G would be the center of gravity of that part of the perimeter represented by ABC. Again, join G*c*, and take *cg* : G*g* : : AB + BC : CD, then *g* is the center of gravity of such part of the perimeter as is represented by ABCD; and so we might proceed till we had found the center of gravity of the whole perimeter.

80. *Pyramid and cone.—The center of gravity of a triangular pyramid is in the line joining the vertex and the center of gravity of the base, at one-fourth of its length from the base.*

FIG. 52.



Let G be the center of gravity of the base BDC; and *g* that of the face ABC. The line AG passes through the center of gravity of every lamina parallel to DBC, on account of the similarity and similar position of all those laminæ; ∴ the center of gravity of the pyramid is in AG. For a similar reason, it is in D*g*; and therefore at their intersection, O. Now EG = $\frac{1}{3}$ED, and E*g* = $\frac{1}{3}$EA; hence, by similar triangles, *g*G = $\frac{1}{3}$AD. But G*g*O and AOD are also similar; ∴ GO = $\frac{1}{3}$AO = $\frac{1}{4}$AG.

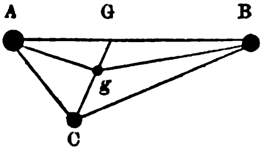
From this it is readily proved, that the center of gravity of *every* pyramid and cone is one-fourth of the distance from the center of the base to the vertex.

The determination of the center of gravity in curvilinear forms, whether lines, surfaces, or solids, generally requires the aid of the calculus. The nature of the process, with a few examples, will be presented at the close of this chapter.

81. *Calculation of the place of the center of gravity.*—If the weights of three bodies are given, and their distances from each other respectively, the place of their common center of gravity is computed by trigonometry.

1. Three bodies, A, B, C (Fig. 53), weighing respectively 3, 2, and 1 pounds, have their centers of gravity joined by the lines AB, BC, CA; of which AB = 5 feet, BC = 4, CA = 2; what is the distance of their common center of gravity from the body C?

Fig. 53.



From the given sides of the triangle ABC, calculate the angles. A will be found equal to 49° $27\frac{1}{2}'$. Find G, the center of gravity of A and B by the proportion A + B : B : : AB : AG = 2 feet. The computation of CG is simplified, because we find AGC to be isosceles. It is found to be 1.673. Considering A and B to be collected at G, we next find C*g* = 1.394.

By a similar process, we could find the center of gravity between *g* and a *fourth* body in the same plane, if its weight and position were given; then between that center and a fifth, and so on.

2. If three equal bodies be placed at the angles of any triangle; show that the common center of gravity of those bodies is in the same point with the center of gravity of the triangle.

3. The bodies A, B, C, weighing respectively 5, 3, and 12 pounds, are so placed that AB = 8 feet, AC = 4 feet, and the angle BAC is a right angle; what is the distance of their common center of gravity from the body C? *Ans.* 2 feet.

4. Find the center of gravity of five equal heavy particles placed at five of the angular points of a regular hexagon.

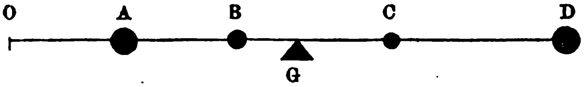
Ans. $\frac{1}{5}$ of the distance from the center to the third particle.

5. A regular hexagon is bisected by a line joining two opposite angles; where is the center of gravity of one-half?

Ans. $\frac{4}{9}$ of the distance from the center to the middle of the second side.

82. *Center of gravity of a system of bodies referred to a point or a plane.*—If several bodies are in a straight line, their common center of gravity may be referred to a point in that line; and its distance from that point is obtained by *multiplying each weight into its own distance from the same point, and dividing the sum of the products by the sum of the weights.* Let A, B, C, and D, represent the weights of several bodies, whose centers of gravity are in the straight line, OD (Fig. 54). Required the

Fig. 54.



distance of their common center of gravity from any point O assumed in the same line. Let G be their common center of gravity, then the moments of A and B must be equal to the opposing moments of C and D with reference to the point G (Art. 74). That is,

$$A \times AG + B \times BG = C \times CG + D \times DG;$$

$$A \times (OG - OA) + B \times (OG - OB) = C \times (OC - OG) + D \times (OD - OG);$$

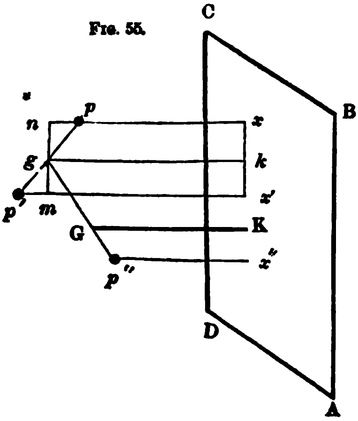
$$\therefore A \times OG + B \times OG + C \times OG + D \times OG = A \times OA + B \times OB + C \times OC + D \times OD.$$

$$\text{Hence } OG = \frac{A \times OA + B \times OB + C \times OC + D \times OD}{A + B + C + D}.$$

83. If the bodies are not in a straight line, they may be referred to a plane, which is assumed at pleasure. The distance of their common center of gravity from that plane is expressed as before: *multiply each weight into its own distance from the plane, and divide the sum of the products by the sum of the bodies.*

Let p, p', p'' (Fig. 55), represent the weights of several bodies, whose centers of gravity are at those points respectively, and let ABCD be the plane of reference. Join pp', and let g be the common center of gravity of p and p'; draw px, gk, $p'x'$ at right angles to the plane ABCD, and consequently parallel to each other; join xx', and since the points p, g, p' are in a straight line, the points x, k, x' will also be in a straight line, and therefore xx' will pass through k. Join gp'', and let G be the common center of gravity of p, p', p''; draw GK, $p''x''$, perpendicular to the plane; and through g draw mn parallel to xx' meeting px produced in n.

FIG. 55.



Now $p : p' :: p'g : pg ::$ (by sim. triangles) $p'm : pn$;

$$\therefore p \times pn = p' \times p'm, \text{ or}$$

$$p \times (nx - px) = p' \times (p'x' - mx');$$

$$\text{but} \quad nx = gk = mx',$$

$$\therefore p \times (gk - px) = p' \times (p'x' - gk),$$

$$\text{and } (p + p') \times gk = p \times px + p' \times p'x'$$

$$\therefore gk = \frac{p \times px + p' \times p'x'}{p + p'};$$

for the same reason, if $p + p'$ is placed at g, we have

$$GK = \frac{(p + p') \times gk + p'' \times p''x''}{(p + p') + p''} = \frac{p \times px + p' \times p'x' + p'' \times p''x''}{p + p' + p''};$$

and thus we might proceed, whatever be the number of bodies.

1. Four bodies, A, B, C, D, weighing respectively 2, 3, 6, and 8 pounds, are placed with their centers of gravity in a right line, at the distance of 3, 5, 7, and 9 feet from a given point: what is the distance of their common center of gravity from that given point; and between which two of the bodies does it lie? *Ans.* Between C and D; and its distance from the given point $7\frac{2}{19}$ feet.

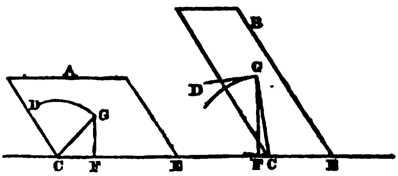
2. There are five bodies, weighing respectively 1, 14, $21\frac{1}{2}$, 22, and $29\frac{1}{2}$ pounds; a plane is assumed passing through the last body, and the distances of the other four from the plane are respectively 21, 5, 6, and 10 feet; how far from the plane is the common center of the five bodies? *Ans.* 5 feet.

84. *Support of a body.*—A body can not rest on a smooth plane, unless it is horizontal; for the pressure on a plane (Art. 69), can not be balanced by the resistance of that plane, except when perpendicular to it; therefore, as the force of gravity is vertical, the resisting plane must be horizontal.

The *base of support* is that area on the horizontal plane which is comprehended by lines joining the extreme points of contact. If there are *three* points of contact, the base is a triangle; if *four*, a quadrilateral, &c.

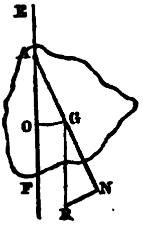
When the vertical through the center of gravity (called *the line of direction*) falls within the base, the body is supported; if without, it is not supported. In the body A (Fig. 56), the force of gravity acts in the line GF, and there are lines of resistance on both sides of GF, as GC and GE, so that the body can not turn on the edge of the base, without *rising* in the arc whose radius is GC. But, in the body B, there is resistance only on one side; and therefore, if GF be resolved on GC and a perpendicular to it, the body is not prevented from moving in the direction of the latter, that is, in the arc whose radius is GC.

FIG. 56.



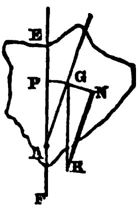
85. *Stable, unstable, and neutral equilibrium.*—If the base is reduced to a *line* or *point*, then, though there is support, there is no *firmness* of support; the body will be moved by the least force. Let the body (Fig. 57) be suspended by the pivot A. While the center of gravity is in the vertical EAF, the force of gravity is counterbalanced by the strength of A. But suppose the body to be disturbed, and left with its center

FIG. 57.



at G. Let GR represent the force of gravity, and resolve it into GN acting from A, and NR perpendicular to it; GN is resisted by the strength of A, and NR moves the body in the arc whose radius is GA. Hence the body swings toward O, and passes beyond it, till gravity destroys its motion. When it comes to rest, let gravity again be resolved, and we shall find a component acting, as before, toward AF. Thus the body vibrates till the motion is destroyed by obstructing forces.

Fig. 58.

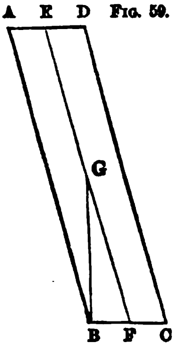


But the body, in the position (Fig. 58), will not thus return to P the place of support. For, resolve GR into GN, NR; then the former acts away from P, instead of toward it. Hence, if the center of gravity should be left at P, and then suffer the least disturbance, it could never return to that position again.

Fig. 57 presents a case of *stable equilibrium*, and Fig. 58 a case of *unstable equilibrium*. Whatever may be the mode of support, a stable equilibrium occurs, if the center, on being disturbed in either direction, begins to *rise;* for then it will return. But, if it begins to *descend* each way from the place of support, the equilibrium is *unstable*, since gravity will only make it descend further. The equilibrium is called *neutral*, if the center of gravity, when disturbed, neither ascends nor descends, as in the case of a uniform sphere on a horizontal plane.

86. *Questions.*—1. A frame 20 feet high, and 4 feet in diameter, is racked into an oblique form (Fig. 59), till it is on the point of falling; what is its inclination to the horizon? *Ans.* 78° 27′ 47″.

Fig. 59.



2. A stone tower of the same dimensions as the former, is inclined till it is about to fall, but preserves its rectangular form; what is its inclination? *Ans.* 78° 41′ 24″.

3. A cube of uniform density lies on an inclined plane, and is prevented by friction from sliding down; to what inclination must the plane be tipped, that the cube may just begin to roll down? *Ans.* 45°.

4. The density of a sphere not being uniform, the center of gravity does not coincide with the center of magnitude; how shall it be placed on a horizontal plane, so as to have a stable, and how, so as to have an unstable equilibrium?

87. ***Motion of the center of gravity, arising from the motions of the bodies.***—When the bodies of a system are in motion, the center of gravity of the system may also be in motion, or it may be at rest. If the motions of the bodies arise entirely from their influence on each other, their common center of gravity will not be disturbed. This follows from the law of equal action and reaction, and is proved as follows.

Let G be the center of gravity of A and B (Fig. 60); and first suppose the bodies to *attract* each other, A moving over Aa, while B moves over Bb. Since the momenta are equal,

$$A \times Aa = B \times Bb.$$

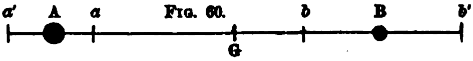


FIG. 60.

But, as the bodies have equal moments relatively to G (Art. 74),

$$A \times AG = B \times BG;\ \therefore$$

subtract first from second, $A \times aG = B \times bG$;

therefore G is still the center of gravity.

If the bodies *repel* each other, A moving over Aa', and B over Bb', in equal times, then, their momenta being equal, we have as before,

$$A \times Aa' = B \times Bb';$$

but (Art. 74) $A \times AG = B \times BG$;

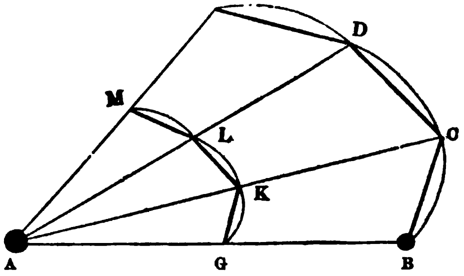
adding equations, $A \times a'G = B \times b'G$;

and the center still remains at rest.

But if any of the bodies of a system are in motion by means of foreign forces, which act upon them, their common center of gravity will generally be in motion also.

88. *One body at rest, and one in motion.*—If one body remain at rest, while another is made to describe any path, then the common center of the two will describe a *similar* path; and its velocity will be to that of the moving body, as the weight of the moving body is to that of both bodies.

Let A (Fig 61) remain at rest, while B describes the straight lines, BC, CD, DE, &c.; the center G will in the same time describe the similar series, GK, KL, LM, &c. When B is in the position B, and the center at G, AG : AB : : B : A+B; when B is at C, AK : AC : : B : A+B; ∴ AG : AB : : AK : AC. Hence GK is parallel to BC; and GK : BC : : B : A+B. In like manner KL : CD : : B : A+B; and LM : DE : : B : A+B. Thus all the lines of one path are

FIG. 61.


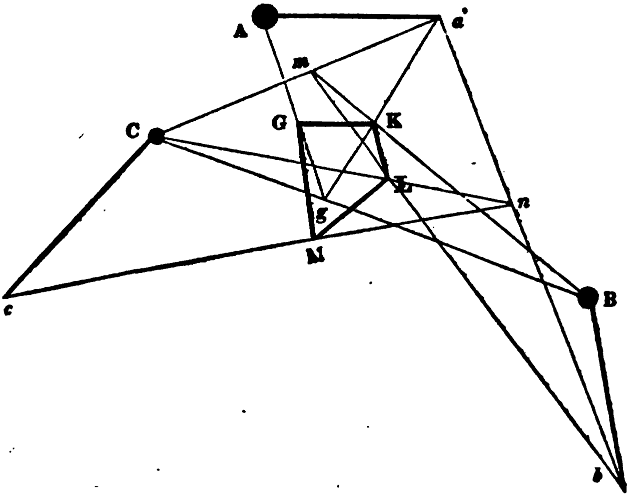
parallel to the corresponding ones of the other, and have a constant ratio to them. Therefore the paths are similar. As the several lines of the two paths are described in equal times respectively, their lengths are as the velocities. But the lengths are proved to be as $B : A + B$; therefore, the velocity of the common center is to that of the moving body, as the moving body to the sum of the bodies.

The same reasoning is applicable to the curved paths GKLM and BCDE.

89. *All the bodies in motion.*—When the bodies of a system are in motion, their common center of gravity will move in the the same manner as if a body equal to the sum of the bodies were placed in that point, and the same motions were communicated to it as are communicated to the bodies separately.

Let us take the case of three bodies, A, B, C (Fig. 62), moving with uniform velocities, in equal successive parts of time, through the spaces Aa, Bb, Cc. Let G be the position of the common center of gravity of the three bodies, and g that of B and C, before they begin to move; then (Art. 75),

FIG. 62.



$$Gg : Ag :: A : A + B + C.$$

While A moves from A to a, $B + C$ may be considered as at rest in g, therefore (Art. 88) the common center of gravity G will in the same time describe GK parallel to Aa, and

$$GK : Aa :: Gg : Ag :: A : A + B + C.$$

When A has arrived at a, join aC and BK; produce BK till it meets *a*C in *m*, then *m* will be the center of gravity of A and C;* join *mb*, then while B moves from B to *b*, the common center of gravity will describe KL parallel to B*b*, and

$$\mathrm{KL} : \mathrm{B}b :: m\mathrm{K} : m\mathrm{B} :: \mathrm{B} : \mathrm{A} + \mathrm{B} + \mathrm{C}.$$

When B has arrived at *b*, join *ab*, CL; produce CL till it meets *ab* in *n*, then *n* will be the center of gravity of A and B; join *nc*, then while C moves from C to *c*, the common center of gravity will describe LM parallel to C*c*, and

$$\mathrm{LM} : \mathrm{C}c :: n\mathrm{L} : n\mathrm{C} :: \mathrm{C} : \mathrm{A} + \mathrm{B} + \mathrm{C}.$$

While the bodies A, B, C, therefore, in equal successive parts of time, move uniformly through the spaces A*a*, B*b*, C*c*, their common center of gravity will in the same time describe the polygon GKLM, whose sides GK, KL, LM, are respectively parallel to A*a*, B*b*, C*c*, and bear to them the ratio of A, B, and C to A + B + C.

90. If, instead of moving in successive intervals of time, the three bodies A, B, C, were all to begin to move at the same instant, and describe the lines A*a*, B*b*, C*c*, cotemporaneously; let us then consider what effect would be produced upon their common center of gravity. Now since GK, A*a*, are described in the same time, calling the velocity of the common center of gravity v, and that of the body A, V, then $v : \mathrm{V} :: \mathrm{GK} : \mathrm{A}a :: \mathrm{A} : \mathrm{A} + \mathrm{B} + \mathrm{C}$; hence $\mathrm{A} \times \mathrm{V} = (\mathrm{A} + \mathrm{B} + \mathrm{C}) \times v$; i. e., the momentum of A is equal to the momentum of a body equal to A + B + C moving with the velocity v; the same force which causes the body A to move over A*a*, would in the same time cause a body equal to A + B + C to move over GK. For the same reason, the forces which impel B and C over B*b*, C*c*, are such as would in the same time cause a body equal to A + B + C to move over KL, LM. Hence it appears that the motion of the common center of gravity along the sides of the polygon GKLM, is analogous to the motion of a body equal to A + B + C acted upon by three forces which would carry it over GK, KL, LM in the same time that they would carry the bodies A, B, C over the spaces A*a*, B*b*, C*c*, respectively. But a body acted upon at once by these forces would (Art. 48) describe the other side GM of the polygon GKLM in the same time that it would describe either of the sides GK, KL, LM, when the forces act separately; if the bodies A, B, C, therefore, move cotemporaneously, their common center of gravity will describe the line GM, while the bodies themselves describe the three lines Aa, Bb, Cc,* and the same reasoning is applicable to any number of bodies.

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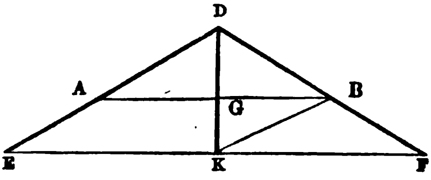
* For, when A moves to *a*, the center of gravity of A and C is somewhere in the line C*a*. But when A moves to *a*, the center of gravity of the three bodies moves to K; therefore the center of gravity of A and C must also be in the line BK produced, since it must be such a point that A and C, when placed there, shall balance B. It must therefore be in the intersection of C*a* and BK, or at *m*.

91. Hence, in the first place, if the bodies which compose a system move uniformly in right lines, then their common center of gravity will either remain at rest, or will move uniformly in a right line; for if a body equal to the sum of the bodies were placed in that center, and then acted upon by the same forces which cause the bodies to move separately in right lines, it would either remain at rest (viz., when the forces counteract each other), or would describe uniformly the remaining side of a polygon, whose other sides represent the quantity and direction of the several forces acting upon it. In the second place, the common center of gravity of the system will not be affected by the mutual action of the bodies upon each other; for action and reaction being equal, the effect produced upon the common center of gravity by such mutual action will only be that of two equal and opposite forces acting upon a body equal to the sum of the bodies placed in that center; which would evidently not disturb its state, either of motion or quiescence. Lastly, if the motion of the bodies in these right lines were to cease, and they were left to the mutual attraction of each other, then their common center of gravity would remain at rest, and the bodies would approach each other, in lines drawn to it from their respective centers of gravity, and all collect together in that common center.

92. *Calculation of the motion of the center of gravity.*—

1. Let two *equal* bodies, A and B (Fig. 63), move from the point D with the same uniform velocity, along DE and DF, which make a given angle EDF with each other; in what line, and with what relative velocity, does their common center of gravity move?

FIG. 63.



From the conditions, it is obvious that the path of the center of gravity DK bisects all the lines AB, &c., which join the bodies at the same moments of time. Hence, DF : DK : : rad : cos FDK, which is half the given angle. Thus DK is known in direction and length.

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* We have here supposed the bodies A, B, C, to keep their centers of gravity in the same plane; in which case it is evident that the motion of their common center of gravity will be in the same plane. If the motion of the bodies be in different planes, then the value of each line GK, KL, LM, might be found as before; but as they will then lie in different planes, the resulting quantity GM must be ascertained according to the principles laid down in Articles 42 and 48.

Or, in another way; suppose B first describes DF, and then A describes DE; the successive motions of the center are DB $(=\frac{1}{2}DF)$, and BK $(=\frac{1}{2}DE)$, parallel to the lines described by the bodies (Art. 88); DK is then computed in the triangle DBK.

2. *The foregoing question generalized.*—The given bodies are unequal, and their given uniform velocities unequal, and the angle EDF given.

After a given time has elapsed, DF and DE are known, and the angle between them; from which EF and the angle F are determined. As the bodies are given, the position of the center K, between them, is obtained; hence, in the triangle DFK, DF, FK, and angle F, are known, to find DK, the path of the center, both in length and direction.

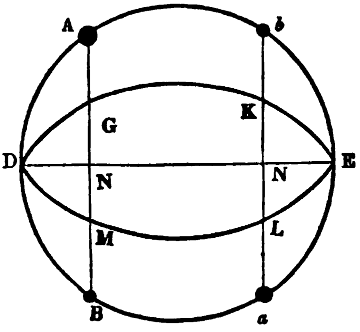
3. Two bodies, A and B, begin to move in opposite directions at the same instant from the extremity D of the diameter DE of the circle DAEB, and continue to move on with the same uniform velocity till they meet in E; they pass each other at E, and then continue to move on till they arrive at the point D, whence they set off; what is the course of the common center of gravity during this revolution of the two bodies?

Suppose the bodies arrived at the position AB (Fig. 64), then since DA = DB, the line AB will be bisected by DE in N. Let G be the common center of gravity of A and B, then

$$\begin{aligned} A : B &:: BG : AG; \\ \therefore\ A+B : A-B &:: BG+AG : BG-AG; \\ &:: \quad AB \quad : \quad GM; \\ &:: \quad AN \quad : \quad GN. \end{aligned}$$

Therefore AN, the ordinate of the circle, is to GN, the corresponding ordinate of the figure described by the center, always in the same constant ratio, of the sum of the bodies to their difference. But this is a property of the ellipse, that, when its axis is the diameter of a circle, the corresponding ordinates of the two figures are in a constant ratio. Hence the center of gravity of A and B describes an ellipse, while they move, in the manner before stated, round the circle.

FIG. 64.



If the bodies approach equality, their difference grows less, and therefore the ellipse more eccentric, till, when the bodies are equal, the path of the center is a straight line, as it evidently should be, in order to bisect the chords AB, &c.

4. Three bodies, A, B, C, at the same instant begin to move uniformly from the three angles of a given triangle, and in the same time change places in the direction ABC: how will their common center of gravity be affected by this motion of the bodies?

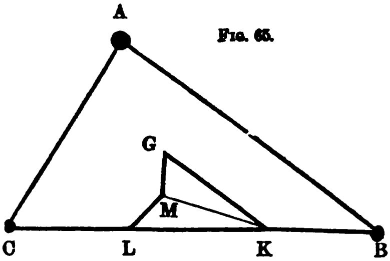


Fig. 65.

Let G (Fig. 65) be their common center of gravity, and suppose the bodies first to move in succession. While A moves from A to B, their common center of gravity will (by Art. 88) describe GK parallel to AB, and

$$GK : AB :: A : A+B+C, \therefore GK = \frac{A \times AB}{A+B+C}.$$

While B moves from B to C, the center of gravity will describe $KL = \frac{B \times BC}{A+B+C}$; and if LM be drawn parallel to CA, and equal to $\frac{C \times AC}{A+B+C}$, while C moves from C to A, the center of gravity will describe LM. Suppose now the bodies to move cotemporaneously, then their common center of gravity will describe GM (the remaining side of the polygon GKLM), while the bodies change places in the direction ABC.

To find the actual value of GM, we have KL, LM, and the angle KLM (= ACB) given, from which MK and MKL may be found; but GKM = GKL (or ABC) − MKL; in the triangle GKM there are therefore given GK, KM, and GKM, from which GM may be determined.

If $A = B = C$, then $GK = \frac{1}{3}AB$, $KL = \frac{1}{3}BC$, and $LM = \frac{1}{3}AC$, $\therefore$ GK, KL, LM, are to each other as AB, BC, AC; and since the angles GKL, KLM are respectively equal to ABC, BCA, the three lines GK, KL, LM will form a triangle similar to the triangle ABC. GM, therefore, in this case is equal to 0, and the body remains at rest. This also follows from the general theorem in Art. 89; for the common center of gravity, being under the same circumstances as a body acted upon by three forces which are to each other as the three sides of a triangle taken in order, will remain at rest.

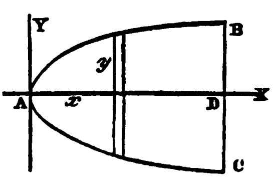
DETERMINATION OF THE CENTER OF GRAVITY BY MEANS OF THE CALCULUS.

93. In order to apply the processes of the calculus to the determination of the center of gravity, the principle is used, which was proved (Art. 83), that if every particle of a body be multiplied by its distance from a plane, and the sum of the products be divided by the sum of the particles, the quotient is the distance of the common center from the same plane.

The product of any particle or body by its distance from the plane, is conveniently called its *moment* with respect to that plane.

94. *General formulæ.*—Let BAC (Fig. 66) be any symmetrical curve, having AX for its axis of abscissas, and AY, at right angles to it, for its axis of ordinates. It is obvious that the center of gravity of the line BAC, of the area BAC, of the solid of revolution around the axis AX, and of the surface of the same solid, are all situated on AX, on account of the symmetry of the figure. It is proposed to find the formula for the distance of the center from AY, in each of these cases.

FIG. 66.



Let G in every instance represent the distance of the general center of gravity from the axis AY, or the plane AY, at right angles to AX. The distance G would plainly be the same for the *half* figure BAD, as for the whole BAC; expressions may therefore be obtained for either, according to convenience.

1. *The line* AB.—Let x be the abscissa, and y the ordinate; then $(dx^2+dy^2)^{\frac{1}{2}}$ is the differential of the line AB. For brevity, let $s=$ the line, and ds, its differential. If we now multiply this differential by its distance from AY, xds is the moment of a minute portion of the line; and the integral of it, $\int xds$, is the moment of the whole. Dividing this by the line itself, i. e., by s, we have $\frac{\int xds}{s}$ for the distance G.

2. *The area* BAD.—The differential of the area is ydx; the differential of its moment is $xydx$; hence the moment itself is $\int xydx$; and the distance $G=\frac{\int xydx}{\text{area}}$.

3. *The solid of revolution.*—The differential of the solid, generated by the revolution of AB on AX, is πy^2dx; the dif-

ferential of its moment is πxy^2dx; and the moment is $\int \pi xy^2dx$; hence the distance $G = \frac{\int \pi xy^2dx}{\text{solid}}$.

4. *The surface of revolution.*—The differential of the surface is $2\pi yds$; the differential of its moment is $2\pi xyds$; and therefore the moment is $\int 2\pi xyds$; and the distance $G = \frac{\int 2\pi xyds}{\text{surface}}$.

95. *Application of formulæ.*—We proceed to determine the center of gravity in a few cases, by the aid of these formulæ.

1. *A straight line.*—Imagine the line placed on AX, with one extremity at the origin A. The moment of a minute part of it is xdx, and that of the whole is $\int xdx$, while the length of the whole is x; $\therefore\ G = \frac{\int xdx}{x} = \frac{\frac{1}{2}x^2 + C}{x} = \frac{1}{2}x$, as it evidently should be. In all the cases considered here, $C = 0$, because the function vanishes when x does.

2. *The arc of a circle.*—By Formula 1st we have $G = \frac{\int xds}{s}$; but $ds = (dx^2 + dy^2)^{\frac{1}{2}}$; by the equation of the circle, $y^2 = 2ax - x^2$;

$$\therefore\ ydy = (a - x)\,dx;\ \therefore\ dy^2 = \frac{(a - x)^2dx^2}{y^2} = \frac{(a - x)^2dx^2}{2ax - x^2};$$

$$\therefore\ (dx^2 + dy^2)^{\frac{1}{2}} = \frac{adx}{(2ax - x^2)^{\frac{1}{2}}};$$

$\therefore \frac{\int xds}{s} = \int \frac{x}{s} \times \frac{adx}{(2ax - x^2)^{\frac{1}{2}}} = \frac{a}{s}\int \frac{xdx}{(2ax - x^2)^{\frac{1}{2}}} = \frac{a}{s}\left\{ \text{vers}^{-1}x - (2ax - x^2)^{\frac{1}{2}} \right\} = \frac{a}{s}(s - y) = a - \frac{ay}{s} = a - \frac{ac}{t}$, if the arc is doubled and called t, and c (chord) put for $2y$. As $a - \frac{ac}{t}$ is the distance from the origin A, and a = radius of the arc; $\therefore$ the distance from the center of the circle to the center of gravity of the arc, is $\frac{ac}{t}$, which is a fourth proportional to the *arc*, the *chord*, and the *radius*.

3. *The area of a parabola.*—The equation of the curve is

$$y^2 = px, \text{ or } y = p^{\frac{1}{2}}x^{\frac{1}{2}};$$

therefore the Formula 2 for moment,

$$\int xydx = \int p^{\frac{1}{2}}x^{\frac{3}{2}}dx = \tfrac{2}{5}p^{\frac{1}{2}}x^{\frac{5}{2}}\ (+ C = 0);$$

but the area of the half parabola $= \frac{2}{3}p^{\frac{1}{2}}x^{\frac{3}{2}}$;

$$\therefore \text{G} = \tfrac{2}{5}p^{\frac{1}{2}}x^{\frac{5}{2}} \div \tfrac{2}{3}p^{\frac{1}{2}}x^{\frac{3}{2}} = \tfrac{3}{5}x.$$

To find the distance of the center of gravity of the semi-parabola from the axis AX, proceed as follows: the differential of the area, as before, equals ydx; and the distance of its center from AX is $\frac{1}{2}y$. Therefore its moment with respect to AX is $\frac{1}{2}y^2dx = \frac{1}{2}pxdx$; and the moment of the whole is $\int \frac{1}{2}pxdx = \frac{1}{4}px^2$; $\therefore$ the distance of the center from

$$\text{AX} = \tfrac{1}{4}px^2 \div \tfrac{2}{3}p^{\frac{1}{2}}x^{\frac{3}{2}} = \tfrac{3}{8}p^{\frac{1}{2}}x^{\frac{1}{2}} = \tfrac{3}{8}y.$$

4. *The area of a circular segment.*—The equation of the circle is, $y = (2ax - x^2)^{\frac{1}{2}}$. Therefore (Formula 2),

$$\int xydx = \int x\,(2ax - x^2)^{\frac{1}{2}}dx.$$

Add and subtract $a\,(2ax - x^2)^{\frac{1}{2}}dx$, and it becomes

$$\int a\,(2ax - x^2)^{\frac{1}{2}}dx - \int (a - x)\,(2ax - x^2)^{\frac{1}{2}}\,dx =$$

$$a\int ydx - \frac{(2ax - x^2)^{\frac{3}{2}}(a - x)dx}{\frac{3}{2}(2a - 2x)dx} = a\,.\,\text{area ABD} - \tfrac{1}{3}(2ax - x^2)^{\frac{3}{2}}.$$

$$\therefore \text{G} = a - \frac{(2ax - x^2)^{\frac{3}{2}}}{3\text{ area ABD}}.$$

When $x = a$, $\text{G} = a - \frac{4a}{3\pi}$; and the distance of the center of gravity of a semicircle from the center of the circle $= \frac{4a}{3\pi}$. When $x = 2a$, $\text{G} = a$, as it plainly should be.

5. *A spherical segment.*—The equation of the circle is $y^2 = 2ax - x^2$. Therefore (Formula 3),

$$\int \pi xy^2dx = \int \pi xdx(2ax - x^2) = \int 2a\pi x^2dx - \int \pi x^3dx = \tfrac{2}{3}a\pi x^3 - \tfrac{1}{4}\pi x^4;$$

$$\therefore \text{G} = \frac{\frac{2}{3}a\pi x^3 - \frac{1}{4}\pi x^4}{a\pi x^2 - \frac{1}{3}\pi x^3} = \frac{8ax - 3x^2}{12a - 4x}.$$

When $x = a$, $\text{G} = \frac{5}{8}a$; that is, the center of gravity of a hemisphere is $\frac{5}{8}$ of radius from the surface, or $\frac{3}{8}$ of radius from the center of the sphere. If $x = 2a$, $\text{G} = a$.

6. *A right cone.*—In this case AB (Fig. 66) is a straight line, and its equation is $y = ax$, where a is any constant.

$$\therefore y^2 = a^2x^2;\ \therefore \int \pi xy^2dx = \int \pi a^2x^3dx = \frac{\pi}{4}a^2x^4;\ \therefore \text{G} = \frac{\frac{1}{4}\pi a^2x^4}{\frac{1}{3}\pi a^2x^3} = \tfrac{3}{4}x.$$

Hence the center of gravity of a cone is three-fourths of the axis from the vertex, as shown Art. 80.

7. *The convex surface of a right cone.*—The equation is

$$y=ax\,;\ \therefore\ dy^2=a^2dx^2\,;\ \text{and}\ (dx^2+dy^2)^{\frac{1}{2}}=(a^2+1)^{\frac{1}{2}}dx.$$

Therefore (Formula 4),

$$\int 2\pi xyds=\int 2\pi xy(dx^2+dy^2)^{\frac{1}{2}}=\int 2\pi ax^2(a^2+1)^{\frac{1}{2}}dx=\tfrac{2}{3}\pi ax^3(a^2+1)^{\frac{1}{2}}$$

= the moment of the surface. The surface itself,

$$=\pi y(x^2+y^2)^{\frac{1}{2}}=\pi ax^2(a^2+1)^{\frac{1}{2}}.\quad \therefore\ G=\frac{\frac{2}{3}\pi ax^3(a^2+1)^{\frac{1}{2}}}{\pi ax^2(a^2+1)^{\frac{1}{2}}}=\tfrac{2}{3}x.$$

The center of gravity of the convex surface of a right cone is on the axis, at a distance equal to two-thirds of its length from the vertex.

CHAPTER V.

THE COLLISION OF BODIES.

96. *Elastic and inelastic bodies.*—*Elastic* bodies are those which when compressed, or in any way altered in form, tend to return to their original state. Those which show no such tendency are called *inelastic* or *non-elastic.* No substance is known, which is entirely destitute of the property of elasticity; but some have it in so small a degree, that they are called inelastic, such as lead and clay. Elasticity is *perfect*, when the *restoring* force is equal to the *compressing* force. Air, and the gases generally, seem to be perfectly elastic; ivory, glass, and tempered steel, are imperfectly, though highly, elastic; and in different substances, the property exists in all conceivable degrees between the above-named limits.

The *perfection* or *imperfection* of elasticity has no reference to the *strength* or *energy* of the elastic force. If a body restores itself with a force equal to that by which it is compressed, it is called *perfectly elastic*, whether that force is small or great. Rarefied air, or a filament of glass, may be almost perfectly elastic, though they yield to the minutest force. The elasticity of a stick of timber is hundreds of thousands of times greater, and yet it is imperfect, since the restoring force is not fully equal to the compressing force. Experiments on collision are usually made with balls of the same density, moving in the line which joins their centers of gravity.

97. *Collision of inelastic bodies.*—Such bodies after impact always move as one mass; and the common velocity is equal to the *algebraic sum of the momenta divided by the sum of the bodies.*

Let A, B, represent the quantities of matter in two bodies, and a, b, their respective velocities. Considering a as positive, if B moves in the opposite direction, its velocity must be called $-b$. Let v be the common velocity after impact.

1. *Same directions.*—The momentum of A is Aa, that of B is Bb; and the momentum of the whole after collision is $(A+B)v$. But according to the third law of motion,

$$Aa+Bb=(A+B)v\,; \quad \therefore v=\frac{Aa+Bb}{A+B}.$$

If B is at rest before impact, $b=0$, and $v=\frac{Aa}{A+B}$.

To find the *loss* or *gain* of velocity for either body, multiply the *other* body by the *difference* of velocities, and divide by the sum of the bodies. For, A's velocity before impact was a, after impact it is $\frac{Aa+Bb}{A+B}$; therefore the *loss* $=a-\frac{Aa+Bb}{A+B}=\frac{B(a-b)}{A+B}$. But B's *gain* is the velocity after impact diminished by the velocity before, i. e., $\frac{Aa+Bb}{A+B}-b=\frac{A(a-b)}{A+B}$. When B is at rest, these expressions become

$$\frac{Ba}{A+B} \text{ for A's loss; } \frac{Aa}{A+B} \text{ for B's gain.}$$

2. *Opposite directions.*—Since b is negative, $v=\frac{Aa-Bb}{A+B}$.

To find *loss* or *gain* in this case, multiply the *other* body by the *sum* of the velocities, and divide by the sum of the bodies. For, A's *loss* $=a-\frac{Aa-Bb}{A+B}=\frac{B(a+b)}{A+B}$;

and B's *gain* $=\frac{Aa-Bb}{A+B}-(-b)=\frac{Aa-Bb}{A+B}+b=\frac{A(a+b)}{A+B}$.

In the case of opposite motions, the formula for v becomes zero, when $Aa=Bb$; but in that case, $A : B :: b : a$. Hence, if bodies which meet each other have velocities inversely as the quantities, they will *be at rest* after the collision.

1. A, weighing 3 oz., and moving 10 feet per second, overtakes B, weighing 2 oz., and moving 3 feet per second; what is the common velocity after impact? *Ans.* 7⅕ feet per second.

2. A weight of 7 oz., moving 11 feet per second, strikes upon another at rest weighing 15 oz.; required the velocity after impact? *Ans.* 3½ feet per second.

3. A weighs 4 and B 2 pounds; they meet in opposite directions, A with a velocity of 9, and B with one of 5 feet per second; what is the common velocity after impact?

Ans. $4\frac{1}{3}$ feet per second.

4. A = 7 pounds, B = 4 pounds; they move in the same direction, with velocities of 9 and 2 feet per second; required the velocity lost by A and gained by B? *Ans.* A $2\frac{6}{11}$, B $4\frac{5}{11}$.

5. A body moving 7 feet per second, meets another moving 3 feet per second, and thus loses half its momentum; what are the relative magnitudes of the two bodies?

Ans. A : B :: 13 : 7.

6. A weighs 6 pounds and B 5; B is moving 7 feet per second, in the same direction as A; by collision B's velocity is doubled; what was A's velocity before impact?

Ans. $19\frac{5}{6}$ feet per second.

98. *Collision of elastic bodies.*—In the impact of *elastic* bodies, the *loss* and *gain* are twice as great as they would be if the bodies were *inelastic*.

According to the definition of elasticity, the body restores itself with a force equal to that which compresses it; consequently as much momentum is exerted in the restitution as in the compression. In a given body, therefore, the velocity of restitution is equal to that of compression. Suppose, for example, that an inelastic ball A strikes upon another, B; then what B gains, A loses by reaction, and both bodies move on together; but when an elastic ball impinges on another, it not only loses the momentum which it at first imparted to B, but the bodies, in restoring themselves, exert a force equal to that of impact, and therefore destroy as much more of the motion of A, and double the effect of the impulse upon B.

Distinguishing the corresponding elastic body by an accent, since, the direction being the same, the velocity lost by A is

$$\frac{B\,(a-b)}{A+B};\quad \therefore \text{ that lost by } A' \text{ is } \frac{2B\,(a-b)}{A+B}.$$

Subtract this last from A′'s original velocity, and we have

$$a-\frac{2B\,(a-b)}{A+B}=$$

$$1.\quad \ldots\ldots\quad \frac{(A-B)\,a+2Bb}{A+B}.$$

So the velocity of B′ after impact is

$$2.\quad \ldots\ldots\quad \frac{(B-A)\,b+2Aa}{A+B}.$$

When the directions are opposite,

The velocity lost by $A = \frac{B(a+b)}{A+B}$. By $A' = \frac{2B(a+b)}{A+B}$.

Hence, velocity of A′ after impact $= a - \frac{2B(a+b)}{A+B} =$

3. . . . . $\frac{(A-B)a - 2Bb}{A+B}$.

And the velocity of B′ after impact is

4. . . . . $\frac{(A-B)b + 2Aa}{A+B}$.

99. *Equal elastic bodies.*—After the impact of *equal* elastic bodies, *each takes the original velocity of the other.* When A = B, Formula 1 is reduced to b; and Formula 2, to a; that is, A′ has B′'s former velocity, and B′ has A′'s. The same is true if they move in opposite directions. For, when A = B, Formula 3 becomes $-b$, which was B′'s original velocity, and Formula 4 becomes a, which was A′'s. Therefore, in the *opposite* motions of equal elastic bodies, collision causes *each* to rebound, since $+a$ is exchanged for $-b$, and $-b$ for $+a$.

If we reduce these four formulæ for the case in which A = B, and B is at rest, we find the same interchange of conditions; for 1 becomes 0, and 2 becomes a; so 3 becomes 0, and 4 becomes a.

100. *Unequal elastic bodies.*—If a *greater* elastic body impinges on a *less* one *at rest*, the impinging body still proceeds, though with a diminished velocity, and the other precedes it with a greater velocity than the impinging body first had. According to these conditions 1 becomes $\frac{(A-B)a}{A+B}$, which is positive, and therefore A′ proceeds. But Formula 2 becomes $\frac{2Aa}{A+B}$, which is more than a.

If a *less* body impinges on a *greater* one at rest, it rebounds, and the other goes forward with a less velocity than the impinging body had. For in this case, $\frac{(A-B)a}{A+B}$ is negative, and $\frac{2Aa}{A+B}$ is less than a.

101. *Series of elastic bodies.*—Let there be a row of equal elastic balls A, B, C, &c. . . X (Fig. 67), placed contiguous to each other; then (by Art. 99), if A is moved from its position and made to impinge upon B, it will rest after impact, and B will have a tendency to move on with A's velocity; after the impact of B upon C, it will remain at rest, and C have a tendency to move on with A's velocity; after the impact of C upon D, it will remain at rest, and D will have a tendency to move on with the same velocity; and so the motion will be propagated through the whole row, and the last body X will move forward with the velocity of A, all the others remaining at rest.

FIG. 67.

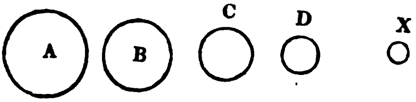


FIG. 68.

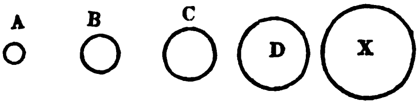


FIG. 69.

If the bodies decrease in magnitude (Fig 68), then, since A is greater than B, the velocity communicated to B will be greater than that of A; and the velocity communicated from B to C greater than that of B, &c; so that the last body will move forward in the direction of A's motion with a velocity much greater than that of A, and the other bodies will follow it in such a manner that the velocity of each body shall be greater than that of the body impinging on it. On the contrary, if the bodies increase in magnitude (Fig. 69), since A is less than B, the velocity communicated to B will be less than that of A, and A will be reflected back by B; for the same reason the velocity communicated from B to C will be less than that of B, and B will be reflected back by C; so that in this case all the bodies will move backward except the last, and that will move forward in the direction of A's original motion, but with a velocity much less than that of A.

If the bodies in Fig. 68 are in geometrical progression, the velocity of the first to that of the last is as $1 : \left(\frac{2}{1+r}\right)^{n-1}$.

Let the series be $A, Ar, Ar^2 \ldots . Ar^{n-1}$.

By Art. 90, when A impinges on B at rest the velocity communicated to B is $\frac{2Aa}{A+B} = \frac{2Aa}{A+Ar} = \frac{2a}{1+r} = b$.

Again, the velocity imparted to C is

$$\frac{2Bb}{B+C}=\frac{2Ar}{Ar+Ar^2}\times\frac{2a}{1+r}=\frac{2^2a}{(1+r)^2}.$$

Hence the successive velocities are a, $\frac{2a}{1+r}$, $\frac{2^2a}{(1+r)^2}$, &c., from which it appears that any term in the series is found by multiplying the original velocity by 2, raised to a power one less than the number of terms and divided by $1+r$ raised to the same power. Consequently, the last term is $\frac{2^{n-1}a}{(1+r)^{n-1}}$. Hence,

$$V_y \text{ of the first} : V_y \text{ of the last} :: a : \frac{2^{n-1}a}{(1+r)^{n-1}} :: 1 : \left(\frac{2}{1+r}\right)^{n-1}.$$

102. *Questions on elastic bodies.*—

1. A, weighing 10 lbs. and moving 8 feet per second, impinges on B, weighing 6 lbs. and moving in the same direction, 5 feet per second; what are the velocities of A and B after impact? *Ans.* A's = $5\frac{3}{4}$. B's = $8\frac{3}{4}$.

2. A : B :: 4 : 3; directions the same; velocities 5 : 4; what is the ratio of their velocities after impact? *Ans.* 29 : 36.

3. A, weighing 4 lbs., velocity 6, meets B, weighing 8 lbs., velocity 4; required their respective directions and velocities after collision? *Ans.* A is reflected back with a velocity of $7\frac{1}{3}$, and B with a velocity of $2\frac{2}{3}$.

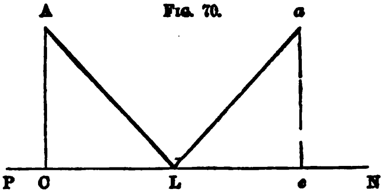
4. A and B move in opposite directions; A equals 4B, and $b=2a$; how do the bodies move after collision? *Ans.* A returns with $\frac{1}{5}$, B with $1\frac{2}{5}$ its original velocity.

5. There are ten bodies whose magnitudes increase geometrically by the constant ratio 3, and the first impinges on the second with the velocity of 5 feet per second; required the motion of the last body? *Ans.* The last body would move with the velocity of $\frac{5}{512}$ feet per second.

$V_f : V_l$ last :: 1 : $\left(\frac{2}{1+3}\right)^9$ =

103. *Impact upon an immovable plane.*—If an inelastic

Fig. 70.



body strikes a plane perpendicularly, its motion is simply *destroyed;* in strictness, however, an infinitesimal velocity is imparted to the body which is called immovable. If it strikes obliquely, and the plane is smooth, it slides along the plane with a diminished velocity. Let AL (Fig. 70) represent the motion of the body before impact, and resolve it into AC, perpendicular, and CL, parallel to the plane. Then AC, as before, is destroyed, but CL is not affected; hence the former velocity is to its velocity on the plane, as AL : CL : : radius : cosine of the inclination.

If a perfectly elastic body impinges perpendicularly upon a plane, then, after its motion is destroyed, the force by which it resumes its form causes an equal motion in the opposite direction; that is, the body rebounds in its own path as swiftly as it struck. But if the impact is oblique, the body rebounds at an equal angle on the opposite side of the perpendicular. Resolve AL, as before, into AC and CL; the latter is unaffected; but, for the component AC, there is substituted an equal opposite motion. Therefore, if L*c* is made equal to CL, and *ca* equal to AC, the resultant is L*a*, which is equal to AL, and has the same inclination to the plane. Hence, the angles of incidence and reflection are equal, and on opposite sides of the perpendicular.

104. *Imperfect elasticity.*—The formulæ of Art. 97 and Art. 98, and the statements of Art. 103, are correct only on the supposition that bodies are, on the one hand, entirely destitute of elasticity, or on the other, perfectly elastic. As no solid bodies are known, which are strictly of either class, these deductions are found to be only near approximations to the results of experiment. In all practical cases of the impact of movable bodies, the loss and gain of velocity are *greater* than if they were inelastic, and *less* than if perfectly elastic. And in cases of impact on a plane, there is always *some* velocity of rebound, but less than the previous velocity; and therefore, if the collision is oblique, the angle between the plane and the path of the body is less after reflection than before.

CHAPTER VI.

SIMPLE MACHINES.

105. *Classification of machines.*—In the preceding chapters, the motion of bodies has been supposed to arise either from collision, or from the immediate action of one or more forces. We now proceed to consider the effects produced, when these forces are made to act by the intervention of other bodies. These intermediate bodies are called *machines;* and by means of them the effect of a given force may be modified in many ways. Machines are divided into *simple* and *compound.*

The following are the most common *simple* machines. 1. *The Lever;* 2. *The Wheel and Axle;* 3. *The Pulley;* 4. *The Rope Machine;* 5. *The Inclined Plane;* 6. *The Wedge;* 7. *The Screw;* 8. *The Knee-Joint, or Combined Levers.* But this list does not include all the forms of the elements of machinery. As to principle, however, these eight, and all others, may be reduced to three: (1.) The law of *equal moments,* which is applicable wherever the machine turns on a pivot or axis, as in the lever, and the wheel and axle; (2.) the law of *transmitted tension,* to be applied wherever the force is exerted through a cord, as in the pulley and the rope machine; (3.) the principle of *oblique action,* which explains the relation of forces in the inclined plane, the wedge, the screw, the knee-joint, and other machine-elements not here enumerated.

Compound machines are formed from the combination of two or more simple ones. But it is not the object of this treatise to enter upon a full description of the nature and use of compound machinery; our intention is rather to explain, upon mathematical principles, the general theory of mechanical action.

I. The Lever.

106. The *lever* is a bar of any form, free to turn on a fixed point, which is called the fulcrum.

The straight lever.—This is the simplest form, and is of three orders; in the 1st, the *fulcrum* is between the power and the weight (Fig. 71); in the 2d, the *weight* is between the fulcrum and power (Fig. 72); in the 3d, the power is between the fulcrum and weight (Fig. 73).

If P and W, in either of these figures, represent forces acting in vertical lines, then the circumstances of equilibrium are determined on the principle of *parallel forces* (Art. 56). In Fig. 71, if P and W are in equilibrio, their resultant will be so situated at C, that $P : W :: BC : AC$; and the fulcrum F must be at that point, and bear a pressure equal to $P + W$. In Fig. 72, P and the reaction of F at C, are two upward forces, whose resultant is counterbalanced by W; then W is represented by the whole line AC, and P by the part BC; $\therefore P : W :: BC : AC$, as before. The pressure on $F = W - P$. In Fig. 73, W and the reaction of F, are downward forces, whose resultant is at A, in equilibrio with P. Here P is represented by the whole line BC, and W by the part AC; $\therefore P : W :: BC : AC$. The upward pressure from $F = P - W$.

FIG. 71.

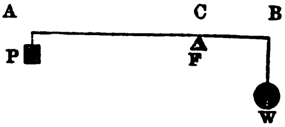
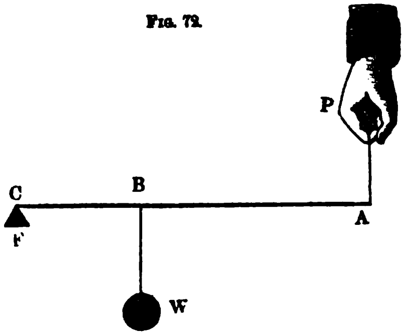
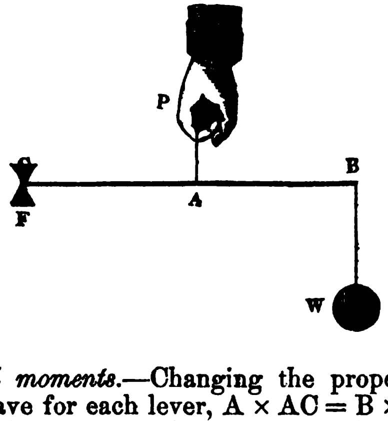


FIG. 72.



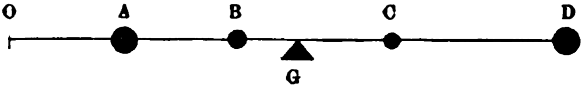
Hence, in each order of the straight lever, where the forces are parallel, *the power and weight are inversely as the lengths of the arms on which they act.*

FIG. 73.



107. *Equal moments.*—Changing the proportion into an equation, we have for each lever, $A \times AC = B \times BC$; that is, *the moment of the power equals the moment of the weight, with respect to the fulcrum.* The moment of either force is the measure of its efficiency to turn the lever; for, since the lever is in equilibrio, the efficiency of the power to turn it in one direction must equal the efficiency of the weight to turn it in the opposite direction. We may therefore use $A \times AC$ to represent the former, and $B \times BC$, the latter. If several weights, as in Fig. 74, are in equilibrio, some tending to turn the bar in one direction, and others in the opposite, then A and B must

FIG. 74.

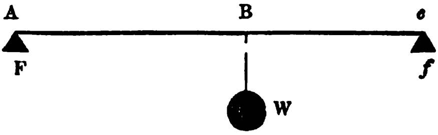


have the same efficiency to produce one motion, as C and D have to produce the opposite; that is, $A \times AG + B \times BG = C \times CG + D \times DG$; or *the sum of the moments* of A and B equals *the sum of the moments* of C and D.

In order to allow for the influence of the weight of the lever itself, consider it to be collected at its center of gravity, and add its moment to that of the power or weight, according as it aids the one or the other. In Fig. 71, let the weight of the lever $= w$, and its distance from C on the side of $P = m$; then $P \times AC + mw = W \times BC$. In the 2d and 3d orders, the moment of the lever necessarily aids the weight; and hence, in each case, $A \times AC = W \times BC + mw$.

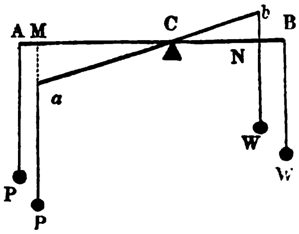
If a weight hangs on a bar between two supports, as in Fig. 75, it may be regarded as a lever of the 2d order, the pressure on either support being considered as a power. By the theorems of parallel forces, we have the pressures at F and f inversely as their distances from W, and $W = F + f$.

FIG. 75.



108. *The acting distance.*—In the three orders, as above described, the equilibrium is not destroyed by inclining the lever to any angle whatever with the horizon, provided the center of motion C is at the centre of gravity of the bar, and not *above* or *below* it, and provided the directions of the forces remain vertical. For, by the principle of parallel forces, bC to aC, on the *inclined* bar (Fig. 76), as truly expresses the ratio of P to W, as does BC to AC, on the same bar, in the *horizontal* position. Hence, the resultant of P and W remains at C, in every position of the lever. By similar triangles, $bC : aC :: CN : CM$; $\therefore P : W :: CN : CM$: $\therefore P \times CM = W \times CN$. The lines CM and CN, which are drawn from the fulcrum perpendicular to the lines in which the forces act, are called the *acting distances* of the po[illegible]nd

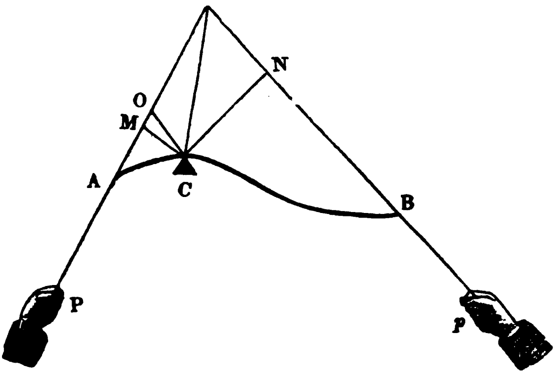
FIG. 76.



weight respectively. And as they may be employed in levers of irregular form, the moments of power and weight are usually measured by the products, P × CM and W × CN; therefore, *the power multiplied by its acting distance equals the weight multiplied by its acting distance;* or, more briefly, the moment of the power equals the moment of the weight, as in Art. 107. In figures 71, 72, and 73, the acting distances are in each case identical with the arms of the lever.

109. *Lever of irregular form, and forces not parallel.*—Let ACB (Fig. 77) be a lever of any form, and let it be in equilibrio by the forces P and *p*, acting in the same plane, in any oblique directions. Produce PA and *p*B till they meet in S;

Fig. 77.



then if the fulcrum is at C, the resultant of P and *p* must be in the direction SC, otherwise the reaction of the fulcrum cannot keep the system in equilibrio (Art. 65). Therefore,

$$P : p :: \sin pSC : \sin PSC.$$

Draw CM perpendicular to PS, and CN to *p*S, and they are sines of PSC and *p*SC to the same radius, SC.

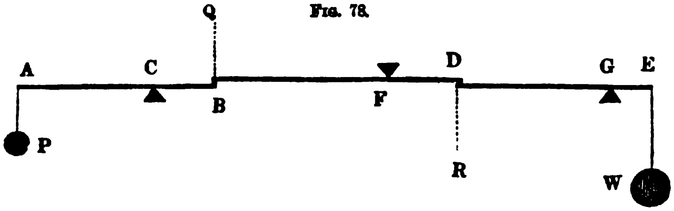
$$\therefore P : p :: CN : CM; \text{ and } P \times CM = p \times CN.$$

The lines CM and CN are the acting distances of P and *p*; therefore, we have, as the law of the lever in all cases, *the moment of the power equals the moment of the weight.*

The pressure on the fulcrum is less than the sum of P and *p*, when they act obliquely; for, if CO is parallel to *p*S, then SO, OC, and CS represent the three equilibrating forces. But CS is less than SO and OC.

110. *The compound lever.*—When a lever acts on a second, that on a third, &c., the machine is called a *compound lever.* Th[illegible]w of equilibrium is, *power is to weight as the product of* *acting distances on the side of the weight is to the product of acting distances on the side of the power.*

FIG. 78.



Let the force exerted by AB on BD (Fig. 78) be called Q, and that of BD on DE be called R. Then

$$P : Q :: BC : AC;$$
$$\text{and} \quad Q : R :: DF : BF;$$
$$\text{and} \quad R : W :: EG : DG.$$

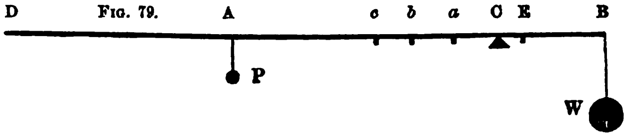
Compounding these proportions, and dividing the first couplet by the common factors, we have

$$P : W :: BC \times DF \times EG : AC \times BF \times DG.$$

If the levers were of irregular forms, the acting distances might not be identical with the arms, as they are in the figure.

111. *The lever as a weighing machine.*—When the lever is employed for weighing, either in the form of the balance or steelyard, the fulcrum is placed slightly *above* the center of gravity of the bar; because if it coincides with it, there is equilibrium in every position (Art. 108); and if *below*, the equilibrium is unstable (Art. 85). In the *balance*, the arms are equal, and equal increments of weight are indicated by equal increments of the poise. But in the *steelyard*, the arms are unequal, and equal increments of weight are indicated by equal increments of the acting distance of the poise. Suppose (Fig. 79) that the bar is so constructed that its center of gravity is just

FIG. 79.



below the fulcrum C. Then $P \times AC = W \times BC$; and, as P and BC are constant, $AC \propto W$; hence, if W is successively 1 lb., 2 lbs., 3 lbs., &c., the distances of the notches *a*, *b*, *c* from C are as 1, 2, 3, &c.; in other words, the bar CD is graduated in equal parts.

But suppose, what is usually true, that the center of gravity of the whole bar is in the arm CD, and that P placed at E would balance it; then the moment of the bar is equal to P × CE on the side CD. Therefore,

$$P \times AC + P \times CE = W \times BC; \text{ or } P \times AE = W \times BC.$$

Therefore, $W \propto AE$; and the graduation must begin at E. To find the length of the divisions for each unit of weight, divide AE by the number of units in the weight W, which P would balance when suspended at A. But, since $\frac{AE}{W} = \frac{BC}{P}$, the latter fraction gives also the length of one division.

If the two arms of a balance are not precisely equal, so that an article seems to have different weights when weighed at different ends, the *true* weight is found by taking the *geometrical mean* between the two apparent weights. Let $x =$ true weight; $a =$ the length of the arm on which p is suspended to balance it; and $b =$ the length of the other, on which q is suspended to balance it. Then, $ap = bx$, and $bq = ax$; $\therefore x = \sqrt{pq}$.

112. *Questions on the lever.*—

1. AB (Fig 80) is a uniform bar, 2 feet long, and weighs 4 oz.; where must the fulcrum be put, that the bar may be balanced by P, weighing 5 lbs.? *Ans.* $\frac{4}{7}$ of an inch from A.

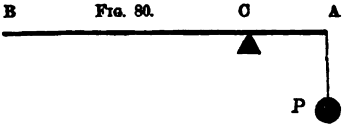


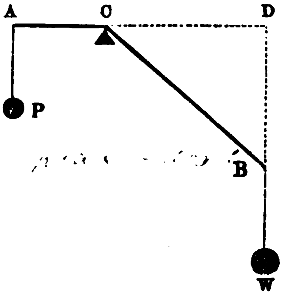
FIG. 80.

2. A lever of the second order is 25 feet long; at what distance from the fulcrum must a weight of 125 pounds be placed, so that it may be supported by a power able to sustain 60 pounds, acting at the extremity of the lever? *Ans.* 12 feet.

3. A and B are of the same height, and sustain upon their shoulders a weight of 150 pounds, placed on a pole $9\frac{1}{3}$ feet long; the weight is placed $6\frac{2}{3}$ feet from A; what is the weight sustained by each person?
Ans. A sustains $42\frac{6}{7}$ pounds, and B sustains $107\frac{1}{7}$ pounds.

4. The longer arm of a steelyard is 2 feet 2 inches in length, and the shorter $2\frac{2}{3}$ inches; and its apparatus of hooks, &c., is so contrived, that a weight of two pounds placed upon the longer arm, at the distance of 10 inches from the center of motion, will balance 8 pounds placed at the extremity of the shorter arm; the movable weight (of 2 pounds) cannot conveniently be placed nearer to the fulcrum than $\frac{2}{3}$ of an inch; what must be the graduation of the steelyard that it may weigh ounces, and what will be the greatest and least weights that can be ascertained by it? *Ans.* The graduation is to 12ths of an inch; and it will weigh from 1 to 20 pounds.

FIG. 81.



5. A *bent lever*, ACB (Fig. 81), has the arm AC = 3 feet, CB = 8 feet, P = 5 lbs., and the angle ACB = 140°; what weight, W, must be attached at B, in order to keep AC horizontal? *Ans.* 2.4476 lbs.

6. A cylindrical straight lever is 14 feet long, and weighs 6 lbs. 5 oz.; its longer arm is 9, and its shorter 5 feet; at the extremity of its shorter arm a weight of 15 lbs. 2 oz. is suspended; what weight must be placed at the extremity of the longer arm to keep it in equilibrio? *Ans.* 7 lbs.

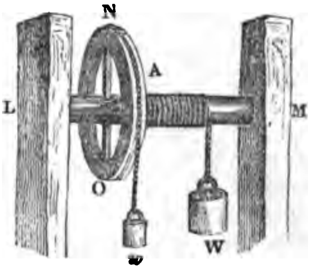
7. A body weighs 11 lbs. at one end of a false balance, and 17 lbs. 3 oz. at the other; what is its real weight? *Ans.* 13 lbs. and 12 oz.

8. A uniform bar 12 feet long, weighs 7 lbs.; a weight of 10 lbs. hangs on one end, and 2 feet from it is applied an upward force of 25 lbs.; where must the fulcrum be put to produce equilibrium? *Ans.* 1 foot from the 10 lbs.

II. THE WHEEL AND AXLE.

113. The wheel and axle consists of a cylinder and a wheel, firmly united, and free to revolve on a common axis. The power acts at the circumference of the wheel in the direction of a tangent, and the weight in the same manner, at the circumference of the cylinder or axle; so that the acting distances are the radii at the two points of contact. As the system revolves, the radii successively take the place of acting distances, without altering at all the relation of the forces to each other. The wheel and axle is therefore a kind of endless lever.

FIG. 82.



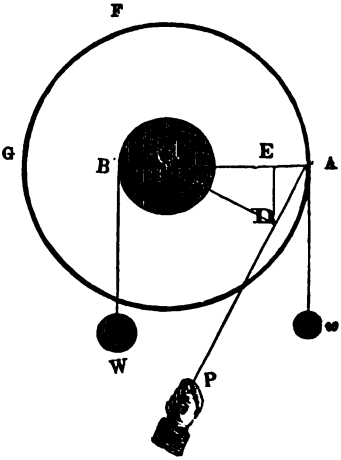
Let the weight W (Fig. 82) be the force tending to turn the system on the axis LM; and w, the force which opposes that motion. The acting distances are respectively the radius of the axle, and that of the wheel ON. In case of equilibrium, the moment of W equals the moment of w; i. e.,

$$W \times \text{rad. axle} = w \times \text{rad. wheel}.$$

Instead of the weight w, a power P may be applied at any point on the circumference of the wheel; and if the line of its action is

perpendicular to the radius at that point, then equilibrium takes place when $W\,.\,\text{rad. axle} = P\,.\,\text{rad. wheel}$;
or, $P : W :: \text{rad. axle} : \text{rad. wheel}$.

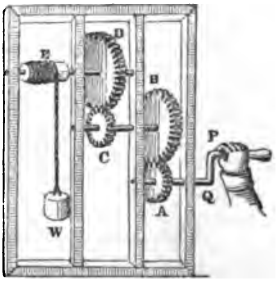
Fig. 83.



If the power does not act at right angles to the radius of the wheel, but in some oblique direction, as AP (Fig. 83), then let fall CD perpendicular to AP. The moment of P is now $P \times CD$; hence, when there is equilibrium, $P \times CD = W \times BC$; $\therefore P : W :: BC : CD$. So that the efficiency of the power is diminished, its present value being to its former value as sin CAD : rad. The figure represents the string as fastened at A, and slipped off upon the end of the wheel. Precisely the same effect would be produced, if AP were drawn away from the wheel (being confined at A), at an equal angle on the other side of A*w*. The perpendicular would then fall above CA.

Hitherto we have not considered the thickness of the rope; when that is taken into the account, we must add the half of it to the distance at which W and *w* respectively act. Let therefore $2t$ = diameter of the rope, and let R = rad. of the wheel, r = rad. of the axle; then, if the thickness of the rope be taken into consideration, we have $w : W :: r+t : R+t$; and since in this case the same quantity (t) is added to each term of the ratio $r : R$, w must bear a greater ratio to W than that of $r : R$, or of the radius of the axle to the radius of the wheel; and therefore the efficiency of the power (w) is diminished.

Fig. 84.



114. *The compound wheel and axle.*—When a train of wheels, like that in Fig. 84, is put in motion, those which *communicate* motion by the circumference are called *driving wheels*, as A and C; those which *receive* motion by the circumference, are called *driven wheels*. And the law of equilibrium is, *the power is to the weight, as the product of the radii of the driving wheels to the product of the radii of the driven wheels.* The crank PQ is to be reckoned among driven wheels; the axle E among driving wheels.

Let the radius of B be called R; of D, R′; of A, r; of C, r'; of E, r''. Call the force exerted by A on B, x; that of C on D, y. Then

$$\begin{aligned} P : x &:: r : PQ; \\ x : y &:: r' : R; \\ y : W &:: r'' : R'; \\ \hline \therefore\ P : W &:: r \times r' \times r'' : PQ \times R \times R'. \end{aligned}$$

If the driving wheels are equal to each other, and also the driven wheels, and the number of each is n, then

$$P : W :: r^n : R^n.$$

115. *Questions on the wheel and axle.*—

1. A power of 12 lbs. balances a weight of 100 lbs. by a wheel and axle; the radius of the axle is 6 inches, what is the *diameter* of the wheel? *Ans.* 8 ft. 4 in.

2. $W = 500$ lbs.; $R = 4$ ft.; $r = 8$ in.; the weight hangs by a rope 1 inch thick, but the power acts at the circumference of the wheel without a rope; what power will sustain the weight? *Ans.* 88.54 lbs.

3. $R = 1$ foot; $r = 2$ inches; the well-stone weighs 256 lbs.; the bucket, empty, weighs 18 lbs.; the bucket, filled, weighs 65 lbs.; what force must a person apply to the bucket rope, in each case, for equilibrium? *Ans.* 1st, down, $24\frac{2}{3}$ lbs.; 2d, up, $22\frac{1}{3}$ lbs.

III. The Pulley.

116. The pulley consists of a wheel or roller, with a cord passing over its edge; the axis of the roller is in a *block*, which may be fixed, or may move with the weights; and the pulley is accordingly called a *fixed pulley* or a *movable pulley*. The principle, which explains the relation of power and weight in every form of pulley, is this: *whatever strain or tension is applied to one end of a cord, is transmitted through its whole length, however much its direction is changed.* The ropes of a pulley are supposed, in every case, to be parallel.

117. *The fixed pulley.*—In the fixed pulley, A (Fig. 85), the force P produces a tension in the string, which is transmitted through its whole length, and which can be balanced only when W equals P. Hence, in the fixed pulley, *the power and weight are equal.* This machine is useful for changing the *direction* in which the force is applied to the weight.

Fig. 85.

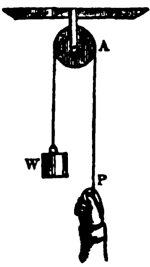
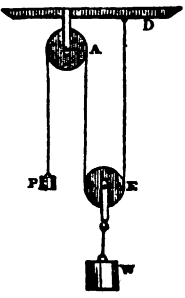


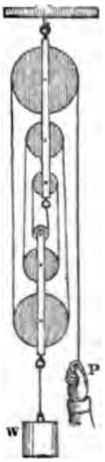
Fig. 86.



118. *The movable pulley.*—In Fig. 86, the tension produced by P is transmitted down to E, and thence up to D; so that W is sustained by *two* portions of the rope, each of which exerts a force equal to P. $\therefore W = 2P$.

The same reasoning applies, where the rope passes between the upper and lower blocks any number of times, as in Fig. 87; the force P causes a tension in the rope, which is transmitted to every portion that passes back and forth between the blocks. If n is the number of such portions, then W is sustained by their united tensions, which is nP; and if there is equilibrium, $P : W :: 1 : n$. In the figure, the weight equals *six times* the power. The law of equilibrium, therefore, for the movable pulley with one rope, is this: *power is to weight as one to the number of times the rope passes between the blocks.*

Fig. 87.



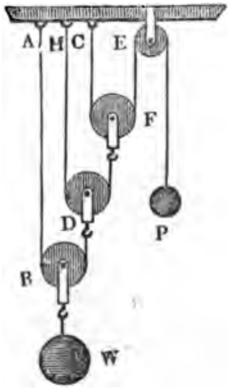
119. *The compound pulley.*—Wherever a system of pulleys has separate ropes, the machine is to be regarded as compound, and its efficiency is calculated accordingly. Figures 88 and 89 are examples. In Fig. 88, call the weight sustained by F, x, and that sustained by D, y. Then (Art. 118),

$$P : x :: 1 : 2;$$
$$x : y :: 1 : 2;$$
$$y : W :: 1 : 2.$$
$$\therefore P : W :: 1 : 2^3 :: 1 : 8.$$

And if n is the number of ropes,

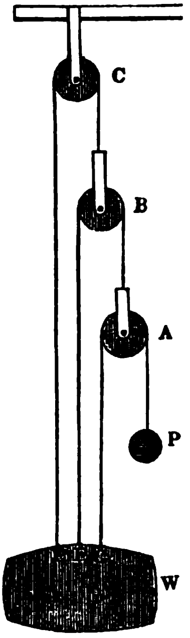
$$P : W :: 1 : 2^n.$$

Fig. 88.



In Fig. 89, the tension P is transmitted directly to the weight at G; the wheel A is loaded, therefore, with 2P, and a tension of 2P comes upon the second rope ABF; and for the same reason, a tension of 4P acts on the third rope BCE. The sum of all these is applied to the weight, which must therefore be equal to that sum, in case of equilibrium. Therefore $P : W :: 1 : 1 + 2 + 4 +$, &c. But the sum of this geometrical series to n terms is $2^n - 1$; $\therefore P : W :: 1 : 2^n - 1$. This combination is therefore a little less efficient than the preceding.

Fig. 89.



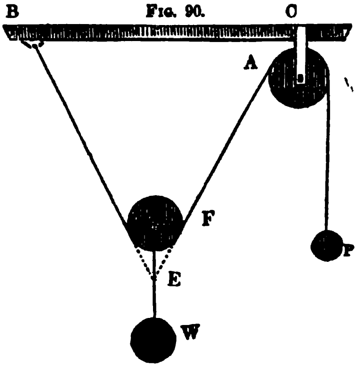
Since the several ropes have different tensions, the weight can not be balanced upon them, unless those of greatest tension are nearest the direction of the center of gravity. For example, if the rope F is directed toward the center of gravity of the weight, the rope G should be attached four times as far from it as the rope E, in order to prevent the weight from tipping.

The pulley owes its efficiency as a machine to the fact, that the tension produced by the power is applied *repeatedly* to the weight. The only use of the wheels is to diminish friction. Were it not for friction, the rope might pass round fixed pins in the blocks, and the ratio of power to weight would still be in every case the same as has been shown.

IV. The Rope Machine.

120. The rope machine is one in which the power and weight are in equilibrio by the tension of a rope. According to this general definition, the pulley would be included. And, in truth, the pulley may be regarded as a rope machine of a particular kind, namely, that in which the portions of rope are parallel among themselves.

Fig. 90.

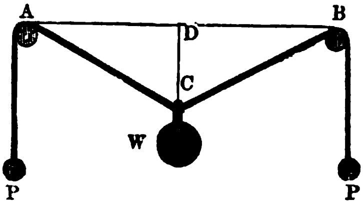


If the two portions of rope which sustain W become oblique, as in Fig. 90, then W is no longer equal to the sum of their tensions. Let EF represent the tension of P, and resolve it into ED vertical and DF horizontal; ED is the component employed in sustaining W, and the tension of EB on the other side produces the same effect; so that

P : W :: EF : 2ED :: rad. : 2 cos. FED or FEW.

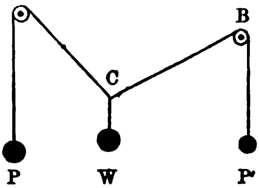
If, in Fig. 91, the end of the cord, instead of being attached to the beam, is carried over another fixed pulley, and a weight equal to P is hung upon it, the equilibrium will be preserved, and as before, P : W :: BC : 2CD, or 2P : W :: BC : CD. If P is given, all the possible values of W are included between W = 0, and W = 2P. When W = 0, CD, by the above proportion, must equal *zero;* that is, the rope is straight from A to B. As W increases, it descends; and when it equals P, the proportion gives DC = $\frac{1}{2}$BC; ∴ DCB = 60°, and CA, CB, CW, diverge at 120° from

Fig. 91.



each other, as they plainly should do, since each of the three equal forces, P, P′, and W, is as the sine of the angle between the lines of the other two. But when W = 2P, it descends to an infinite distance; for then CD = BC; and we have the case (Fig. 86), where the two portions of the rope are parallel, and ½ W (= P) balances P. It appears, from the foregoing, that a perfectly flexible rope, *having weight*, can not be drawn horizontally into a straight line by any force, however great, for C can not coincide with D, except when W = 0.

Fig. 92.

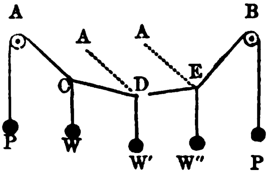


If the pulleys are not in the same horizontal line, or if P and P′ are unequal (Fig. 92), C will, in any given case, assume a determinate position, according to the principle in Art. 65, that

P : W : : sin BCW : sin ACB, and
P′ : W : : sin ACW : sin ACB.

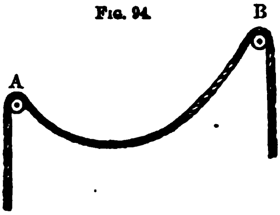
121. *The funicular polygon.*—If several weights are attached at fixed points along the cord ACB (Fig. 93), the combination is called the funicular polygon; and the fact that there are opposite and equal tensions in any portion of the cord, will enable us to transfer all the forces to one point.

Fig. 93.



Let the tension of CD = T; and that of DE = T′. C is kept at rest by P, T, and W; hence T is equal to the resultant of P and W. But the same T (in the opposite direction) equals and balances the resultant of T′ and W′. Suppose, now, CD to vanish, by removing C to D; draw AD parallel to AC; let P act in the line DA, and W in the line DW′. D will now be in equilibrio as before, because there has merely been made a substitution of P and W in their original directions for T, their equivalent. Now consider D to be acted on by three forces, T′, P, and W + W′; ∴ T′ = the resultant of P and W + W′, and the two latter can be transferred as before to E, AE being parallel to AC or AD. E is therefore kept at rest by the three forces, P, P′, and W + W′ + W″. We can now use the triangle of forces to determine the directions of BE and AE, or its parallel, AC, and hence, of the parts, CD and DE.

Fig. 94.

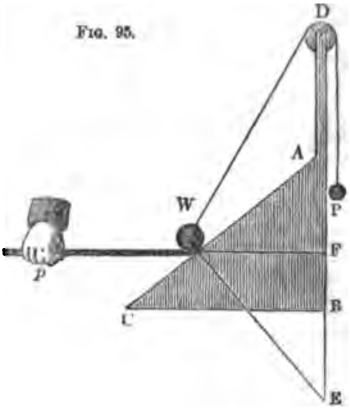


If the weights are all equal, and their number = *n*, the three forces at E, are P, P′, and *n*W. An extreme case of this kind is the chain or flexible rope suspended loosely over two supports, which assumes the form of a curve called the *catenary* (Fig. 94).

The directions of the curve at the ends A and B, as well as the law of the entire curve, may be determined by the calculus, in conformity with the principles presented above. Another example of a very similar character is seen in the suspension bridge.

V. The Inclined Plane.

FIG. 95.



122. The mechanical efficiency of the inclined plane is explained on the principle of *oblique action;* that is, it enables us to apply the power to balance or overcome only *one component* of the weight instead of the whole. The weight W (Fig. 95) is kept at rest by three forces; P acting in the line WD; its own gravity in a line parallel to DE; and the resistance of the plane in the line EW. These forces may therefore be represented by the three sides of the triangle WDE; and the particular relation of P to W will depend on the direction of WD.

123. *Power most efficient when parallel to the plane.*—If the power acts in the line WA, it sustains the greatest possible weight on the plane. For, let AE represent the force of gravity, and resolve it into AW on the plane, and WE perpendicular to it. The latter is destroyed by the reaction of the plane; the former, AW, tends to carry the weight down the plane. Hence, the power is most efficient when acting in the opposite direction. In this case P : W : : WA : AE; but, by similar triangles, WA : AE : : AB : AC. This is the usual formula for the plane, and is stated thus; when the power acts parallel to an inclined plane, *the power is to the weight, as the height to the length of the plane.*

If the power acts parallel to the *base* of the plane, the triangle of forces is WFE; and P : W : : WF : FE; but by similar triangles, WF : FE : : AB : BC; ∴ P : W : : AB : BC; that is, as height to base.

124. *Place of equilibrium on a plane.*—If the pulley D (Fig. 96) were placed at A, then W, when acted on by a given force P, would rest indifferently at any point along the plane; since the direction of the forces would not change, by any change of position in the weight. But if the pulley in the line DB is at any point above or below A, there is but one

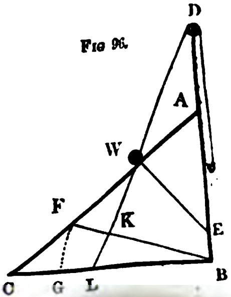


Fig 96.

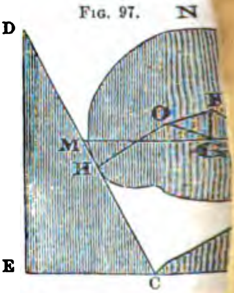
place where W can rest. It is found thus: Make BG to BC as P to W; revolve BG about B, till it meets AC in F; draw DL perpendicular to BF, produced if necessary. The point W, where DL intersects AC, is the only point of equilibrium. WDE and FBC are similar, because their sides are respectively perpendicular to each other.

∴ FB : BC :: WD : DE; but
P : W :: FB : BC; ∴ P : W :: WD : DE;
and this proportion is the condition of equilibrium.

125. *Equilibrium between two planes.*—If a b
represented in Fig. 97, between two smooth pla
forces which retain it are its weight, and the resistances of the planes. Draw HF and LF through the points of contact, and GF vertically through the center of gravity of the body. If these three lines do not pass through the same point, there will be no equilibrium (Art. 65), and the body will roll until the lines do intersect in one point. Let that point be F, and draw GO parallel to LF, and MK parallel to the horiz
similar to KCM. Therefore, (since Pressure on
DC :: OG : FO),

Pressure on AC : Pr. on DC :: KC : MC,
:: sin M : sin K,
:: sin DCE : sin

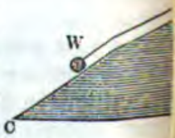
FIG. 97.



That is, when a body rests between two planes, it
ures on them which are inversely as the sines of
tions to the horizon.

126. *Bodies balanced on two planes by a cord*
the ridge.—Let P and W balance each other on th
and AC (Fig. 98), which have the common height AB, by means of a cord passing over the fixed pulley A. The tension of the cord is the common power which prevents each body from descending; and, as the cord is parallel to each plane, we have (calling the tension *t*),

Fig



$$t : P :: AB : AD;$$
$$\text{and} \quad t : W :: AB : AC;$$
$$\therefore \quad P : W :: AD : AC;$$

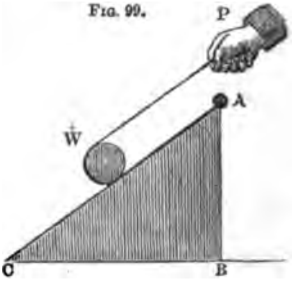
that is, the weights, in case of equilibrium, are directly as the lengths of the planes.

127. *Pulley and inclined plane combined.*—Let a cord, attached at A (Fig. 99), pass round a pulley connected with the weight, and the power be applied at its other extremity; to find the relation of P to W. The force by which W tends to descend, is a component of its weight; let this force be called F; then, by the principle of tension,

$$P : F :: 1 : 2;$$
$$\text{but} \quad F : W :: AB : AC;$$
$$\therefore \quad P : W :: AB : 2AC;$$

or, power is to weight, as height to twice the length.

Fig. 99.



This example illustrates the mode of procedure, in all cases of compound machines, for finding the ratio of the original force to the last effect produced; it being understood that the *effect* of the first machine is the *power* for the second, the effect of the second, the power for the third, &c. *First determine the ratio of* P : W, *for each simple machine, and then compound those ratios.* This was done for the compound lever (Art. 110), the compound wheel and axle (Art. 114), and the compound pulley (Art. 119); but the rule is equally applicable to machines compounded of *different* kinds of simple machines.

128. *Questions on the inclined plane.*—

1. If a horse is able to raise a weight of 440 lbs. perpendicularly, what weight can he raise on a railway having a slope of five degrees? *Ans.* 5048.5 lbs.

2. The grade of a railroad is 20 feet in a mile; what power must be exerted to sustain any given weight upon it? *Ans.* 1 lb. for every 264 lbs.

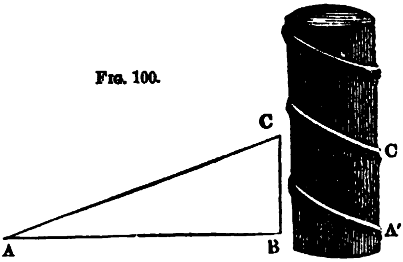
3. What force is requisite to hold a body on an inclined plane, by pressing perpendicularly against the plane? *Ans.* An infinite force.

4. A certain power was able to sustain 500 tons on a plane of $7\frac{1}{2}°$; but on another plane, it could sustain only 400 tons: what was the inclination of the latter? *Ans.* $9° \, 23' \, 25''$.

VI. The Screw.

129. The screw is a cylinder having a spiral ridge or thread around it, which cuts, at a constant oblique angle, all the lines of the surface parallel to the axis of the cylinder. A hollow cylinder, called the *nut*, having a similar spiral within it, is fitted to move freely upon the thread of the solid cylinder. In Fig. 100, let the base AB of the inclined plane be equal to the circumference of the cylinder; then, supposing the plane to be wrapped about the cylinder, A and B will coincide at A′, and BC will take the position A′C, parallel to the axis of the cylinder, and the length AC will form one revolution of a screw-thread. The mechanical relations of the screw, therefore, are the same as of the inclined plane.

Fig. 100.



If we suppose a weight resting on the thread, and free to move only in a vertical direction, and apply a force at the circumference of the cylinder, so as to revolve it, the case is analogous to that of raising a weight on a plane by a force parallel to the base. Since, in the plane,

$$P : W :: BC : AB,$$

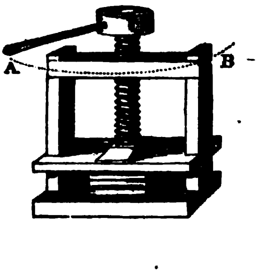
$$\therefore \text{ in the screw, } \quad P : W :: A'C : 2\pi r,$$

if r = the radius of the cylinder. Hence, the law of equilibrium in the screw is this; *power is to weight, as the distance between the threads, measured parallel to the axis, is to the circumference of the screw.*

In Fig. 101, instead of a weight resting on the thread, and elevated by revolving the screw, the screw itself is moved longitudinally in the nut, against a resistance. Since action and reaction are equal, it is immaterial whether the force tends to move the screw within the nut, or the nut upon the screw.

130. *The screw and lever combined.*—The screw is so generally combined with the lever in practical mechanics, that it is important to present the law of the compound machine. Let AFG (Fig. 102) be the section of a screw, and suppose BC, a lever of the second order, to be applied to turn it. The fulcrum is at C, the power acts at B, and the effect produced by the lever is at A, the surface of the

Fig. 101.



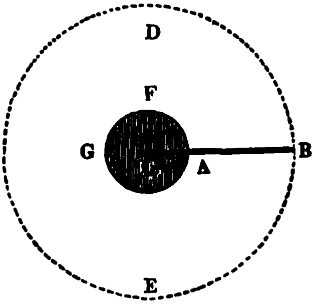
cylinder. Call that effect x, and let d = the distance between the threads; then,

$$P : x :: AC : BC$$
$$:: 2\pi AC : 2\pi BC;$$
and $$x : W :: d : 2\pi AC;$$
$$\therefore\ P : W :: d : 2\pi BC;$$

that is, *power is to weight, as distance between threads parallel to the axis to the circumference described by the power.*

Fig. 101 represents a combination of the screw and lever as commonly used.

Fig. 102.

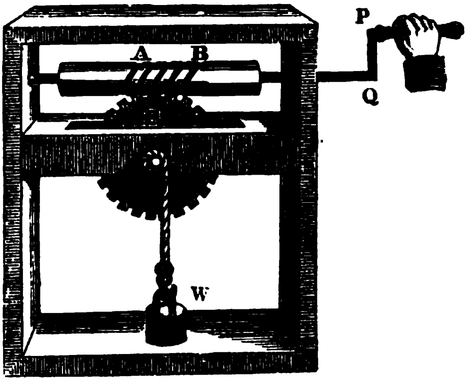


131. *The endless screw.*—This name is given to the compound machine, in which the thread of a screw works in the teeth of a wheel.

In Fig. 103, let d = distance between threads; R = rad. of wheel; r = rad. of axle; and x = the force communicated by the screw to the wheel. Then,

$$P : x :: d : 2\pi \times PQ;$$
and $$x : W :: r : R;$$
$$\therefore\ P : W :: dr : 2\pi \times R \times PQ.$$

Fig. 103.



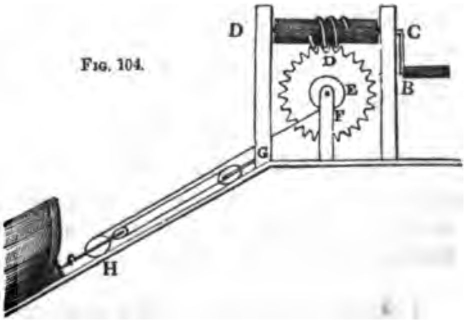
132. *Questions on the screw.*—

1. The distance between the threads of a screw is one inch, the bar is two feet long from the axis, and the power 30 lbs.; what is the pressure?

Ans. 4523.76 lbs.

2. The bar is three feet long, reckoned from

Fig. 104.



the axis, $P = 60$ lbs., $W = 2240$ lbs.; what is the distance between the threads? *Ans.* 6.048 inches.

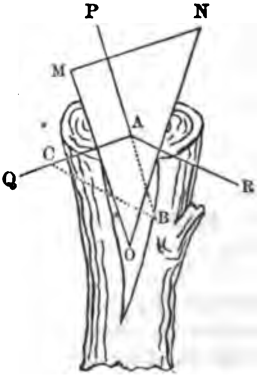
3. In the compound machine represented in (Fig. 104), $P = 100$ lbs.; $BC = 18$ inches; $d = 1$ inch; $R = 2$ feet; $r = 6$ inches; the same cord passes 4 times between the blocks; and the inclination of the plane $= 30°$; what is the weight balanced by the power? *Ans.* 361911.168 lbs.

VII. The Wedge.

133. All those instruments which are used for the separation of the parts of bodies, such as knives, axes, and chisels, come under the general denomination of the wedge; but this term is usually employed to designate a triangular prism, one of whose edges is introduced between two obstacles so as to exert upon them lateral pressures to separate them. This is effected by a blow, or some similar means, which produces a violent pressure for a short time, sufficient to overcome a very great resistance.

134. *Law of equilibrium.*—Whatever be the direction of the blow or force, we may suppose it to be resolved into two components, one perpendicular to the back of the wedge, and the other parallel to it. The latter produces no effect. The same is true of the resistances; we need consider only those components of them which are perpendicular to the sides of the wedge.

Fig. 105.



Let MNO (Fig. 105) represent a section of the wedge perpendicular to its faces; then PA, QA, and RA, drawn perpendicular to the faces severally, show the directions of the forces which hold the wedge in equilibrio. Taking AB to represent the power, draw BC parallel to RA, and we have the triangle ABC, whose sides represent these forces. But ABC is similar to MNO, as their sides are respectively perpendicular to each other. Hence, calling the forces P, Q, and R, respectively,

$$P : Q :: MN : MO;$$
$$\text{and}\quad P : R :: MN : NO;$$

that is, there is equilibrium in a wedge, when *the power is to the resistances as the back of the wedge to the sides on which the resistances respectively act.*

135. If the triangle MNO is isosceles, the two resistances are equal, as the proportions show; and P is to one of them as the back to one of the sides, which in this case may be called the length of the wedge.

If the resisting surfaces touch the sides of the wedge only in one point each, then QA and RA, drawn through the points of contact, must meet AP in the same point (Art. 65); otherwise the wedge will roll, till one face rests against the resisting body in two or more points.

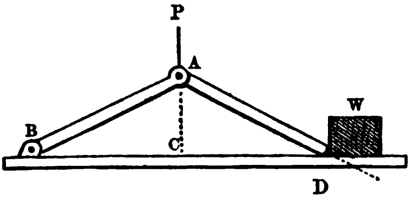
The efficiency of the wedge is usually very much increased by combining its own action with that of the lever, since the point where it acts, generally lies at a distance from the point where the effect is to be produced. Thus, in splitting a log of wood, the resistance to be overcome is the cohesion of the fibers; and this force is exerted at a distance from the wedge.

VIII. THE KNEE-JOINT, OR COMBINED LEVERS.

136. This is a machine consisting of two bars, usually equal, hinged together at one end, while the others can separate, one from the other, in a straight line. The power is applied at the hinge, tending to thrust the bars into a straight line; and the weight is the resistance to be overcome by the moving extremity.

In Fig. 106, suppose AB and AD to be equal bars hinged together at A; and that the end B is pivoted to the frame BD. If a power P urges A perpendicularly toward the frame, the end D will press against the weight W. It is required to find the ratio of P to W when in equilibrio.

FIG. 106.



Consider A to be kept at rest by three forces, the power P, the resistance along BA, and that along DA. The two resistances are equal, since they are (Art. 65) as the sines of PAB and PAD; and these angles are equal by the supposition that the bars are equal, and that PA is perpendicular to BD. Let $\alpha = \mathrm{BAC} = \frac{1}{2}\mathrm{BAD} =$ supplement of PAB; and let R = the resistance in the direction DA. Then,

$$P : R :: \sin 2\alpha : \sin \alpha.$$

But W, which is not free to move in the line AD, furnishes only one component of that resistance; the other arises from the reaction of the frame BD. Therefore, resolving R, we have (Art. 54),

$$R : W :: \text{rad} : \cos \mathrm{ADC} = \sin \alpha;$$

and $R = \frac{W}{\sin \alpha}$. Hence, by substitution,

$$P : \frac{W}{\sin \alpha} :: \sin 2\alpha : \sin \alpha;$$

$$\therefore \quad P : W :: \sin 2\alpha : \sin \alpha^2.$$

By trig. anal. $\sin 2\alpha = 2 \sin \alpha \cos \alpha$;

$$\therefore \quad P : W :: 2 \sin \alpha \cos \alpha : \sin \alpha^2;$$

$$:: 2 \text{ rad} : \frac{\sin \alpha^2}{\sin \alpha \cos \alpha} = \tan \alpha.$$

Hence, the law of equilibrium for the knee-joint is, *power is to weight as twice radius to the tangent of the half angle between the bars.* In the lines of the figure, this proportion is

$$P : W :: 2AC : CD.$$

137. *Ratio of power to weight variable.*—It is obvious that this ratio changes with every different position of the bars. As A is raised higher, BAD diminishes; and when BAD $= 0$, then $\alpha = 0$, and $\tan \alpha = 0$;

$$\therefore \quad P : W :: 2 \text{ rad} : 0,$$

and the power has no efficiency. But as A approaches C, α approaches 90°; therefore $\tan \alpha$ increases, and the power is more efficient. When A, B, and D are in a straight line, $\alpha = 90°$, and $\tan \alpha$ is infinite, $\therefore P : W :: 2 \text{ rad} : \infty$. Hence, the efficiency of the power is immeasurably great. The indefinite increase of efficiency in the power, which occurs during a single movement, renders this machine one of the most useful for many purposes, as printing and coining. (See Part II, Art. 287.)

Questions on the knee-joint.—

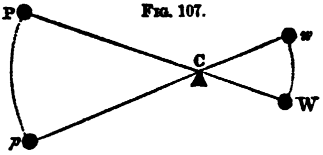
1. A power of 50 lbs. is exerted on the joint A (Fig. 106); compare the weight which will balance it, when BAD is 90°, and when it is 160°. *Ans.* 25 lbs. and 141.78 lbs.

2. When the angle between the bars is 110°, a certain power just overcomes a weight of 65 lbs.; what must be the angle, in order that the weight overcome may be five times as great? *Ans.* 164° 3′ 22″.

138. *Principle of virtual velocities.*—In examining the simple machines, we have in each instance simply inquired for the relative *magnitude* of the forces, called the power and the weight, when in equilibrio. There is another important particular to be noticed, namely, the relative *velocity* of the power and weight, when they begin to move. It can be shown, in every case, that *the velocities, when reckoned in the direction in which the forces act, are inversely as the weights, which represent those forces.*

In the *straight lever* (Fig. 107), which is in equilibrio by the weights P and W, suppose a slight motion to exist; then the velocity of each will be as the arc described in the same time; but the arcs are similar, since they subtend equal angles. Therefore, if $V =$ velocity of P, and $v =$ velocity of W,

FIG. 107.



$$V : v :: Pp : Ww :: PC : WC;$$

But (Art. 106), $$P : W :: WC : PC;$$
$$\therefore V : v :: W : P;$$

that is, the velocity of the power is to the velocity of the weight, as the weight to the power. Hence, $P \times$ its velocity $= W \times$ its velocity.

In the *wheel and axle*, let R and r be the radii, and suppose the machine to be revolved; then while P descends a distance equal to the circumference of the wheel $= 2\pi R$, the weight ascends a distance equal to the circumference of the axle $= 2\pi r$.

Therefore, $$V : v :: 2\pi R : 2\pi r :: R : r;$$
but (Art. 113), $$P : W :: r : R;$$
$$\therefore V : v :: W : P;$$

or, the velocities are inversely as the weights; and $P \times V = W \times v$, the momentum of the power equals the momentum of the weight.

In the *fixed pulley* the velocities are obviously equal; and we have before seen, that the power and weight are equal; then the proportion holds true, $V : v :: W : P$; and the momenta are equal.

In the *movable pulley*, if n is the number of times which the cord passes between the blocks, if W rises any distance $= x$, each portion of cord is shortened by the distance x, and all these n portions pass over to P, which therefore descends a distance $= nx$.

Hence, $$V : v :: nx : x :: n : 1;$$
but (Art. 118), $$P : W :: 1 : n;$$
$$\therefore V : v :: W : P;$$

as in all the preceding cases.

In the screw (Fig. 102), while the power describes the circumference $= 2\pi \times BC$, the weight moves only the distance $= d$;

therefore, $$V : v :: 2\pi \times BC : d;$$
but (Art. 130), $$P : W :: d : 2\pi \times BC;$$
$$\therefore V : v :: W : P;$$

therefore the momentum of the power equals the momentum of the weight, as before.

139. The cases thus far noticed are the most obvious ones, because the points of application of power and weight *actually move* in the directions in which their force is exerted. But the principle we are considering is that of *virtual velocities.* If the force is exerted in one line, and the motion of the point of application is in a different line, then its *virtual velocity* is merely that component which lies in the former line. A single example will illustrate the principle.

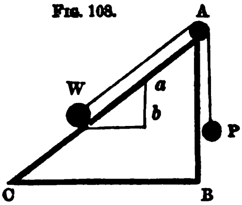
In the *inclined plane* (Fig. 108), if the power P moves, the *actual* velocity of the weight is equal to it; but its *virtual* velocity is to be reckoned in the line in which the force of gravity would move it. If P descends a distance equal to Wa, then W, though it passes over Wa, has a *virtual* velocity of only ba, measured in a vertical line. Estimating the velocities of the power and weight, therefore, in the lines in which they respectively act, we have, $V : v :: Wa : ba :: AC : AB$;

but (Art. 123), $P : W :: AB : AC$;

$\therefore V : v :: W : P$;

or, the *virtual* velocities are inversely as the forces.

FIG. 108.



We learn from the foregoing principle, that a machine does not enable us to obtain any *greater* effect than the power could produce without its aid, but only to produce an effect in a *different form.* A given power, for instance, may move a much *greater quantity* of matter by the aid of a machine, but it will move it as much more slowly. On the other hand, a power, by means of a machine, may produce a far *greater velocity* than would be possible without such aid; but the quantity moved would be proportionally less. By machines, therefore, we do not increase the effects of a power, but only modify them.

140. *Loss of effect by obstructions.*—No allowance has been made in the discussions of this chapter for the obstructions to motion, which always exist. On account of friction, rigidity of ropes, and the resistance of air, the effect produced by the operation of any machine is always less than what is given by theory. In many cases, as large a fraction as one-fourth or one-third must be deducted from the calculated effect. The subject of friction will be considered in Part II. of Mechanics.

CHAPTER VII.

MOTION ON INCLINED PLANES.—THE PENDULUM.

141. *The force which moves a body down a plane.*—In Art. 123 it was shown, that if a weight W be sustained upon an inclined plane AC (Fig. 109), by another weight W′ acting upon it in a direction parallel to the plane, then W′ : W : : AB : AC. Suppose now the string WAW′ to be cut in two, then it is evident that the weight W would descend down the plane with a force which bears to its own weight the ratio of AB : AC; and since this force is constant through every part of the plane, the body thus descending may be considered as acted upon in every point of its descent by a constant force, which bears to the force of gravity the given ratio of the height of the plane H to the length of the plane L; call this force F, and let the force of gravity be represented by unity, then F : 1 : : H : L, and $F=\frac{H}{L}$; i. e., the force which accelerates the motion of a body down an inclined plane, is such a part of the force of gravity as may be represented by the fraction $\frac{H}{L}$; this force, therefore, differs not from the force of gravity in kind, but in degree; the effects produced by it must consequently be analogous to the effects produced by gravity.

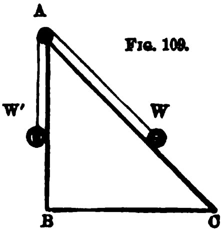


FIG. 109.

142. *Formulæ for motion on a plane.*—In the formulæ of Art. 28, g was employed as the measure of the force of gravity, since it expresses the rate of motion acquired in a second of time. But, on a plane whose height $=$ H, and length $=$ L, the force (Art. 141) is only $\frac{H}{L}g$. If, therefore, $\frac{H}{L}g$ be substituted for g in those formulæ, they will be adapted to motion on an inclined plane.

	Formulæ for free fall.	Formulæ for fall down a plane.
1.	$S=\frac{1}{2}gT^2$	$S=\frac{gHT^2}{2L}$
2.	$T=\sqrt{\frac{2S}{g}}$	$T=\sqrt{\frac{2LS}{gH}}$.

	Formulæ for free fall.	Formulæ for fall down a plane.
3.	$S=\frac{V^2}{2g}$	$S=\frac{LV^2}{2gH}$.
4.	$V=\sqrt{2gS}$.	$V=\sqrt{\frac{2gHS}{L}}$.
5.	$T=\frac{V}{g}$.	$T=\frac{LV}{gH}$.
6.	$V=gT$	$V=\frac{gHT}{L}$.

143. *Nature of motion on a plane.*—Since $S=\frac{gHT^2}{2L}$, therefore, on a given plane, $S \propto T^2$; that is, the space described varies as the square of the time when a body falls from rest down an inclined plane, as well as when it descends freely by the force of gravity; the spaces described from rest in equal successive portions of time will therefore be as the odd numbers 1, 3, 5, 7, &c.; and if the body be projected upward with the velocity acquired in falling through any space upon the plane, it will ascend to the point from which it fell, the spaces described in equal successive portions of time being as the numbers, 1, 3, 5, 7, &c., taken in the inverted order. If, moreover, at any point of its descent, it moves forward with the velocity acquired continued uniformly, it will describe twice the space in the same time as that in which it has fallen to acquire the velocity; and if it be projected downward or upward with the velocity V, and moves for the time T, the space described in that time will be equal to $TV \pm \frac{gHT^2}{2L}$. All this follows from the law of acceleration and retardation of bodies moving upon inclined planes, being the same as that which regulates the motion of bodies descending or ascending freely by the force of gravity. (See Arts. 23–27.)

144. *Formulæ for the whole length of a plane.*—If a body falls the whole length of a plane, then $S=L$; therefore Formula 4 gives $V=\left(\frac{2gHS}{L}\right)^{\frac{1}{2}}=(2gH)^{\frac{1}{2}}$; and this (Art. 28) is the velocity acquired in falling freely through H. Therefore, in descending the whole length of a plane, a body acquires *the same velocity as in falling freely through its height.*

Also, as $V=(2gH)^{\frac{1}{2}}$, $\therefore V \propto H^{\frac{1}{2}}$; or, the *velocity* acquired in descending a plane varies as *the square root of the height.*

Again, Formula 2 gives $T = \left(\frac{2LS}{gH}\right)^{\frac{1}{2}} = L\left(\frac{2}{gH}\right)^{\frac{1}{2}}$. But the time of falling freely through the height (Art. 28) is $\left(\frac{2H}{g}\right)^{\frac{1}{2}}$;

$\therefore$ T down a plane : T through height $:: L\left(\frac{2}{gH}\right)^{\frac{1}{2}} : \left(\frac{2H}{g}\right)^{\frac{1}{2}}$

$:: L\left(\frac{2}{g}\right)^{\frac{1}{2}} : H\left(\frac{2}{g}\right)^{\frac{1}{2}}$

$:: L : H.$

$L\sqrt{\frac{2}{gH}} : \sqrt{\frac{2H}{g}}$ mul by H

$\sqrt{\frac{2H}{gH}} : \sqrt{\frac{2H^2}{g}} = L\sqrt{\frac{2}{g}} : H\sqrt{\frac{2}{g}}$

That is, *the time of descending a plane is to the time of falling freely through its height, as the length to the height.*

And, as $T = L\left(\frac{2}{gH}\right)^{\frac{1}{2}}$, $\therefore T \propto \frac{L}{\sqrt{H}}$; or, the *time* of falling down a plane is as *the length directly*, and *the square root of the height inversely.*

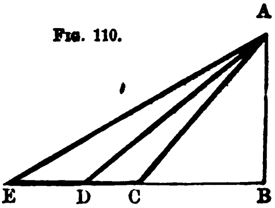
It follows, that if several planes have a common height, as AC, AD, AE (Fig. 110), the *same velocity* is acquired in descending them; for the velocity on each is equal to that acquired in falling down AB.

But the *times* of describing them are *as the lengths;* for

T down each plane : T down AB :: L : H;

and as the 2d and 4th terms are constant, the 1st varies as the 3d, i. e., $T \propto L$.

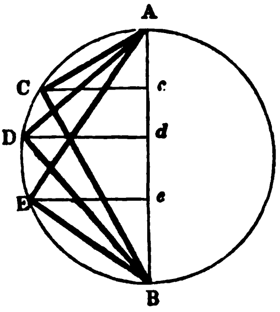
FIG. 110.



145. *Descent on the chords of a circle.*—If several chords of a circle terminate in the extremities of a vertical diameter, as in Fig. 111, then the *times* of describing them are *equal* to each other, and to the time of falling through the diameter; but the *velocities* acquired are *as the lengths.*

From the ends of the chords, C, D, E, draw ordinates to the vertical diameter, AB. By geometry, $AC^2 = Ac \times AB$; $\therefore AC = \sqrt{Ac} \times \sqrt{AB}$; and so of each chord. Now the *time* of descending a plane is as $\frac{L}{\sqrt{H}}$ (Art. 144); $\therefore$ times down AC, AD, AE, are as

FIG. 111.

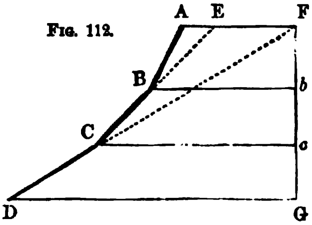


$\frac{AC}{\sqrt{Ac}}$, $\frac{AD}{\sqrt{Ad}}$, $\frac{AE}{\sqrt{Ae}}$, each of which equals $\sqrt{AB}$. Hence, the *times* of descent are *equal* to each other, and to the time of falling through AB.

Again, the *velocity* acquired on a plane, is as $\sqrt{H}$; ∴ velocities on AC, AD, AE, are as $\sqrt{Ac}$, $\sqrt{Ad}$, $\sqrt{Ae}$; which are as $\frac{AC}{\sqrt{AB}}$, $\frac{AD}{\sqrt{AB}}$, $\frac{AE}{\sqrt{AB}}$; and these are as AC, AD, AE. Hence, the *velocities* acquired are *as the lengths* of the chords.

146. *Velocity acquired on a series of planes.*—If no velocity should be lost in passing from one plane to another, the velocity acquired in descending a series of planes, would be equal to that acquired in falling through their perpendicular height. For, in Fig. 112, the velocity at B is the same, whether the body comes down AB or EB, as they are of the same height, F*b*. If, therefore, the body enters on BC with the acquired velocity, then it is immaterial whether the descent is on AB and BC, or on EC; in either case, the velocity at C is equal to that acquired in falling F*c*. In like manner, if the body can change from BC to CD without loss of velocity, then the velocity at D will be the same, whether acquired on AB, BC, and CD, or on FD, which is the same as down FG.

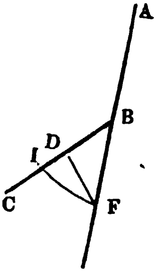
FIG. 112.



147. *The loss in passing from one plane to another.*—The condition named in the foregoing article is not fulfilled. A body *does* lose velocity in passing from one plane to another. And the *loss* is to the *whole previous velocity*, as the *versed sine of the angle* between the planes, to *radius*.

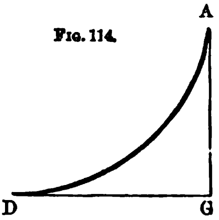
Let BF (Fig. 113) represent the velocity possessed at B. Resolve it into BD on the second plane, and DF perpendicular to it. BD is the initial velocity on BC; and, if BI = BF, DI is the loss. But DI is the versed sine of the angle FBD, to the radius BF; and ∴ the loss is to the velocity at B, as DI : BF : : ver. sin. B : rad.

FIG. 113.



148. *No loss on a curve.*—Suppose now the number of planes in a system to be increased, and their lengths and inclinations to each other to be diminished *ad infinitum*, then the system becomes a curve (Fig. 114). As the angle of two successive elements of the curve is infinitely small, its chord is also infinitely small; but its versed sine is *infinitely smaller still*, i. e., an infinitesimal of the *second* order; for (by Geom.) diam. : chord : : chord : ver. sin. Therefore, when the infinitesimal angle is multiplied so as to equal the whole change of direction in the curve, the versed sine, or loss of velocity, being multiplied by the same, is still an infinitesimal—being now of the *first* order. Hence, a body loses no velocity on a curve, and therefore acquires at the bottom the same velocity as in falling freely through its height.

Fig. 114.



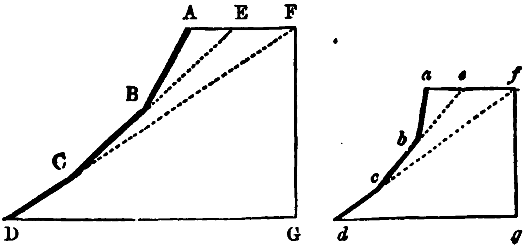
It appears, therefore, that whether a body descends *vertically*, or on an *inclined plane*, or on a *curve* of any kind, the *acquired velocity is the same*, if the height is the same.

149. *Times of descending similar systems of planes and similar curves.*—If two planes are equally inclined to the horizon, the *times* of describing them are as *the square roots of their lengths*. For (Art. 144), $T \propto \frac{L}{\sqrt{H}}$; and if the inclination is given, $\sqrt{H} \propto \sqrt{L}$ by similar triangles; $\therefore T \propto \frac{L}{\sqrt{L}} \propto \sqrt{L}$.

If two *systems* of planes are similar, i. e., if the corresponding parts are proportional and equally inclined to the horizon, it is still true that the times of descending them are as the square roots of their lengths.

Let ABCD and *abcd* (Fig. 115), be similar, and let AF and *af* be drawn horizontally, and the lower planes produced to meet them, then it is readily proved that all the homologous lines of the figures are proportional, and their square roots also proportional. Then (letting t,AB signify time down AB, &c.),

Fig. 115.



we have $t,\mathrm{AB} : t,ab :: \sqrt{\mathrm{AB}} : \sqrt{ab}$;

$t,\mathrm{EB} : t,eb :: \sqrt{\mathrm{EB}} : \sqrt{eb} :: \sqrt{\mathrm{AB}} : \sqrt{ab}$;

and $t,\mathrm{EC} : t,ec :: \sqrt{\mathrm{EC}} : \sqrt{ec} :: \sqrt{\mathrm{AB}} : \sqrt{ab}$;

∴ (by subtraction) $t,\mathrm{BC} : t,bc :: \sqrt{\mathrm{AB}} : \sqrt{ab}$.

In like manner, $t,\mathrm{CD} : t,cd :: \sqrt{\mathrm{AB}} : \sqrt{ab}$.

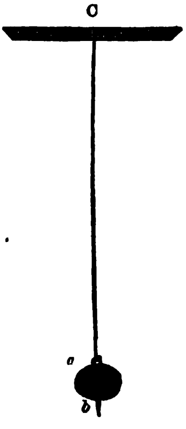
∴ (by addition)

$$t,(\mathrm{AB}+\mathrm{BC}+\mathrm{CD}) : t,(ab+bc+cd) :: \sqrt{\mathrm{AB}} : \sqrt{ab}$$
$$:: \sqrt{(\mathrm{AB}+\mathrm{BC}+\mathrm{CD})} : \sqrt{(ab+bc+cd)}.$$

This reasoning is applicable when the number of planes is increased and their lengths and inclinations to each other diminished *ad infinitum;* i. e., when they have become *curves*, similar, and similarly inclined to the horizon. Suppose these curves to be *circular* arcs; then, as they are similar, they are proportional to their radii. Hence, the times of descending similar circular arcs are as the square roots of the radii of those arcs.

150. *The pendulum.*—A weight attached by an inflexible rod to a point of suspension, about which it is free to move, is called *a pendulum.* If it is drawn aside from a vertical line, and left to the influence of gravity, it descends, and by the momentum acquired, rises on the opposite side to the same height, when gravity again causes its descent, as before. If unobstructed, its vibrations would never cease. One *vibration* is the motion from the highest point on one side to the highest point on the other. The *center of oscillation* of a pendulum is such a point, that, were all the matter of the pendulum collected in it, the quantity of motion (or momentum) would be equal to the sum of the momenta of all the parts taken separately.

Fig. 116.



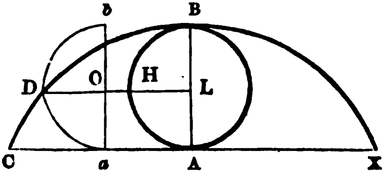
Thus (Fig. 116), the parts of the pendulum about b move faster than those about a, and consequently each particle has more momentum; but there is a point about which the momenta balance each other, and therefore in the investigations relating to the pendulum, all the parts of which it consists may be considered as concentrated in that point.

The center of oscillation is below the center of gravity; for, since the parts more remote from the center of suspension have more velocity than the parts that are nearer to it, the quantity of matter below the center of oscillation must be less than the quantity of matter above it. The *length* of the pendulum is the distance from the point of suspension to the center of oscillation.

151. *The cycloid.*—One of the methods of investigating the theory of the pendulum, is by means of the properties of the cycloid. This curve is described by a point situated on the circumference of a circle, as it rolls on a straight line.

Let the circle AHB (Fig. 117) make one revolution upon the line CAX, equal to its circumference; the curve line CDBX, traced out by that point of the circle which was in contact with C when the circle began to revolve, is called a cycloid. If CX be bisected in A, and AB be drawn at right angles to it, it is evident, from the manner in which the curve is generated, that it will have similar branches on both sides of AB, and that its vertex B will be so placed as to make its *axis* AB equal to the diameter of the generating circle. Its properties, as applied to the vibration of the pendulum, are the following.

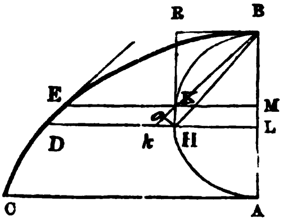
FIG. 117.



152. The *cycloidal ordinate* DH *equals the circular arc* BH.—For, let *b*D*a* (Fig. 117) be the position of the circle when the generating point is at D; draw the diameter *ba* parallel to BA, and from D draw DHL parallel to CA; then the arc D*a* = arc HA, ∴ sin DO = sin HL, and consequently DH = OL; but from the mode in which the cycloid is generated, C*a* = arc D*a*, and CA = semicircle BHA; hence DH = OL = *a*A = CA − C*a* = semicircle BHA − arc HA = arc BH.

153. *A tangent to the cycloid at any point,* E (Fig. 118), *is parallel to the corresponding chord* BK *of the generating circle.*—Draw DHL indefinitely near to EKM; join BK, and produce it to *k*. The indefinitely small triangle HK*k* is similar to the triangle KRB formed by the tangents (KR, BR) to the circle at the points K, B, and is consequently isosceles; ∴ KH = H*k*. Now by Art. 152, arc BKH = DH, ∴ BKH − KH (= arc BK = EK) = DH − H*k* = D*k*; ∴ EK = D*k*. Hence, since EK and D*k* are equal and parallel, ED and K*k* must also be equal and parallel; and as the tangent at the point E may be considered as coinciding with ED, it must therefore be parallel to the chord BK.

FIG. 118.



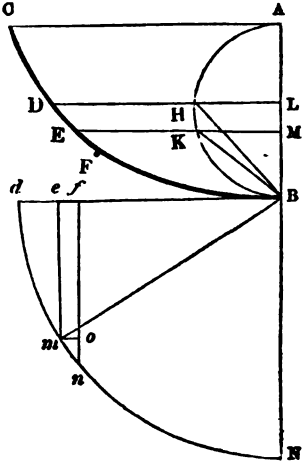
Hence the ends of the cycloid meet the base at right angles; for the tangent at C is parallel to BA, the axis.

154. *The cycloidal arc* BE *is equal to twice the correspond-*

ing chord BK *of the generating circle.*—Draw Ho perpendicular to Kk; and since the triangle KHk is isosceles, Ho bisects the base Kk, ∴ Kk or $\mathrm{ED} = 2\mathrm{K}o$; and since H$o$ may be considered as a small circular arc described with radius BH, $\mathrm{K}o = \mathrm{B}o - \mathrm{BK} = \mathrm{BH} - \mathrm{BK}$; hence ED and K$o$ are cotemporaneous increments of the cycloidal arc BE and the chord BK; and as the arc and chord begin together from the point B, and the former increases by ED or Kk, while the latter increases by $\mathrm{K}o = \frac{1}{2}\mathrm{ED}$, the arc BE must be equal to twice the chord BK; consequently, the whole arc BC = twice the diameter AB; and the whole curve CBX (Fig. 117) : the base CX : : 4 : π.

155. *Descent by gravity on a cycloid.*—AHB (Fig. 119) is a circle, whose diameter AB is perpendicular to the horizon, and CDB a cycloidal arc. Now, by Art. 148, if a body begins to descend from any point D, its velocity at the point E will be the same as the velocity at the point M of a body falling freely through the perpendicular height LM; and its velocity at every other point during its descent through the cycloidal arc DEB will be the same as the velocity of the body falling freely at every other corresponding point of the line LMB. By Art. 144, therefore, the velocity V of the body thus descending along the cycloidal arc DE will vary as $\sqrt{(\mathrm{LM})} \propto \sqrt{(\mathrm{BL} - \mathrm{BM})} \propto \sqrt{(\mathrm{AB} \times \mathrm{BL} - \mathrm{AB} \times \mathrm{BM})} \propto \sqrt{(\mathrm{HB}^2 - \mathrm{KB}^2)} \propto \sqrt{(\mathrm{DB}^2 - \mathrm{EB}^2)}$. Hence, let B$d$ be drawn parallel to AC and equal to BD, and upon Bd describe the quadrant of a circle dmN; take Be equal to BE, and draw em at right angles to Bd; then will $em^2 = \mathrm{B}m^2 - \mathrm{B}e^2 = \mathrm{B}d^2 - \mathrm{B}e^2 = \mathrm{DB}^2 - \mathrm{EB}^2$, and consequently V, which varies as $\sqrt{(\mathrm{DB}^2 - \mathrm{EB}^2)}$, will vary as em, the sine of the arc dm, whose versed sine is de or DE, the space fallen through.

FIG. 119.



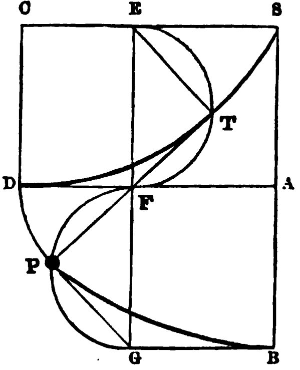
156. Let EF be an indefinitely small part of the cycloidal arc, and make ef equal to it; draw fn at right angles to Bd, and mo parallel to it. Since EF is very small, it may be considered as described with the velocity V at E continued uniformly, and therefore the *time* of describing EF $\left(\text{since } \mathrm{T} = \frac{\mathrm{S}}{\mathrm{V}}\right)$ will be represented by $\frac{\mathrm{EF}}{\mathrm{V}}$ or $\frac{ef}{em} = \frac{om}{em}$. Now, since the sine em

represents the velocity at any point E, the whole velocity acquired in falling down DB will be represented by the radius BN or Bm; if, therefore, a body were to describe the quadrantal arc dmN with the velocity at the lowest point continued uniformly, the time of describing any small part (mn) of it would be represented by $\frac{mn}{Bm}$. But by similar triangles, Bme, mno, Bm : em : : mn : om; $\therefore \frac{om}{em} = \frac{mn}{Bm}$; hence the time of describing the small cycloidal arc EF is equal to the time of a body's moving through the corresponding small circular arc mn, with the velocity in B continued uniformly; *the whole time of descent, therefore, through* DEB *will be equal to the time of a body's describing the quadrantal arc dm*N, with the velocity at B continued uniformly.

157. We wish now to compare this time of descending DB, with the time of falling freely down AB. In the time of falling down AB, a body would descend the chord HB (Art. 145), and, in so doing, would acquire the same velocity as down DB, i. e., BN. And, with this velocity continued uniformly, it would pass over 2HB = BD = Bd (Art. 25). Therefore it is proved, that while DB is described, a body could describe the quadrantal arc dN, with a certain uniform velocity, BN; and while AB is fallen through, a body could describe the radius Bd with the same uniform velocity BN. ∴ time down DB : time down AB : : a quadrantal arc : radius : : semi-circumference : diameter.

158. *The involute of a semi-cycloid.*—The *involute* of any curve is another curve described by the extremity of a tangent as it unwinds from the former, which is called the *evolute*. If, for example, a tangent of a circle unwinds from it, the circumference of the circle is the *evolute*, and the spiral described by the end of the tangent is the *involute* of the circle. The involutes of most curves are different from their evolutes; but in the case of the semi-cycloid, the involute and evolute are of the same form and size.

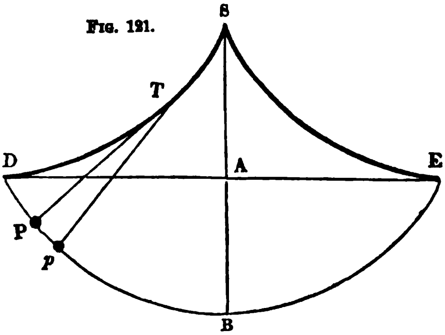
Fig. 120.



Take any line SC (Fig. 120), and draw SA at right angles to it; make SC : SA : : semi-circumference of a circle : its diameter; and complete the parallelogram SCDA. Produce SA to B, making AB = SA; upon SC, AD, describe two semi-cycloids SD, DB, the vertex of the former of which is at D, and the latter at B; then if the tangent unwinds, beginning at D, until the point of contact reaches S, its extremity will always be found in the semi-cycloid DB. For, through any point F on AD, draw EFG perpendicular to SC, and through B draw BG parallel to SC; then EG = SB; on EF, FG, describe the semicircles ETF, FPG, and draw the chords TF, FP, the former of which (Art. 153) is a tangent to the cycloid SD at T. Now SE = arc ET, and SC = ETF; ∴ CE (= DF) = arc TF; but DF = FP; ∴ arcs FT, FP are equal, and also the angles subtended, TEF, FGP. Therefore, as T and P are right angles, EFT = PFG, and PFT is a straight line; moreover, TP = 2TF = (Art. 154) the cycloidal arc TD. Therefore, TP is a tangent unwound from D, and P is its extremity; and P having been assumed anywhere on the semi-cycloid DB, it follows that DPB is the involute of STD.

159. *The cycloidal pendulum.*—A pendulum may be made to vibrate in a cycloid, by attaching the weight P (Fig. 121) to a flexible cord, whose point of suspension is at S, where two semi-cycloids meet. The cord and the semi-cycloid should be of the same length, and then (Art. 158) the weight P will, at each vibration, describe arcs of the cycloid DBE, as involutes of SD and SE. Hence, the conclusion arrived at in Art. 157, applies to this motion; namely, *the time down* PB, *from any point* P : *time down* AB : : *semi-circumference* : *diameter;* ∴ doubling the antecedents, *the time of one vibration* : *time of falling half the length of the pendulum* : : π : 1.

Fig. 121.



160. Since (Art. 157) the time of a vibration is to the time down the axis in the same given ratio, whatever be the point from which the pendulum begins its oscillations, *all the vibrations of a pendulum of the same length are equal to each other.* But from the difficulty of constructing plates of the exact form SD, SE, and from other causes, the cycloidal pendulum is of little or no *practical* utility. In a geometrical point of view, however, this mode of comparing the time of a vibration with the time of falling down a space equal to half the length of the string, is of considerable importance; for the cycloid at the lowest point B may evidently be considered as a circular arc described with the radius SB; if therefore a body be suspended by a string whose length is SB, and vibrates in a circular arc only to a very short distance on each side of the point B, the time of the vibration of a pendulum in small *circular* arcs, is to the time down half the length of the pendulum, as the circumference of a circle to its diameter; and therefore, within moderate limits, the time will be the same, whether the arc of vibration be larger or smaller.

161. *Relation of time, length, and force of gravity.*—Let $L =$ the length of a pendulum from the point of suspension to the center of oscillation. Then the time of falling down half the length $= \left(\frac{2S}{g}\right)^{\frac{1}{2}} = \left(\frac{L}{g}\right)^{\frac{1}{2}}$. Hence, the time of a vibration, $T : \left(\frac{L}{g}\right)^{\frac{1}{2}} :: \pi : 1$; or, $T = \left(\frac{\pi^2 L}{g}\right)^{\frac{1}{2}}$; and $L = \frac{gT^2}{\pi^2}$. Therefore, the length of a pendulum being known, the time of one vibration is found, and the reverse.

From the same formulæ, we obtain $T \propto \sqrt{L}$, or $L \propto T^2$. If, therefore, a *seconds pendulum* $= L$, a pendulum which beats *double seconds* $= 4L$, and one which beats *half seconds* $= \frac{1}{4}L$.

Again, by observing the *length* of a pendulum which vibrates in a given *time*, the *force of gravity*, g, may be found. For, as $L = \frac{gT^2}{\pi^2}$, $\therefore g = \frac{\pi^2 L}{T^2}$. And, if g varies, as it does in different latitudes, and at different altitudes, then $L = \frac{gT^2}{\pi^2} \propto gT^2$; and if the time is constant (as, for example, *one second*), then $L \propto g$. Hence, the *length* of a pendulum for beating seconds, varies as *the force of gravity*.

Also, $T \propto \left(\frac{L}{g}\right)^{\frac{1}{2}}$; that is, the time of a vibration varies directly as the square root of the length, and inversely as the square root of the force of gravity.

Since the number, N, of vibrations in a given time varies inversely as T, $\therefore N \propto \left(\frac{g}{L}\right)^{\frac{1}{2}}$.

1. What is the length of a pendulum to beat seconds, at the place where a body falls $16\frac{1}{12}$ ft. in the first second?

Ans. 39.11 inches, nearly.

2. If 39.11 in. is taken as the length of the seconds pendulum, how long must a pendulum be to beat 10 times in a minute? *Ans.* $117\frac{1}{3}$ feet.

3. In London, the length of a seconds pendulum is 39.1386 inches; what velocity is acquired by a body falling one second in that place? *Ans.* 32.19 feet.

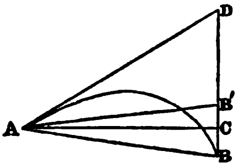
CHAPTER VIII.

PROJECTILES.

162. It has been shown already (Art. 49), that a body projected in any direction not coincident with the vertical, describes a parabola. In swift motions, however, the path of a projectile differs widely from a parabola; and the laws of atmospheric resistance must be employed to obtain corrections for the conclusions deduced in this chapter. (See Part II., Chap. VI.)

163. *Formulæ investigated.*—In order to investigate the *general formula*,* let A (Fig. 122) be the point of projection, AB or AB′ the plane on which the body is projected, passing through A. AB also denotes the *range* or distance to which the body is thrown. Let AC be drawn parallel, and BCD perpendicular to the horizon; let the angle of elevation CAD $= a$, the angle of depression of the plane CAB $= b$, the velocity of projection $= v$, the time of flight $= t$, the range AB $= r$, and the velocity acquired by gravity in one second, or 32⅙ feet $= g$.

FIG. 122.



Then, by the laws of uniform motion, at the end of the time t, if gravity did not act, the body would be found in the point D, while, by the laws of falling bodies, it would in the same time pass through the perpendicular DB; consequently,

$$\text{AD} = tv; \text{ and DB} = \tfrac{1}{2}gt^2.$$

In the right-angled triangles ABC and ADC, the angle B is the complement of b, and the angle D is the complement of a; and, since the sides are as the sines of the opposite angles,

$$\cos b : \sin (a \pm b)\dagger :: tv : tv\frac{\sin (a \pm b)}{\cos b} = \tfrac{1}{2}gt^2.$$

Or,
$$\frac{gt}{2v} = \frac{\sin (a \pm b)}{\cos b}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (1.)$$

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[*] See *Encyc. Metropolitana*, Art. Mechanics, p. 105, from which a large part of this chapter is taken.

[†] *Plus* when the plane descends, *minus* when the plane ascends.

Again, $\cos a : \sin (a \pm b) :: r : \dfrac{r \sin (a \pm b)}{\cos a} = \frac{1}{2}gt^2$.

Or, $$\frac{gt^2}{2r} = \frac{\sin (a \pm b)}{\cos a}. \quad \ldots \ldots \quad (2.)$$

From equation (1) $t = \dfrac{2v \sin (a \pm b)}{g \cos b}$,

and $t^2 = \dfrac{4v^2 \sin^2 (a \pm b)}{g^2 \cos^2 b}$.

From equation (2) $t^2 = \dfrac{2r \sin (a \pm b)}{g \cos a}$.

Hence, $$\frac{r}{v^2} = \frac{2 \sin (a \pm b) \cos a}{g \cos^2 b}. \quad \ldots \ldots \quad (3.)$$

From these three equations, all the relations between the time, velocity, range, and angle of elevation, are readily determined; so that any two of these four quantities being given, the other two may be found. Thus,

By equation (1) $v = \dfrac{gt \cos b}{2 \sin (a \pm b)}$.

By equation (2) $r = \dfrac{gt^2 \cos a}{2 \sin (a \pm b)}$.

If the *range* and *elevation* are given to find the *time* and *velocity*,

By equation (2) $t = \left(\dfrac{2r \sin (a \pm b)}{g \cos a}\right)^{\frac{1}{2}}$.

By equation (3) $v = \left(\dfrac{rg \cos^2 b}{2 \sin (a \pm b) \cos a}\right)^{\frac{1}{2}}$.

If the *velocity* and *elevation* are given to find the *time* and *range*,

By equation (1) $t = \dfrac{2v \sin (a \pm b)}{g \cos b}$.

By equation (3) $r = \dfrac{2v^2 \sin (a \pm b) \cos a}{g \cos^2 b}$.

If any two of the above quantities be given to find the *angle of elevation*, then (the inclination of the plane to the horizon (b) being supposed to be known) in order to find the value of a, we must substitute for $\sin (a \pm b)$, its value involving the sine of each arc separately. Now $\sin (a \pm b) = \sin a \cos b \pm \sin b \cos a$, whence either the $\sin a$ or $\cos a$ may be obtained.

The value of a and b being known, we shall then find in all the preceding cases, $\sin (a \pm b) \cos a$ equal to a known quantity. Let this be denoted by C; then $\cos a \sin a \cos b \pm \sin b \cos^2 a = C$.

Let $x = \sin a$, then $\cos a = (1 - x^2)^{\frac{1}{2}}$,

$$x(1 - x^2)^{\frac{1}{2}} \pm \tan^* b(1 - x^2) = \frac{C}{\cos b}.$$

164. As this is a quadratic equation, the solution will give two values of x, which shows that there are always two different angles of elevation, which equally satisfy the conditions of the problem, except in the case where the two roots of the equation are equal to each other, when only one angle will be found. In this case, as is shown below, $\sin a = \sin \frac{1}{2}(90° \pm b)$, under which limitation the range will be a maximum, or the greatest possible; and all angles of elevation equally above and below that which gives the maximum range, will give ranges equal to each other. For, by the value of r as determined above,

$$r = 2v^2 \frac{\sin(a \pm b) \cos a}{g \cos^2 b}.$$

If v and the angle b are given, the range will vary as $\sin(a \pm b) \cos a$. But

$$\begin{aligned} \sin(a \pm b) \cos a &= \sin a \cos a \cos b \pm \sin b \cos^2 a \\ &= \tfrac{1}{2} \sin 2a \cos b \pm \sin b \left(\tfrac{1}{2} + \tfrac{1}{2} \cos 2a\right)\dagger \\ &= \tfrac{1}{2} \sin 2a \cos b \pm \tfrac{1}{2} \cos 2a \sin b + \tfrac{1}{2} \sin(\pm b) \\ &= \tfrac{1}{2} \sin(2a \pm b) + \tfrac{1}{2} \sin(\pm b); \end{aligned}$$

and since the second part of this expression is constant, the range will vary as $\sin(2a \pm b)$. This quantity will be greatest, when

$$2a \pm b = 90°.$$

$$\therefore \quad a = \tfrac{1}{2}(90° \pm b)$$

$$\text{and} \qquad \sin a = \sin \tfrac{1}{2}(90° \pm b).$$

Therefore, as above, the range will be a maximum where the sine of the angle of elevation equals the sine of $\frac{1}{2}(90° \pm b)$.

And since all angles equally above and below 90° have the same sine, all angles equally above and below that which gives the maximum have equal ranges. Thus, a cannon-ball fired at an angle of 60° (in a vacuum) above a horizontal plane, would reach the plane at the same distance from the point of projection as if fired at an elevation of 30°. When the data of the problem give or require a greater value for $\sin(2a \pm b)$ than 1, that is, than the sine of 90°, it shows the problem, under the proposed conditions, to be impossible.

165. To find the greatest *height* to which the projectile will ascend, we must recollect that a body projected perpendicularly upward, will rise to the same height from which it must have fallen to acquire the velocity of projection (Art. 24). Calling the time of rising to the greatest height t', we shall have $gt'=$ the velocity of descent from gravity; and, v representing the whole velocity of projection in an oblique direction, the perpendicular component will be represented by $v \sin a$; whence

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* $\frac{\sin b}{\cos b} = \tan b$.

† Analytical Trigonometry.

$$gt' = v \sin a, \text{ and } t' = \frac{v \sin a}{g}, \text{ and } t'^2 = \frac{v^2 \sin^2 a}{g^2}.$$

But the space fallen through in the time $t' = \frac{1}{2}gt'^2 = \dfrac{v^2 \sin^2 a}{2g}$. And the distance in the same time, if the velocity, $v \sin a$, is continued uniformly, equals $t'v \sin a = \dfrac{v^2 \sin^2 a}{g}$. Subtracting the former from the latter (Art. 27), we have for the greatest height of the projectile,

$$h = \frac{v^2 \sin^2 a}{g} - \frac{v^2 \sin^2 a}{2g} = \frac{v^2 \sin^2 a}{2g}, \text{ and } \frac{2gh}{v^2} = \sin^2 a. \quad . \quad (4)$$

If therefore the angle of elevation and velocity are given, the greatest height may be determined; or if the range (r) or the time (t) be given (the angles being known), the value of v^2 may first be ascertained by preceding formulæ, and then the height (h) from Equation (4).

166. *Particular formulæ for a horizontal plane.*—All the preceding equations become much more simple when the projection is along a *horizontal* plane; for then $b = 0$, and $\sin b = 0$, and $\cos b = 1$; hence,

$$\text{In Eq. (1) } \frac{gt}{2v}\left(= \frac{\sin(a \pm b)}{\cos b}\right) = \sin a. \quad . \; . \; . \; . \; . \; . \; . \; . \quad (1')$$

$$(2)\ \frac{gt^2}{2r}\left(= \frac{\sin(a \pm b)}{\cos a}\right) = \frac{\sin a}{\cos a} = \tan a. \quad . \; . \; . \; . \quad (2')$$

$$(3)\ \frac{r}{v^2}\left(= \frac{2 \sin(a \pm b) \cos a}{g \cos^2 b}\right) = \frac{2 \sin a \cos a}{g} = \frac{\sin 2a}{g}. \quad (3')$$

$$\text{Hence from } (1')\ t = \frac{2v \sin a}{g} \propto v \times \sin a. \quad . \; . \; . \; . \; . \; . \quad (5)$$

$$(3')\ r = \frac{v^2 \times \sin 2a}{g} \propto v^2 \times \sin 2a. \quad . \; . \; . \; . \; . \quad (6)$$

$$(4)\ h = \frac{v^2 \times \sin^2 a}{2g} \propto v^2 \times \sin^2 a. \quad . \; . \; . \; . \; . \; . \quad (7)$$

On a horizontal plane, therefore (the most usual case), we have the following THEOREMS.

I. *The* TIME OF FLIGHT *varies as the velocity of projection multiplied by the sine of the angle of elevation.*

II. *The* RANGE[*] *varies as the square of the velocity of projection, multiplied by the sine of twice the angle of elevation.*

III. *The* GREATEST HEIGHT *varies as the square of the velocity of projection, multiplied by the square of the sine of the angle of elevation.*

Moreover, since the sine of twice 45° equals the sine of 90°, which equals radius, hence, by Theorem II,

IV. *The* RANGE IS GREATEST *when the angle of elevation is* 45°, *and is the same at elevations equally above and below* 45°.[†]

And since the square of the sine of the angle of elevation must be greatest when the angle is a right angle, therefore, by Theorem III,

V. *A projectile rises to the* GREATEST HEIGHT *when thrown perpendicularly upward.*

Finally, since the sine of the angle of elevation is greatest when the angle is a right angle, therefore, by Theorem I,

VI. *The* TIME OF FLIGHT IS GREATEST, *when the body is thrown perpendicularly upward.*

167. *Questions on projectiles.*—

1. A gun was fired at an elevation of 50°, and the shot struck the ground at the distance of 4898 feet; with what velocity did it leave the gun, and how long was it in the air?

Ans. Velocity, 400 feet per second.
Time, 19.05 seconds.

2. Random 4898 feet, time of flight 16 seconds; required the angle of elevation and the velocity of projection?

Ans. El. 40° 3′, V. 400 feet per sec.

3. Random 2898 feet, velocity of projection 389.1 feet; what were the elevation and time of flight?

Ans. El. 19° or 71°, T. 7.87 sec.

4. Elevation 40°, random 4898; required the random when the elevation is 29½°? *Ans.* 4263.

5. Elevation 40° 3′, time of flight 16 seconds; required the random and velocity of projection? *Ans.* R. 4898, V. 400 ft.

6. Velocity 510 feet per sec., time of flight 15 seconds, to find the elevation and random. *Ans.* El. 28° 14′, R. 6740.

7. On a side-hill ascending uniformly above a horizontal level at an angle of 10° 20′, a ball was fired at an angle of elevation above the horizon of 34°, and with a velocity of 401 feet per second; what was the range on the hill-side when the gun was directed up the hill, and what when directed downward?

Ans. 3438 and 5985 feet.

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[*] Sometimes called *random.*

[†] For the sine of twice any angle below 45° is the same as the sine of twice the angle of the same number of degrees above 45°.

CHAPTER IX.

STRENGTH OF MATERIALS.

168. THE importance to the architect and the engineer of ascertaining the form and position of the materials which he employs, in order to secure the greatest degree of strength and stability at the least expense, has led mathematicians and writers on mechanics, to devote much attention to this subject. How is the strength of a beam affected by giving to it different shapes and different positions? how must a given quantity of matter be disposed in order that it may have the greatest possible strength? and upon what principles depends the stability of columns, roofs, and arches? these, and many similar inquiries, have been the objects of profound investigation.

The strength of *beams*, or pieces of timber, is the first object of inquiry. STRENGTH is the power to *resist* fracture; STRESS, the power to *produce* fracture.

169. *Longitudinal strength.*—When a force is applied to a rod or bar, to *pull* it asunder, its *strength* is proportional to *the area of its cross-section.* Here each line of particles, in a longitudinal direction, may be considered as exerting a separate force, and therefore the aggregate force will manifestly depend on their number, and of course on the area of its section, the whole being equal to the sum of all its parts. Hence the various shapes of bars make no difference in their absolute strength, since this depends only on the area of the section, and must obviously be the same when the area is the same, whatever be its figure. A rope, therefore, or a wire, to which a weight is appended, is as likely to break in one place as in another, but when the weight of the rope becomes considerable, and the force is applied perpendicularly, the increase of weight, as its length increases, renders it more liable to break in the upper than in the lower parts.

170. *Lateral strength, support at each end.*—When a beam rests horizontally, supported at both ends, and pressed by a weight at the center, its *strength* at that point varies as the *area of the cross-section, multiplied by the depth of the center of gravity.* In Fig. 123, let W represent the weight at the center, and *abcd* the cross-section; then (supposing the weight to be sufficient to break the beam), the fracture will commence at the bottom, and proceed regularly to the top, ending at W.*

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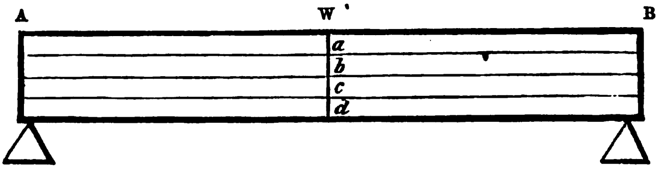
* It is here supposed, according to the views embraced by most writers on Mechanics since the time of Galileo, that the parts of a fractured beam turn

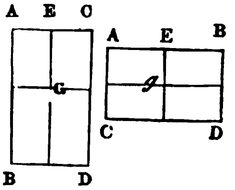
Now since the tendency to resist fracture from cohesion, must depend upon the total amount of all the separate forces acting between contiguous particles, it must evidently depend upon the number of the particles, that is, upon the extent of surface, or area of the section.

FIG. 123.



This would be the case were no other mechanical force involved but cohesion; but the *reaction* of the supports at A and B, being equal to the weight, and severally acting at the longer end of a bent lever AW*a*, AW*b*, &c., consequently, the tendency to fracture, from the leverage, will be lessened, as the shorter arm of the lever is increased, while the longer arm remains the same: therefore the strength being inversely as the stress, will be regularly increased as the distance of any lamina from W is increased, and the whole effect will be as the distance of the center of gravity from that point. Hence from both causes, the strength varies as the area of the section multiplied by the depth of the center of gravity. If A = area of section, G = depth of center, and S = strength, then S ∝ A . G.

FIG. 124.



171. This proposition is general, and applies to a number of distinct cases. In *cylindrical* and *square* beams, since the area of the section varies as the square of its diameter, and the distance of the center of gravity from the point E (Fig. 124) varies as the diameter, their strength is as the *cubes* of the diameters. In beams of an *oblong* section, the strength varies as the *breadth* and *square of the depth*; for here the area being as the product of the two sides, and the distance of the center of gravity from E being equal to half the perpendicular side, and therefore proportioned to that side, the proposition is, that the strength varies as the breadth × depth × depth, or as the breadth into the square of the depth. Hence, the same beam with its narrow side upward, is as much stronger than with its broad side upward, as the depth exceeds the breadth. For the

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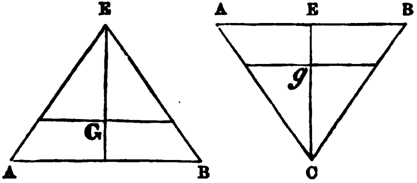
about the line where the fracture terminates; but Mr. Barlow, in his essay on the Strength and Stress of Timber, proves by experiment that the tendency is to turn about a line entirely within the section, the fibers on that side of the line where the fracture begins being extended, and those on the other side compressed. This line he calls the *neutral axis*.

area being the same in both cases, the strengths are proportioned to EG and E*g*, or as AB to AC. Thus if a joist be 10 inches broad and 2½ thick, it will bear four times more weight when laid on its edge than on its side. Hence the modern mode of flooring with thin, but deep pieces of timber.

Again, a *triangular* beam is *twice* as strong when resting on a *side*, as when resting on an *edge*. For, the area being the same in both cases, the strength varies as EG and E*g* (Fig. 125), which are as 2 to 1. (Art. 76.) These principles apply not only to beams, but to bars, and similar forms of every sort of matter.

FIG. 125.



172. Other things remaining the same, the strength of a beam varies *inversely as the product of the length and weight*. Let L = length, and W = weight. In Fig. 123, we may, as before, regard the point W as a fulcrum, on which each half revolves when fracture occurs. The upward force at the end is $\frac{1}{2}$W, since half the weight rests on each support; and the acting distance is $\frac{1}{4}$L, because the half beam has its center of gravity at $\frac{1}{4}$L from the end. Hence, the *stress* is as $\frac{1}{2}W \times \frac{1}{4}L = \frac{1}{8}L.W$. But the *strength* is inversely as the stress; $\therefore S \propto \frac{1}{L.W}$. Combining this result with the former, we have $S \propto \frac{A.G}{L.W}$.

173. If beams are *similar*, then their strength is *inversely as any one of their three dimensions*. Being similar, their length, breadth, and thickness are proportional. Let D represent any one of these three dimensions. Then, in the formula, $S \propto \frac{A.G}{L.W}$, since $A \propto$ breadth into thickness, and $G \propto$ thickness, $A.G \propto D^3$; also $L \propto D$, and $W \propto D^3$;

$$\therefore S \propto \frac{D^3}{D^4} \propto \frac{1}{D}.$$

Hence it is that structures are weaker as they are larger, though all the parts preserve the same proportions. If a model is *three* feet long, and the structure 75 feet long, then the structure is 25 times weaker than the model.

174. *Additional weight at the center*.—Thus far, the weight has been supposed to be that of the beam alone, distributed equally through its length. If we now suppose an extra weight W′, placed at the middle point, the stress produced by it is twice as great as if distributed through the length; for its distance from the support is $\frac{1}{2}$L, instead of $\frac{1}{4}$L. Therefore, the stress of W′ is $\frac{1}{2}L \times \frac{1}{2}W'$, and that of the beam is, as before, $\frac{1}{4}L \times \frac{1}{2}W$; $\therefore$ the whole stress is $\frac{1}{4}LW' + \frac{1}{8}L \,.\, W = \frac{1}{4}L(\frac{1}{2}W + W')$; and the strength, $S = \dfrac{A \,.\, G}{\frac{1}{4}L(\frac{1}{2}W + W')} \propto \dfrac{A \,.\, G}{L(\frac{1}{2}W + W')}$.

If the weight of the beam itself were trifling compared with the weight laid upon it, then $S \propto \dfrac{A \,.\, G}{L \,.\, W'}$.

175. In order that the foregoing general formulæ may be applied to practice, so as to find the actual strength of bars or beams, it is necessary to have some standard of strength ascertained by experiment, which may be employed as the unit of comparison. For example, it is found by experiment that a stick of *oak*, *one foot long* and *one inch square*, is able, when supported at both ends, to sustain a weight of 600 pounds; and that a bar of *iron* of the same dimensions would sustain, in the same circumstances, 2190 pounds. The oak weighs half a pound, and the iron three pounds. With these data applied to the foregoing formulæ, we may perform such problems as the following:

1. What weight might be sustained at the middle point of a prismatic beam of oak, whose length is 6 feet, and its end 4 inches square?

If the weight of the bar is left out of account,

$\dfrac{A \,.\, G}{L \,.\, W'} = \dfrac{1^2 \,.\, \frac{1}{2}}{1 \,.\, 600}$, for one case; and $\dfrac{4^2 \,.\, 2}{6 \,.\, W'}$, for the other.

But these expressions are to be equal, since the strength is supposed to be reduced to *zero*, by the stress which the weight occasions;

$$\therefore \quad \frac{1}{1200} = \frac{16}{3W'}; \quad \therefore W' = 6400 \text{ lbs.} \quad \textit{Ans.}$$

If the weight of the beams be considered, then

$$\frac{1^2 \,.\, \frac{1}{2}}{1 \,.\, (\frac{1}{4} + 600)} = \frac{4^2 \,.\, 2}{6\,(24 + W')}; \quad \therefore W' = 6378\tfrac{2}{3} \text{ lbs.}$$

In this example, the weight of the large beam is known to be 48 lbs., from the given dimensions and weight of the small one.

2. What must be the depth of a beam of an oblong prismatic form, whose breadth is 2 inches and length 8 feet, to support a weight of 6400 pounds, its own weight not taken into consideration?

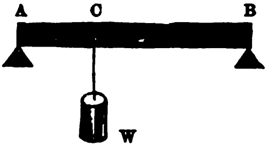
$$\frac{1^2 \,.\, \frac{1}{2}}{1 \,.\, 600} = \frac{2 \,.\, D \,.\, \frac{1}{2}D}{8 \,.\, 6400}; \quad \therefore D = 6.53 \text{ in.} \quad \textit{Ans.}$$

3. What weight might be supported at the middle point of a bar of iron 10 feet long, and the side of whose square end is 3 inches, its own weight not being taken into consideration?

Ans. 5913 pounds.

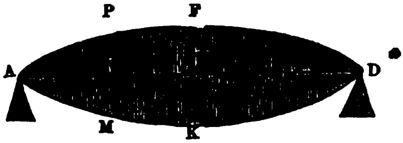
176. *Additional weight, at any point.*—The *stress* produced by a weight, at any point, is as *the product of the two distances from the ends.* In Fig. 126, according to the theorems for parallel forces, pressure at $A = \frac{W \times BC}{AB}$, and pressure at $B = \frac{W \times AC}{AB}$. But the *reaction* of either point of support is equal to the pressure on that point; and this force acts at C with a *leverage* AC on one side, and BC on the other, so that the stress at C = $\frac{W \times BC}{AB} \times AC$, or $\frac{W \times AC}{AB} \times BC$, either of which expressions = stress at C, and $\propto AC \times BC$. And since this rectangle is greatest when $AC = CB$, and diminishes as these lines become more and more unequal in length, so the tendency of a horizontal bar to break is greatest in the middle, and decreases toward the points of support.

Fig. 126.



177. *Form for equal strength.*—Hence a beam, in order to be equally strong throughout, must be thickest in the middle, being thinned off toward the ends; and if the sides of such a beam are parallel planes, the figure of the beam must be *elliptical.*

Fig. 127.



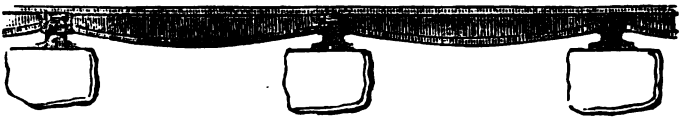
For let the curve APDM (Fig. 127), whose axis is AD, represent this longitudinal section, and let a equal the thickness or breadth of the beam; then a lateral section of the beam at any point C, will be a rectangle, whose breadth is a, depth PM, and the depth of its center of gravity ½PM. Hence, the tendency of the beam to resist fracture at any point C, is as $a \times PM^2$; but the *stress* at C is as $AC \times CD$; therefore

$$\text{the strength at C} \propto \frac{a \times PM^2}{AC \times CD} \propto \frac{PM^2}{AC \times CD};$$

hence, if $PM^2 \propto AC \times CD$, the strength will be the same at every point: but in this case the curve APDM is an ellipse, whose transverse is AD, and conjugate FK.

178. The timbers which compose the horizontal part of the frame of a house, being usually rectangular parallelopipeds of uniform dimensions throughout, it is manifest that a considerable portion of the material is wasted; but, in such cases, the attempt to save the material would be attended with paramount disadvantages. When, however, the material is expensive, or where lightness is important, as in many kinds of machinery, the foregoing principles may be applied with great advantage. A useful application of it is seen in the shape sometimes given to the iron bars of railways, as represented in the following figure.

FIG. 128.

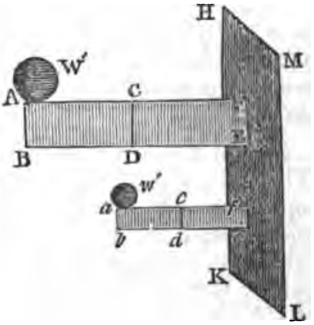


179. *Lateral strength, support at one end.*—When a prismatic beam is secured firmly in a wall at one end, the same general statements hold true, as in relation to beams supported at both ends.

1. The strength varies as *the area of the cross-section multiplied by the depth of the center of gravity.*

Let ABEF, *abef* (Fig. 129), represent the longitudinal sections of two prismatic beams fixed horizontally into the wall HKLM; then the tendency of these beams to *resist* fracture at the ends EF, *ef*, where they are inserted into the wall, will be measured in the same manner as in the preceding cases, that is, *by the area of the lateral section into the depth of its center of gravity*, except that, in this case, the fracture will begin at the upper points F, *f*, and end at the lower points E, *e*; $S \propto A . G$.

FIG. 129.



But the tendency to *produce* fracture will be the weight of the beams, acting at the distance of their centers of gravity from the ends EF, *ef*. Hence,

2. The strength varies *inversely as the length multiplied by the weight;* $S \propto \frac{1}{L . W}$.

It follows from 1 and 2, that $S \propto \frac{A . G}{L . W}$.

3. If an additional weight lies on the end of the beam, then

$$S \propto \frac{A \,.\, G}{L(\frac{1}{2}W + W')}.$$

4. If beams are similar, $S \propto \frac{1}{D}$.

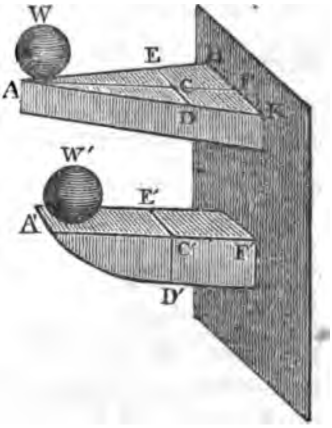
If any other cross-section be taken, as CD, and W represent the weight from it to the end AB, then by the same course of reasoning as before, the strength at CD ($= S$) $\propto \frac{A\,.\,G}{AC\,.\,W}$.

If W' represent a weight at the end, the strength to support W', at any section CD, is as $\frac{A\,.\,G}{AC\,.\,W'}$; or, since W' is constant, $S \propto \frac{A\,.\,G}{AC}$.

180. *Forms of equal strength at every section.*—

1. A beam supported at one end, and having the form of an isosceles wedge, whose triangular sides are parallel to the horizon, has equal strength at every section, for supporting a weight at the extremity. Let the wedge, in Fig. 130, have the uniform depth d; then $G = \frac{1}{2}d$; and the area of the section at ED is $ED\,.\,d$; $\therefore A\,.\,G = \frac{1}{2}ED\,.\,d^2 \propto ED \propto AC$; hence the strength is as $\frac{AC}{AC}$; that is, it is constant.

Fig. 130.



2. A beam, whose vertical sides are parallel, and whose longitudinal section, parallel to the sides, is a semi-parabola, has equal strength at every section. In Fig. 130, let d = the uniform thickness; then $A = d\,.\,C'D'$, $G = \frac{1}{2}C'D'$; $\therefore A\,.\,G = \frac{1}{2}d\,.\,C'D'^2 \propto C'D'^2$. Hence, $S \propto \frac{C'D'^2}{A'C'}$; but $A'C' \propto C'D'^2$; $\therefore S \propto \frac{C'D'^2}{C'D'^2}$, which is constant.

181. *Beams breaking by their own weight.*—Suppose beams of similar cross-sections, one dimension of which is D, and supported either at both ends, or only at one end; then,

$S \propto \frac{A\,.\,G}{L(\frac{1}{2}W + W')} \propto \frac{D^3}{L(\frac{1}{2}W + W')}$. This variation, thrown into a full proportion, is as follows: $S : s :: \frac{D^3}{L(\frac{1}{2}W + W')} : \frac{d^3}{l(\frac{1}{2}w + w')}$; W′ and w′ denoting the additional weights laid on the beams. Let $w' = 0$, then $S : s :: \frac{D^3}{L \times (\frac{1}{2}W + W')} : \frac{d^3}{\frac{1}{2}l \times w}$, which expresses the relative strength of two cylindrical beams whose lengths are L, l, diameters D, d, weights W, w, the former of which supports the given weight W′ at the end of it, and the latter supports only its own weight. Let $d = D$;

then $S : s :: \frac{1}{L \times (\frac{1}{2}W + W')} : \frac{1}{\frac{1}{2}l \times w} :: \frac{1}{L \times (\frac{1}{2}W + W')} : \frac{2L}{W \times l^2}$;

for, since $w : W :: l : L$, $\therefore w = \frac{W \times l}{L}$. In this case, therefore, the ratio of S : s expresses the relative strengths of two cylindrical beams of the same diameter, one of which supports the given weight W′ at its end, and the other supports only its own weight; and if the beam whose length is L breaks when W′ is placed at the end of it, the beam whose length is l will break by its own weight. Hence, let S = s, then $\frac{1}{L\,.\,(\frac{1}{2}W + W')} = \frac{2L}{W \times l^2}$;

$\therefore l^2 = \frac{L^2(W + 2W')}{W}$; $\therefore l = L\left(\frac{W + 2W'}{W}\right)^{\frac{1}{2}}$ = length of the beam of the same diameter that would break by its own weight.

Let the beams be *similar* cylinders, then $D^3 : d^3 :: L^3 : l^3$;

$\therefore S : s :: \frac{L^2}{\frac{1}{2}W + W'} : \frac{l^2}{\frac{1}{2}w} :: \frac{L^2}{\frac{1}{2}W + W'} : \frac{2L^3}{W \times l}$.[*] And when S = s,

$\frac{L^2}{\frac{1}{2}W + W'} = \frac{2L^3}{W \times l}$; $\therefore \frac{1}{2}W + W' = \frac{W \times l}{2L}$, or $l = \frac{L(W + 2W')}{W}$.

If, therefore, a cylindrical beam whose length is L breaks with the given weight W′ placed at the end of it, a similar cylindrical beam whose length is $\frac{L(W + 2W')}{W}$ will break with its own weight.

182. *Comparison of beams supported at one end and at both ends.*—If a horizontal beam be supported at *both ends*, the stress produced by its own weight, W, is measured by $\frac{1}{8}L \times W$ (Art. 172).

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[*] The weights of similar cylinders of the same density are as the cubes of their diameters or lengths; therefore, $w : W :: l^3 : L^3$, $\therefore w = \frac{Wl^3}{L^3}$.

If the beam be supported at *one end* only, the stress is measured by the whole weight applied at the center of gravity, and consequently the stress $= \frac{1}{2}L \times W$.

Therefore a beam supported at both ends has four times the strength of the same beam, supported only at one end. And if a certain beam supported at one end breaks by its own weight, a beam of the same dimensions twice as long will break by its own weight when resting on two supports, the former having just four times the strength it would have if twice as long (Art. 179, 2).

If, however, instead of the weight of the beam itself, this is left out of the account, and a weight W' be appended, then the stress on the beam when supported at *one end* will be measured by $L \times W'$; while, in the case of the beam supported at *both ends* (since the weight, being at the center, is also at the center of gravity of the beam), the stress is measured as before, by $\frac{1}{4}L \times W'$ (Art. 174). Therefore, a weight appended at the end of a beam supported only at one end, produces four times the stress as the same weight applied at the center of the beam when supported at both ends.

1. What must be the length of a beam of oak one inch square, supported at both ends, which is just capable of bearing its own weight?

By Art. 175, a beam of oak 1 foot long and 1 inch square, weighing $\frac{1}{2}$ pound, just supports 600 pounds. And by Art. 181, the expression $l = L\left(\frac{W + 2W'}{W}\right)^{\frac{1}{2}}$ denotes that when a beam whose length is L breaks when W' is placed at the end of it, l is the length of a beam that will break with its own weight; consequently, since here $L = 1$, $W = \frac{1}{2}$, and $W' = 600$,

$$l = \left(\frac{\frac{1}{2} + 1200}{\frac{1}{2}}\right)^{\frac{1}{2}} = (2401)^{\frac{1}{2}} = 49 \text{ feet.}$$

2. Two beams are of equal length and weight, the first being a square prism whose section is 4 inches square, the second a rectangular prism, 8 by 2 inches; how much stronger is the second beam than the first, and how much stronger when laid on the narrow than on the broad side?

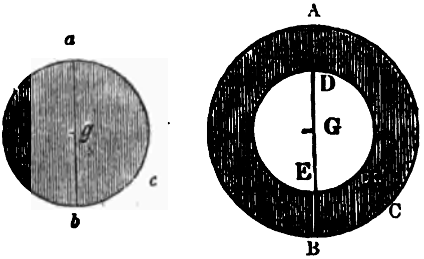
Ans. The second beam is *twice* as strong as the first, and *four* times as strong when laid on the narrow, as on the broad side.

183. On the foregoing principles Dr. Gregory makes the following remarks, most of which were originally suggested by Galileo, to whom we are indebted for the earliest investigation of these propositions. From the preceding deductions (says Gregory), it follows that greater beams and bars are in greater danger of breaking than less similar ones; and that, though a less beam may be firm and secure, yet a greater similar one may be made so long as necessarily to break by its own weight. Hence Galileo justly concludes, that what appears very firm, and succeeds well, in models, may be very weak and unstable, or even fall to pieces by its weight, when it comes to be executed in large dimensions, according to the model. From the same principles he argues that there are necessarily limits in the works of nature and art, which they can not surpass in magnitude; that immensely great ships, palaces, temples, &c., can not be erected, since their yards, beams, bolts, and other parts of their frame would fall asunder by their own weight. Were trees of a very enormous magnitude, their branches would, in like manner, fall off. Large animals have not strength in proportion to their size; and if there were any land animals much larger than those we know, they could hardly move, and would be perpetually subjected to the most dangerous accidents. As to marine animals, indeed, the case is different, as the specific gravity of the water sustains those animals in a great measure; and in fact these are known to be sometimes vastly larger than the greatest land animals. It is (says Galileo) impossible for nature to give bones to men, horses, or other animals, so formed as to subsist, and successfully to perform their offices, when such animals should be enlarged to immense heights, unless she uses matter much firmer and more resisting than she commonly does; or should make bones of a thickness out of all proportion; whence the appearance and figure of the animal must be monstrous. Hence we naturally join the idea of greater strength and force with the grosser proportions, and that of agility with the more delicate ones. The same philosopher likewise remarks, in connection with this subject, that a greater column is in much more danger of being broken by a fall, than a similar small one; that a man is in greater danger from accidents than a child; that an insect can sustain a weight many times greater than itself, whereas a much larger animal, as a horse, could scarcely carry another horse of his own size.

184. *Strength of tubes.*—The *lateral strengths* of two cylinders, of the same matter, and of equal weight and length, one of which is hollow, and the other solid, are to each other as *the diameters of their sections.*

Let ABC, *abc* (Fig. 131), represent sections of two cylinders, of equal length, and containing equal quantities of matter, of which ABC is hollow, and *abc* is solid. Then the area of the ring whose breadth is AD, is equal to that of the circle *abc*. But the strengths of these areas are as the areas multiplied by the distances of their centers of gravity from the points of pressure A, *a* (Art. 170); or, since the areas are equal, the strengths are as AG : *ag*, that is, as the diameters of their sections.

FIG. 131.



The strongest form, therefore, in which a given quantity of matter can be disposed, is that of a hollow cylinder; and leaving out of view the diminished rigidity of their structures or fabrics, there would seem to be no limits to the strength which might be given to such a cylinder by increasing its diameter. But the proposition is true only when the sections are perfectly circular; and this condition, connected with the want of rigidity when the annulus becomes very thin, occasions limits to the actual operation of the principle.

From this proposition Galileo justly concludes, that nature in a thousand operations greatly augments the strength of the substances without increasing their weight; as is manifested in the bones of animals, and the feathers of birds, as well as in most tubes or hollow trunks, which, though light, greatly resist any effort to bend them. Thus (says he), if a wheat straw, which supports an ear heavier than the whole stalk, were made of the same quantity of matter, but solid, it would bend or break with far greater ease than it now does. And with the same reason, art has observed, and experience confirmed the fact, that a hollow cane, or tube of wood or metal, is much stronger or firmer, than if, while it continues of the same weight and length, it were solid; as it would then, of consequence, be not so thick. For the same reason, lances, when they are required to be both light and strong, are made hollow.

An interesting application of the principle has been recently made in the construction of iron tubular bridges.

185. Miscellaneous Problems in Mechanics.

1. Two forces, F and F′, acting in the diagonals of a parallelogram, keep it at rest in such a position that one of its edges is horizontal; show that $F \sec a' = F' \sec a = W \operatorname{cosec}(a + a')$, where W is the weight of the parallelogram, and a and a' the angles between the diagonals and the horizontal side.

2. Four parallel forces act at the angles of a plane quadrilateral, and are inversely proportional to the segments of its diagonals nearest to them; show that the point of application of their resultant lies at the intersection of the diagonals.

3. Find the center of gravity of four equal heavy particles placed at the four angular points of a triangular pyramid.

4. Five pieces of a uniform chain are hung at equidistant points along a rigid rod without weight, and their lower ends are in a straight line passing through one end of the rod; find the center of gravity of the system.

5. A suspended weight of 83 lbs. is drawn aside 9° from the vertical; what pressure does the point of support suffer, and what force urges its descent?

6. A right square pyramid, whose height is 8 feet, and the edge of its base one foot, is tipped on one edge till it is on the point of falling; what angle does its axis make with the horizon?

7. If three uniform rods be rigidly united so as to form half of a regular hexagon, prove that if suspended from one of the angles, one of the rods will be horizontal.

8. A cone of uniform density, whose slant height is 15 inches, is suspended by the edge of its base, when its axis is found to incline 12° to the horizon; required the dimensions of the cone.

9. If ABC be an isosceles triangle, having a right angle at C, and if D and E be the middle points of AC and AB respectively, prove that a perpendicular from E upon BD will pass through the center of gravity of the triangle BDC.

10. An oblique cylinder, inclining $62\frac{1}{2}°$ to the horizon, having slant height = 14 inches, and the diameter of the base = $6\frac{1}{2}$ inches, has a ball of the same material hung upon its edge, which just upsets it; required the diameter of the ball.

11. A body, the lower surface of which is spherical, rests upon a horizontal plane; find in what case the equilibrium is stable, and in what case unstable.

12. A smooth circular ring rests on two pins projecting from a wall, and the pins are not in the same horizontal plane; find the pressure on each pin.

13. A given isosceles triangle is inscribed in a circle; find the center of gravity of the remaining area of the circle.

14. A homogeneous hemisphere rests with its convex surface on a horizontal plane; at what points of the circumference of the plane base of the hemisphere must three weights of 10, 15, and 20 lbs. be suspended, in order that its position be not changed?

15. A weighs *one* pound; B weighs *two* pounds, and lies directly west of A; they move simultaneously, A northward, and B westward, at the same uniform rate of 40 feet per second; required the direction and velocity of the motion of their common center of gravity.

16. Two smooth cylinders of equal radii, just fit in between two parallel vertical walls, and rest on a smooth horizontal plane, without pressing against the walls; if a third equal cylinder be placed on the top of them, find the resulting pressure against either wall.

17. A cylinder, suspended by a point on the side, inclines 40° to the horizon; the point is moved 3 feet lengthwise on the side, and then the cylinder inclines 24° to the horizon, with the other end down; find the point of suspension, that the cylinder may hang horizontal.

18. A flat semicircular board, with its plane vertical, and curved edge upward, rests on a smooth horizontal plane, and is pressed at two given points of its circumference by two heavy rods, which slide freely in vertical guides; find the ratio of the weights of the rods, that the board may be in equilibrio.

19. The radii of a wheel and axle are 12 inches and 3 inches; the power is 30 lbs., the weight 100 lbs.: as the power in this case preponderates, required how many degrees from the bottom of the wheel the end of the rope will be, when the forces are in equilibrio.

20. A frustum is cut from a right cone by a plane bisecting the axis, and parallel to the base; show that it will rest with its slant side on a horizontal plane, if the height of the cone have to the diameter of its base a greater ratio than $\sqrt{7}$ to $\sqrt{17}$.

21. Explain the action of an oar, when used in rowing, and determine the effect produced, having given the distances from the hands to the side of the boat, and from the side of the boat to the point where the oar may be considered as acting on the water.

22. A uniform wheel, free to revolve on its axis, has the weights, 21 lbs. and 13 lbs., attached to the circumference, 100° apart; how far from the bottom will the weights be, respectively, when the system is in equilibrio?

23. Two equal rods, without weight, are connected at their middle points by a pin, which allows free motion in a vertical plane; they stand upon a horizontal plane, and their upper

extremities are connected by a thread, which carries a weight. Show that the weight will rest half way between the pin and the horizontal line joining the upper ends of the rods.

24. A uniform heavy rod, of given length, is to be supported in a given position, with its upper end resting against a smooth vertical wall, by a string fastened to its lower end; find the point in the wall to which the string must be attached.

25. A light cord, with one end attached to a fixed point, passes over a pulley in the same horizontal plane with the fixed point, and supports a weight hanging freely at its other end. A heavy ring being put upon the cord in different places between the fixed point and the pulley, it is required to show that, if the weight of the ring be small compared with the other weight, the positions of the ring, when in equilibrio, will be approximately in the arc of a circle.

26. If particles of unequal weight be placed in the angular points of a triangular pyramid, and G be their common center of gravity, G′, G″, G‴, &c., be the common centers of gravity for every possible arrangement of the particles; show that the center of gravity of equal particles, placed at G, G′, G″, &c., is the center of gravity of the pyramid.

27. Two equal circular disks with smooth edges, placed on their flat sides in the corner between two smooth vertical planes, inclined to each other at a given angle, touch each other in the line bisecting that angle; find the radius of the least disk that may be pressed between them without causing them to separate.

28. A ladder of uniform weight throughout, 36 feet long, weighs 72 lbs., and leans against a vertical wall, making an angle of 66° 40′ with the horizon; a man, weighing 130 lbs., ascends 30 feet on the ladder; required the amount of pressure against the wall.

29. A cylindrical tube of iron 400 feet long and 20 feet external diameter, resting on piers at the ends, just breaks by its own weight; required the thickness of the sides of the tube, taking the lateral strength of iron to be as stated in Art. 175.

PART II.—PRACTICAL APPLICATIONS OF THE PRINCIPLES OF MECHANICS.

CHAPTER I.

THE MECHANICAL PROPERTIES OF MATTER.

186. *Matter* constitutes the great subject of Chemistry; *motion*, that of Mechanical Philosophy. Chemistry inquires, first, whether a given body is simple or compound,—whether it consists of one kind of matter, or of two or more different kinds united in one body; and, secondly, what are the peculiar properties of each individual body. Mechanical philosophy, on the other hand, takes cognizance of those properties of matter only which belong either to all bodies whatsoever, or at least to extensive classes of bodies. The changes it contemplates are those which appertain to the quantity, position, figure, or motion of bodies, while it leaves to chemistry all those changes which alter the constitution of bodies, transforming them into something of a different nature from what they were before.

The leading mechanical properties of matter are *divisibility*, *porosity*, *compressibility*, *elasticity*, *indestructibility*, and *attraction*.

187. *Divisibility.*—Matter is susceptible of mechanical division beyond any known limits. It is said that a grain of *musk* is capable of perfuming for several years a chamber of twelve feet square, without sustaining any sensible diminution of its volume or weight. Such a chamber contains nearly 3,000,000 cubic inches, and as the odor of the musk pervades every part of the room, a certain number of particles is contained in each cubic inch. The air of the room may be, in the mean time, changed many thousand times, and a new supply of odorous particles furnished to each successive portion of air. Hence, the number of particles diffused in the time supposed exceeds all computation, and yet the weight of the material is not sensibly diminished. The thickness of a *soap-bubble*, according to Newton, the moment before it bursts, is only the four-millionth part of an inch. The thread of a silk-worm is a perfectly smooth cylinder, whose diameter is nearly the two-thousandth part of an inch, and yet the spider's web is vastly more attenuated.

188. *Porosity.*—In many bodies the pores, or vacant spaces, are easily distinguishable by the naked eye, as in the case of sponge, wood, and most kinds of stones. Many substances which do not exhibit pores to the naked eye, still betray them to the microscope. Metals do not usually, when pure, appear porous, even under the microscope, but still such a structure may be detected by mechanical means. Thus, if a hollow ball of gold be filled with water, plugged close, and compressed in a vise, the water will exude through the metal. By means of this structure, in animals and vegetables, air and water circulate freely through them, aiding the functions of life, as the sap in trees. A cross-section, or thin slice of wood, viewed with the microscope, shows that the pores occupy usually a much greater space than the solid matter of the wood. Indeed, the solid partitions between the larger cells are themselves seen, by powerful magnifiers, to be full of pores. The surface of the body of a middle-sized man has been estimated to contain more than 20,000,000 pores,* the skin being perforated with a thousand holes to every inch. Wood consists of bundles of fibers of different degrees of fineness, usually aggregated together so loosely, that a free circulation of water is easily maintained between them. Glass is the only solid known which appears, as far as experiments have gone, to be absolutely impermeable to all fluids.†

189. *Compressibility.*—All bodies yield more or less to external pressure, undergoing a diminution of volume proportional, in each case, to the force applied. Aeriform bodies, as common air, yield readily to any compressing force, the diminution of volume being always exactly proportional to that force. Solids, also, as wood and stone, are compressed, in different degrees, under heavy weights. A cork immersed two hundred feet in the sea, is so much compressed and filled with water, as to sink instead of rising; and a pint bottle of fresh water, corked closely, and sunk to a great depth in the ocean, will, when drawn up, be found to be filled with salt water. This remarkable fact is explained by supposing that the cork has been so much contracted in bulk as to admit the salt water, which being heavier than the fresh, displaced it and occupied the bottle. As the cork sustained an equal pressure on all sides, it would not be removed out of its place; and, as the bottle was drawn up, and the pressure was diminished, the cork would regain its original dimensions. Hard mineral substances, as blocks of granite, indicate some contraction of volume when subjected to the pressure of high and massive walls. Liquids resist compression much more than either air or solid bodies. Still, under enormous weights, it may be rendered sensible, as will be more fully explained hereafter.

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* Leslie, Nat. Phil., i, 18.

† Pouillet, El. Phys., t. i, 29.

190. *Elasticity*.—Bodies are said to be *perfectly* elastic when they restore themselves to their original dimensions when released, and with a force equal to that with which they were compressed. *Air* and all gases are of this class; and even *liquids*, as water, are found to conform to the same law, and, in this sense, therefore, they must also be regarded as perfectly elastic substances.* *Metals*, indeed, have the same property, a double extension or compression, requiring twice the force; triple, three times the force, and so on. The elasticity of wood is exemplified in the cross-bow; and that of a mineral substance, peculiarly in mica. The elasticity of *torsion*, or the force by which a wire when twisted endeavors to resume its natural state, is employed as the most delicate test and measure of force known, the force of torsion being always proportional to the angle through which the body has been twisted.

191. *Indestructibility*.—Matter is wholly indestructible. In all the changes we see going on in bodies around us, not a particle of matter is lost; it merely changes its form; nor is there any reason to believe that there is now a particle of matter either more or less than there was at the creation of the world. When we boil water and it passes into the invisible state of steam, this, on cooling, returns again to the state of water without the least loss; when we burn wood, the solid matter of which it is composed, passes into different forms, some into smoke, some into different kinds of airs, or gases, some into steam, and some remains behind in the state of ashes. If we should collect all these various products, and weigh them, we should find the amount of their united weights the same as that of the body from which they were produced, so that no portion is lost. Each of the substances into which the wood was resolved, is employed in the economy of nature to construct other bodies, and may finally reappear in its original form. In the same manner, the bodies of animals, when they die, decay and seem to perish; but the matter of which they are composed merely passes into new forms of existence, and reappears in the structure of vegetables or other animals.

192. *Attraction*.—This property of matter produces or governs a large part of the phenomena of the natural world. By it all matter tends toward all other matter, and by it the particles of matter unite, forming innumerable compounds. It is chiefly this property in different degrees which constitutes the STRENGTH OF MATERIALS,—a subject which has been already considered *theoretically* (Arts. 168–184), but which it will be useful now to consider *practically*, in its relation to the arts.

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* Mosely's Illustrations, Sec. 32.

193. *Longitudinal strength.*—The strength of substances *in the direction of the length* has been determined, experimentally, by suspending small cylinders or wires of each material vertically, and applying weights at the bottom until they broke. Of all substances, that which sustains the greatest load is *iron*. Different materials, before rupture, increase in length in different degrees. Bars of the best wrought iron are elongated about .000082 for a load of one ton to the square inch. Iron in the form of wires admits of a greater extension. A bundle, or cable of small wires, will, under similar circumstances, be elongated .000091; and becomes more extensible in proportion as the wires are smaller. Bar iron will bear to be extended .000714 without injury; and several kinds of wood, as oak, pine, and fir, will bear an elongation three times as great. Iron wire, on account of its extraordinary tenacity, has within a few years been most successfully applied to the construction of suspension bridges. Iron wire $\frac{1}{30}$ inch in diameter, has been found by experiment capable of bearing a load of 60 tons to the square inch without breaking. In one instance, indeed, the load sustained was 90 tons. The suspension bridges at Freyburg in Switzerland, and that over the Niagara, near the great falls, are hung upon wire cables. The principal Freyburg bridge, which is more than 900 feet long, is supported by four cables, four inches in diameter, and each containing more than 1000 wires. The Menai bridge in Wales is suspended on immense iron chains. Russia *bar iron* has a tenacity of 27 tons to the square inch. The best *cast iron* is about one third as strong, having a tenacity of about 9 tons to the inch. The tenacity of platina wire is nearly as great as that of bar iron. The comparative strength of several substances much used in the arts, is thus stated by Mosely: 7 rods of mahogany, taken together; 5 of pine, oak, or beach; 3 of box or of cast iron; 2 of gold; 1½ of silver or copper, have respectively the same tenacity as 1 corresponding rod of wrought iron; or as a rod whose section is $\frac{5}{12}$ made of steel or fine wire cable.

194. *Strength to resist pressure.*—As the materials used in building are liable to give way by the superincumbent weight, it is often important to know the relative power of different materials to resist forces tending to *crush* them. Numerous experiments on this subject have been instituted by taking small blocks of similar shape and size, and loading them respectively with weights until they were crushed. It appears that cast iron is best adapted of all the materials in common use to sustain such pressures; that bar iron is not more than half as strong; granite, one-sixth; Italian marble, one-seventh; freestone, one-tenth; brick-work, still less. Wooden columns have comparatively little power of resisting a force tending to crush them in the direction of their fibers. Short columns, however, bear stronger pressures than long ones, the strength diminishing in a geometrical, while the height is increased in an arithmetical ratio. Slight changes of form are found sometimes greatly to affect the strength of a column. Thus merely *rounding* the ends of a perpendicular column makes its strength only one third that of a column whose extremities are flat. If we take three columns, equal in all respects, and round both ends of the first, one end of the second, and leave both ends of the third flat, their respective strengths will be as the numbers 1, 2, 3. The shape of a column has great influence on its strength. A cylindrical column is weakest in the middle; and it is found that the strength of a column of cast iron, containing a given weight of metal, whether it be solid or hollow, is much greater when it is cast in the form of a double cone, that is, with its greatest thickness in the middle of its height, and tapering to its extremities, than when cast in any other form. In lofty stone columns, however, their own weight may constitute a large part of the load to be sustained, and hence it is the practice of architects to make the swell below the center. In some of the Grecian temples, it was one third from the base; and this rule is now frequently adopted.

In practice, materials cannot safely be subjected to constant strains or thrusts approaching to those which produce rupture. They are liable to various occasional and accidental pressures; and others of a permanent kind, resulting from the settling of the pile. It is therefore regarded as not entirely safe to load any structure of stone more than one-sixth the amount of the pressure that crushes it. Iron, cast or wrought, may be loaded to one-fourth that amount.

195. *Lateral strength.*—The view taken by Barlow of the mode in which a horizontal beam undergoes compression, leads to results somewhat different from those investigated in Articles 168, &c. When a beam is bent in the middle, the fibers on the upper side undergo compression, while those on the under side undergo extension, as in Fig 132, and

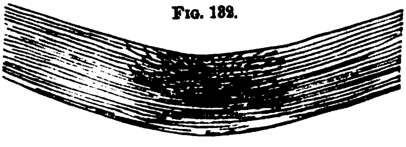
FIG. 132.

between the two is a line that sustains neither, which is called the neutral axis. Since, throughout its neutral axis, the strength of the beam is not at all called into action, this will not be impaired by boring a hole through the beam in the direction of that axis. What constitutes the strength of a beam is its resistance to extension on the lower, and to compression on the upper side. These act as antagonist forces, and if either of them yield, the beam is broken. Hence the strength of the beam is not impaired by sawing it through the upper side as low as to the neutral axis. This extends to about five-eighths of the depth.*

196. Although wood has not intrinsically the strength of iron or stone, yet its lightness in some measure compensates for this, so that large structures of wood have sometimes great power of resistance to external forces. Pine is only one-fifteenth as heavy as cast iron, while it has more than half the tenacity. Sixteen bars of it would weigh only the same as one bar of wrought iron, while they would have three times the strength. Many large structures, when constructed of heavy materials, are weak from their own weight. Iron roofs have been known to fall in by their own pressure. Trees often resist the action of external forces which overturn works of art apparently of much more stable materials; and nothing is more deserving the attention of the architect than the rules which nature has observed, both in the selection and distribution of the materials of which trees are constructed. The tapering form of their trunks; the increased diameter and density of their bases; the buttresses that frequently, in a large tree especially, support the trunk on every side; the comparative lightness of the extended top; and the universal symmetry of form that pervades the entire structure; these qualities, severally and collectively, add to the strength of trees, and fit them to encounter the most violent winds. Indeed, mechanical writers, when they have descended to a minute investigation of the structure of trees, have found the most refined use made of such mechanical principles as tend both to the greatest strength and greatest economy of material. Thus, in a cylinder of wood, like the trunk of a tree, the neutral axis is near the center; hence the resistance to compression on one side, and the resistance to extension on the other, act in opposition to each other, when a tree is bent by the wind, and thus the trunk is prevented from breaking.

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* Mosely.

CHAPTER II.

GENERAL OBSERVATIONS ON MOTION.

197. MOTION and REST are accidental states of bodies, nor is a body naturally prone to one state more than to the other. If it is found at rest, it is because it is kept in equilibrium by opposite and equal forces; and if it is found in motion, it is because it has been put in motion by some force extrinsic to itself. The resistances to motion which exist near the surface of the earth, particularly gravity, create a seeming tendency to a state of rest; but, in reality, rest is no more the natural state of bodies than motion is.

198. *Absolute and relative motion.*—*Absolute motion* is a change of place in space with respect to any fixed point; *relative motion* is a change of place in bodies with respect to each other. A body may be at the same time in a state of absolute motion, and of relative rest. Thus, all the different articles contained in a ship under sail, have a motion in common with the ship, but may be at rest with respect to each other. When a man walks toward the stern of a ship at the same rate as that of the ship, he is in motion with respect to the ship, but at rest with respect to the earth. When a balloon, carried along by the wind, attains the same velocity as the wind, it is relatively at rest, and appears to the aeronaut to be in a perfect calm, although it may be actually moving one hundred miles an hour. Since the earth, in its annual revolution around the sun, is moving eastward at the rate of nineteen miles, or 100,000 feet per second, were a cannon-ball, at a certain time of day, fired eastward at the rate of 2000 feet per second, the only effect would be to add 2000 feet to the velocity which the ball had before in common with the earth: and were it fired westward, the effect would be merely to stop 2000 out of 100,000 parts of its previous motion, while the cannon would proceed onward, leaving it behind.* Did not the atmosphere partake of the diurnal motion of the earth, but were it to remain at rest with respect to this motion, the progress of any place to the east, would cause a relative motion of the air, or a wind, westward, which would blow with a violence far surpassing that of the most terrible hurricanes.†

Indeed, we can not be sure that we have ever seen a body absolutely at rest. In our stillest moments, we are revolving with the earth on its axis; we are accompanying the earth in its annual revolution from west to east around the sun; and are perhaps attending the solar system around a common center of motion.*

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* Robinson's Mechanical Phil., i. 81.

† Winds are in fact frequently produced by this cause, viz., by their having a relative velocity different from that of the part of the earth over which they blow.

199. *Apparent motion*, as distinguished from relative, is that in which the body that seems to be moving is quiescent, and the motion is owing to a real motion in the spectator. Thus, the backward motion of the trees to one riding rapidly, the receding of the shore to one who is sailing from it with a fair wind, and the diurnal motions of the heavenly bodies from east to west, in consequence of the revolution of the spectator in the opposite direction; these are severally examples of apparent motion. It is often a very difficult problem to deduce the real from the apparent motion. While a planet, as Venus, is revolving about the sun in an orbit nearly circular, its motions, as seen from the earth, are extremely irregular; and to make all these irregularities consistent with the real motion, has been a perplexing problem in astronomy. We can sometimes decide that a given motion is real, because we observe a *cause* in operation, which is competent to produce it. The impulse of the wind, or the direction of the current, will satisfactorily account for a ship's receding from a given object, while no cause appears why the object should recede from the ship; the revolution of the earth on its axis is a cause competent to explain the apparent revolution of the heavens, while we can find no cause for their actual revolution. The *effects* also of a given motion, enable us to decide whether it is real or apparent. Thus a constant tendency to move in a straight line is characteristic of real motion.†

200. The *Laws of Motion* have been already recited in Chapter I, and concise illustrations of them were added in that place. It was necessary to proceed thus far at the beginning of this work, since these constitute the fundamental principles of Mechanics. By their great comprehensiveness, they furnish the most convenient classification of the various phenomena of motion, and it will therefore be useful to resume the consideration of them. They are very remarkable examples of a happy generalization; but their very comprehensiveness renders them difficult to be understood by the young learner; nor can they be thoroughly mastered in all their relations, until after considerable proficiency is made in the science of Mechanics. These laws, indeed, are the chief foundation of Newton's Principia.‡

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* Young's Natural Phil., i, 19.

† Wood's Mechanics, p. 22.

‡ Young's Nat. Phil., i, 26.

201. FIRST LAW.—*A body continues always in a state of rest, or of uniform rectilinear motion, until, by some external force, it is made to change its state.*—This law contains the doctrine of *inertia* expressed in four particulars. First, that unless put in motion by some external force, a body always remains at rest; secondly, that when once in motion, it continues always in motion, unless stopped by some force; thirdly, that this motion, arising from inertia, is uniform; and, fourthly, that this motion is in right lines. The proofs by which this and the other laws of motion are established, have been already stated in Part I. It is our present object to make the application of these laws to various phenomena of nature and art.

202. *Bodies at rest.*—The operation of this principle is seen, when a horse starts suddenly forward, and his rider is thrown backward. "When we desire a person, with suspected disease of the brain, to shake his head, and tell whether he feels pain, we are doing nearly the same as if we touched the naked brain with the finger to find the tender part, for the inertia of the brain, when the skull is moved, causes a momentary pressure between it and the skull, almost equivalent for our purpose to such a touch."* In consequence of the inertia of matter, before a body can be brought to the required velocity, this velocity must be impressed upon every particle of matter it contains. Hence, the more numerous its particles, the greater its inertia, which is therefore proportioned to the quantity of matter. But the weight also is proportioned to the quantity of matter, and therefore the inertia is proportioned to the weight. Yet it must be carefully distinguished from weight, having in fact nothing in common with it, except that both are proportioned to the quantity of matter, and of course to each other. But were we to strike with a hammer upon the top of a body falling toward the earth, the resistance from inertia would be the same as if the body were struck with the same force on the side; or in whatever direction the blow were applied, a similar resistance would be felt. This is what we commonly understand by the reaction of a body; but we conceive this reaction itself to depend upon an inherent property in matter, to which we give the name of inertia. Inertia is the *cause*, and reaction the *effect*. A vast weight may be moved on a horizontal railway by a comparatively small force, provided it can be got in motion with the required velocity. In transporting large quantities (eighty tons, for instance) of coal, the weight is distributed into a number of different cars, connected together by a loose chain, in order that the inertia of the several parts may be overcome successively.

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* Arnott's El. Phys., p. 50.

203. In consequence of the inertia of matter, it is not easy to give great velocity to large masses in an instant, or to stop them at once, if moving swiftly. If the attempt is made, either the body, or that by which the force is applied, will be broken, the cohesion being too weak to overcome so great a degree of inertia. This explanation accounts for several familiar facts. When a team starts suddenly with a heavy load, the effort is either wholly ineffectual, or some part of the harness gives way. If we draw a heavy weight by a slender string, a slow and steady pull will move the weight, when a sudden twitch would break the string without starting the mass. The same principle applies to bodies already in motion. Thus, when a horse in a carriage starts suddenly forward, he may break loose as well when the carriage was previously in motion, as when it was at rest. The inertia of a body is in fact the same in both cases, having as great a tendency to continue in its present rate of motion as to remain at rest. Several singular phenomena result from the same cause, showing that time is necessary in order that motion may be communicated to all parts of a mass. A pistol-ball fired through a pane of glass, frequently makes a smooth, well-defined hole, and does not fracture the other parts of the glass. Here, the momentum of the ball is communicated to the particles of glass immediately before it. Had the force been applied gradually, the cohesion of the glass would have been great enough to communicate the motion to all parts of the pane. A ball fired through a board delicately suspended, causes no vibrations in the board. A cannon-ball, having very great velocity, passes through a ship's side, and leaves but little mark, while one with less speed, splinters and breaks the wood to a considerable distance around. A near shot thus often injures a ship less than one from a greater distance.* A soft substance, as clay or tallow, may be fired through a plank,—the body, by its great momentum, forcing its way through the plank, because the strength of the board is not sufficient to impart so rapid motion to the other portions. The whole momentum being concentrated upon the part immediately before the body, the cohesion of that part is overcome, rather than the inertia of the whole.

204. *Bodies in motion.*—All bodies in contact with each other acquire a common motion; as, for example, a horse and his rider, a ferry-boat and its passengers, a ship and every thing within it, the earth and all things on its surface. Whenever either of these bodies stops suddenly, the movable bodies connected with it are thrown forward. Were the revolution of the earth on its axis to be suddenly arrested, the most dreadful consequences would ensue; every thing movable on its surface, as water, rocks, cities, and animals, not receiving instantaneously this backward impulse, would fly off eastward in promiscuous ruin. Were the diurnal motion of the earth, however, very gradually diminished, until it finally ceased, so that time should be afforded to communicate the loss by slow degrees to the bodies on its surface, no such effects would take place. If a passenger leaps from a carriage in rapid motion, he will fall in the direction in which the carriage is moving at the moment his feet meet the ground; because his body, on quitting the vehicle, retains, by its inertia, the motion which it had in common with it. When he reaches the ground, this motion is destroyed by the resistance of the ground to the feet, but it is retained in the upper and heavier part of the body, so that the same effect is produced as though the feet had been tripped.

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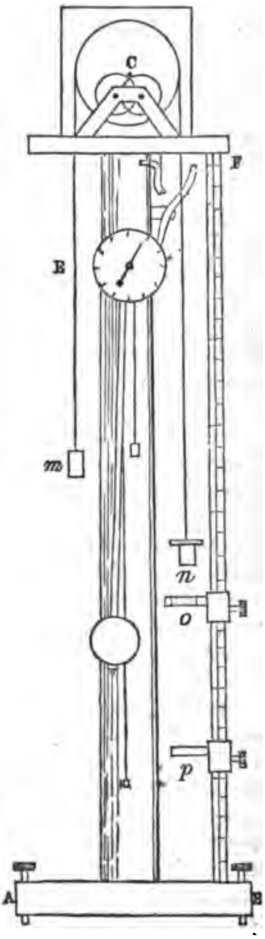
* Arnott's El. Phys., p. 104.

Fig. 183.



205. *Undisturbed motion is uniform.*—Although, on account of the numerous impediments to motion which exist on the surface of the earth, bodies are unable to maintain for any considerable time the motion they have acquired, yet we see the first law of motion, so far as it respects the tendency of bodies to persevere in motion, fully confirmed in the continued and unaltered revolution of the heavenly bodies. These are impelled by no renewed forces, but revolve from age to age in an undeviating course, simply because they meet with no impediments.

In consequence of inertia, bodies have a tendency to move over equal spaces in equal times, that is, to move uniformly. In a ball rolled on ice, in a pendulum continuing to vibrate after the moving force is withdrawn, and in numerous cases similar to these, we observe in nature and art this tendency to uniform motion; but in all these cases, the motion is not absolutely uniform, but more or less retarded by the resistances encountered. A much nearer approximation to the truth is obtained by means of a piece of apparatus called *Atwood's machine*, the essential parts of which are represented in Fig. 133. AB is the base of the instrument, furnished with leveling screws. A substantial pillar, about seven feet high, supports a small table on the top. C is a grooved wheel, very delicately suspended on friction-wheels, and protected from dust by a glass case; *m*, *n* are equal poises attached to the ends of a fine cord, which passes over the grooved wheel. E is a simple clock attached to the pillar, for measuring seconds and dropping the poise and bar exactly at the beginning of a beat of the pendulum; *o* is a ring; and *p* a stage, which can be clamped to any point of the scale, BF. When the loaded poise *n* passes through the ring *o*, the bar is left on the ring, and the poise proceeds with the velocity, which has been acquired, till it strikes upon the stage. The instrument is used principally for illustrating the laws relating to the fall of bodies. To show uniformity of motion, place the ring and stage so that just one second elapses after the bar is left till the poise strikes the stage. Then clamp the stage *twice* as far below the ring, and *two* seconds will be the interval of time, and so on, as far as the scale extends.

206. *Undisturbed motion is rectilinear.*—Moving bodies have a constant tendency to proceed in right lines. In nature there occur indeed but few examples of rectilinear motion, but almost every moving body describes a curve. Thus, the heavenly bodies move in ellipses; projectiles describe parabolas; or if their direction is so altered by a resisting medium, as the atmosphere, that their path is no longer a parabola, it is still changed to some other curve; and a ship sailing across the ocean describes a curvilinear path on the surface of the earth. The waving of trees and plants, the courses of rivers, the spouting of fluids, the motions of winds and waves, are likewise more or less curvilinear. Bodies falling toward the earth by gravity, present almost the only examples we observe in nature of a motion purely rectilinear. But notwithstanding the deviation from a right line, observable in actual motions, yet we find there is always some extraneous cause in operation, which accounts for such deviation.

207. *Centrifugal force.*—In consequence of this tendency of moving bodies to proceed in right lines, when a body revolves in a curve, around some center of motion, it constantly tends to fly off in a straight line, which is a tangent to its orbit. This tendency, so far as it is *directed from the center*, is called the *centrifugal force*. A stone from a sling, water escaping from the periphery of a revolving wheel, and water receding from the center of a tumbler or pail when the vessel is whirled, are familiar instances of the tendency of bodies when revolving in circles to fly off in straight lines. The action of the centrifugal force may be studied more advantageously by means of an apparatus called the *whirling tables.* These consist of two small circular tables, to which is communicated a horizontal revolution around their centers. Bodies laid on the tables in different ways are made to participate in their rotary motions, and thus the laws of the centrifugal force may be observed. By means of this apparatus the following propositions are illustrated.

208. The centrifugal force of bodies revolving in a given circle, is proportional to their *densities or specific gravities.* If quicksilver, water, and cork, be whirled together in a suspended pail or glass vessel, these bodies will arrange themselves in the order of their specific gravities, so that the cork will be at the least, and the quicksilver at the greatest distance from the center of motion.

209. When bodies revolve in the same circle with different velocities, the centrifugal forces are as the *squares of the velocities.* By doubling the velocity of a revolving body, its centrifugal force is quadrupled. Millstones, revolving horizontally, communicate their circular motion to the corn that is introduced between them near the center. The corn, by the centrifugal force which it gradually acquires, recedes from the center and passes out at the circumference. If too great velocity is given to millstones, they sometimes burst with violence. A horse in swift motion, on suddenly turning a corner, throws his rider; and a carriage turning swiftly is overset on the same principle. In feats of horsemanship, when the equestrian rides rapidly round a small ring, he inclines his body inward in different degrees, according to the velocity with which he is moving, and thus counteracts his tendency to fall outward by the centrifugal force.*

210. When bodies revolve in different circles, in the same time, the centrifugal forces are as the *radii of the circles.* Hence, when spherical bodies revolve on their axes, the equatorial parts, being furthest from their centers of motion, and consequently moving faster, have a proportionally greater centrifugal force. If the revolving body is soft so as to yield, it is elevated in the equatorial and depressed in the polar parts. Thus a mass of clay revolving on a potter's wheel, swells out in the central parts and becomes flattened at the two ends. The earth itself, by its figure, which is an oblate spheroid, indicates the operation of this principle, its equatorial exceeding its polar diameter by 26 miles; and the planet Saturn, which has a far more rapid revolution on its axis, indicates the same modification of its figure in a still higher degree, being strikingly elevated at the equator and depressed at the poles.

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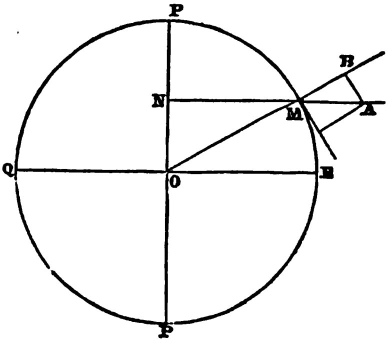
* Arnott's El. Phys., p. 62.

FIG. 134.



211. The centrifugal force is continually opposed to the action of gravity, so that, except at the pole, where this force becomes nothing, the weight of a body is diminished by it in the ratio of *the square of the cosine of the latitude*. For, let EPP (Fig. 134) represent the earth, EQ the equator, PP the earth's axis, and MN the radius of a parallel of latitude, which equals the cosine of the latitude, since EM is the latitude, MP its complement, and MN the sine of MP. Now, on account of the different velocities of two bodies at E and M respectively, their centrifugal forces would be as EO to MN, or as radius to the cosine of the latitude. But, at the equator, the centrifugal force being exerted in the direction of OE is directly opposed to gravity, while at M the force is exerted in the direction of NM, and is therefore not directly opposed to gravity. Produce MN to A, and let MA represent the centrifugal force at M, and upon OM produced let fall the perpendicular AB; then MB will represent the part of the force which acts in opposition to gravity. But since the angle AMB is equal to the latitude (being equal to MOE), therefore, MA is to MB as radius to the cosine of the latitude. Hence, the centrifugal force, so far as it opposes gravity, is diminished twice in the ratio of the cosine of the latitude, and consequently is as the square of the cosine of the latitude. At the equator, the centrifugal force is $\frac{1}{289}$ of the force of gravity, and of course it so much diminishes the weight of bodies; and since this force varies as the square of the velocity, were the earth to revolve with 17 times its present velocity (the square of which is 289), the centrifugal force would be equal to that of gravity, and bodies would lose all their weight; and were the velocity greater than this, they would fly off in the direction of tangents to the surface.*

212. We have hitherto considered the doctrine of central forces only in respect to bodies moving in *circular* orbits; but

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* At the equator the loss of weight by the centrifugal force is $\frac{1}{289}$; and by increased distance from the center of the earth, $\frac{1}{590}$; hence the entire loss is $\frac{1}{194}$.

Newton extended the theory to all possible curves, and established the general laws of motion in every sort of curve;—observing that we can always make a circle pass through three points taken in any curve, and the nearer the points are to each other, the more nearly the circumference of the circle will coincide with the curve. If the three points be taken infinitely near each other, the circle will form what is called the *osculating circle.* We can then suppose that at each point of the curve the body is moving in the osculating circle; and, consequently, its centrifugal force is measured by the square of the velocity divided by the radius of the osculating circle. As the radius changes at each point of the supposed curve, the centrifugal force also is continually changing, while in a circular orbit it remains always the same.*

213. The consideration of centrifugal force proves, that if a body be observed to move in a curvilinear path, some efficient cause must exist which prevents it from flying off, and which compels it to revolve round the center. Thus the bodies of the solar system are constantly impelled or drawn toward the sun by a force which we denominate gravity. If this force did not act constantly, they would resume their motion in the right line in which they were originally projected, when they were first launched into space, and continue moving in it forever.

214. SECOND LAW.—*Motions produced in a body by different forces, are proportional to those forces, and in their directions respectively; and they coexist with each other, so that the final position of the body is the same, whether the forces act separately or together.*

First, motion is proportional to the force impressed. This is very satisfactorily shown by means of Atwood's machine. When the poise *n* is loaded with small bars of different weights (the bars being left on the ring *o*, as in Art. 205), the poise descends along the scale, in consequence of the motion given it by the bars, with velocities exactly proportional to the weights of the bars respectively, provided their fall has continued an equal length of time.

Secondly, motion is in the *direction* of the force impressed. Notwithstanding the diversity of motions to which every terrestrial body is constantly subject, the effect of any force to produce motion is the same, when the spectator has the same motion as the body, as though the body were absolutely at rest. In other words, all motions coexist, and are independent of each other; each remaining, relatively, the same as if there were no others. Since, for example, by the diurnal motion of the earth, places toward the equator move faster than those toward the poles, if the foregoing principle were not true, the same forces would produce different quantities of motion in different latitudes; and a body struck in a direction north or south, would not move in that direction, but would deviate to the east or west. A pendulum also would vibrate differently according as it moved in a north and south, or in an east and west direction, whereas not the slightest difference of time can now be detected. If we are in a ship, moving equally, any force which we can exert will produce the same motion relatively to the vessel, whether it be or be not in the direction of the vessel's motion. If we stand on the deck, supposed to be level, and roll a body along it, the same effort will produce the same velocity along the deck whether the motion be from head to stern, or from stern to head, or across the vessel. Also, a body dropped from the top of the mast will not be left behind by the motion of the ship, but will fall along the mast as it would if the mast were at rest, and will reach the foot of it in the same time. If a body be thrown perpendicularly upward, it will rise directly over the hand and fall perpendicularly upon it again; and if it be thrown in any other direction, the path and motion relatively to the person who throws it will be the same as if he were at rest. Therefore the motion which the body has in common with the ship co-exists with every other motion given to the body.

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* Pontecoulant, El. Astron., p. 502.

215. Since force is measured by the momentum which it produces (Art. 11), it follows that the *smallest force* is capable of moving the *largest body*. Agreeably to this doctrine, a blow with a hammer upon the earth ought to move it, and that it would do so may be inferred from the following reasons.

1. We can conceive the earth to be divided into parts so small, that the blow would produce upon one of them even a sensible motion. Then it would produce on two of the parts half as much velocity; and upon all the parts together a velocity as much less than upon one, as their number was greater than unity. This velocity might be appreciable in numbers, although too small to be observed by the senses.

2. Very heavy weights may be actually put in motion by small forces. Leslie asserts that a ship of any burden, in calm weather and smooth water, may be gradually pulled along even by the exertions of a boy.*

3. The repetition of very small blows finally produces sensible effects upon large bodies. The wearing away of stone by the dropping of water, the abrasion of marble images by the

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* Leslie, El. Nat. Phil., i, 80.

kisses of pilgrims, and, especially, the demolition of the strongest fortresses by repeated blows of the battering-ram, are examples of powerful effects produced by small impulses, each of which must have contributed its share, since the addition of any number of nothings is nothing still.

216. THIRD LAW.—*When bodies act upon each other, action and reaction are equal, and in opposite directions.*

If I strike one hand upon the other at rest, I perceive no difference in the sensations experienced by each. The resistance to the hand which gives the blow is equal to the impulse given. A boatman presses against the bank with his oar, and receives motion in the opposite direction, which being communicated to the boat, makes it recede from the shore. He strikes the water, the reaction of which, at every impulse, carries the boat forward in the opposite direction. An infirm old man presses the ground with his staff, and thus, by lightening the pressure on his lower limbs, makes his arms perform a part of the labor of walking. A bird beats the air with his wings, and by giving a blow whose reaction is more than sufficient to balance the weight of his body, rises with the difference. When the wings are small and slender, as those of the humming-bird, and disproportioned to the weight of the body, the defect is compensated by more frequent blows, giving nimble motions, suited to their short but swift excursions; while the long wings of the eagle are equally fitted, by their less rapid but more effectual blows, for his distant journeys through the skies. Hence, propelling and rowing a boat, flying, and swimming, are processes analogous to each other, depending on the principle of reaction.

217. If a man stands in a boat and pulls upon a rope which is fastened to a post on the shore, the force of the man is expended on the post in one direction, and the post, by its reaction, draws the man in the opposite direction, namely, toward the shore. (Art. 18, Qu. 12.) Call the man A, and let another man B take the place of the post. If B pulls with a force just equal to that of A, he will do nothing more than what the post did before, and therefore the two men together will bring the boat ashore no sooner than A would have done alone in the former case. If A pulls with more force than B, he pulls B toward him, and the reaction, or the force which carries the boat ashore, is the same as before, namely, the force of B. If B were to pull with more force than A, he would pull A out of the boat, were not A attached firmly to the boat, in which case the velocity of the boat would be augmented. By attentively considering this and all analogous cases, we shall perceive, that whenever two bodies act against each other, they give and receive equal momenta, and the momenta being in opposite directions, it follows, that bodies do not alter the quantity of motion they have, estimated in a given direction, by their mutual action on each other. This principle is well explained in Emerson's Mechanics, as follows:

*The sum of the motions of any two bodies in any one line of direction, toward the same part, can not be changed by any action of the bodies upon each other; whatever force these actions are caused by, or the bodies exert among themselves.** Hence it follows, that the sum of the motions of all the bodies in the world, estimated in one and the same line of direction, and always the same way, is eternally and invariably the same. Whatever motion, therefore, one body receives toward another, whether it is drawn toward it by attraction, or by a rope, or by any other method, precisely the same quantity of motion it imparts to the other body in the opposite direction.† If a man in a boat pulls at a rope attached to another boat of equal size, the boats will move toward each other with equal velocities; but a man in a boat pulling a rope attached to a large ship seems only to move the boat, but he really moves the ship a little, although its velocity is as much less than that of the boat as its weight is greater. A pound of lead and the earth attract each other with equal force, and the two bodies approach each other with equal momenta.‡

218. Since momentum is proportioned to the joint product of the velocity and quantity of matter, a great momentum may be obtained either by giving a slow motion to a great mass, or a swift motion to a small body. A striking illustration of this is afforded by Question 9, Art. 18, where, on the supposition that a grain of matter moving with the velocity of light, were to impinge directly against a mass of ice floating at its ordinary slow rate, the one grain would be competent to stop about 44½ tons of ice. Islands of ice move with such vast momentum, that they instantly demolish the largest ship of war, if it comes in their way.

219. If a body in motion strikes a body at rest, the striking body must sustain as great a shock from the collision as if it had been at rest, and struck by the other body with the same force. For the loss of force which it sustains in one direction is an effect of the same kind as if, being at rest, it had received as much force in the opposite direction. If a man walking rapidly, or running, encounters another standing still, he suffers as much from the collision as the man against whom he strikes. When two bodies moving in opposite directions meet, each body sustains as great a shock as if, being at rest, it had been struck by the other body with the united forces of the two. For this reason, two persons walking in opposite directions, receive from their encounter a more violent shock than might be expected. If they be of nearly equal weight, and one be walking at the rate of three and the other four miles an hour, each sustains the same shock as if he had been at rest, and struck by the other running at the rate of seven miles an hour.*

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* Emerson's Mechanics, 4to, p. 17.

† Quantitas motus quæ colligitur capiendo summam motuum factorum ad eandem partem, et differentiam factorum ad contrarias, non mutatur ab actione corporum inter se. (Principia, Lex III, cor. 3.)

‡ The pound of lead does indeed attract the earth only half as much as two pounds would do; nor does it *receive* from the earth but half as much: the power of attracting and of being attracted is the same.

This principle accounts for the destructive effects arising from ships running foul of each other at sea. If two ships of 500. tons burden encounter each other, sailing ten knots an hour, each sustains the shock which, being at rest, it would receive from a vessel of 1000 tons burden sailing at the same rate. It is a mistake to suppose that when a large and a small body meet, the small body suffers a greater shock than the large one. The shock which they sustain must be the same; but the large body may be better able to bear it. When the fist of a pugilist strikes the body of his antagonist, it sustains as great a shock as it gives; but the part being more fitted to endure the blow, the injury and pain are inflicted on his opponent. This is not the case, however, when fist meets fist. Then the parts in collision are equally sensitive and vulnerable, and the effect is aggravated by both having approached each other with great force. The effect of the blow is the same as if one fist, being held at rest, were struck by the other with the combined force of both.

220. The question may be asked, Why are the effects so much more severe when we fall from an eminence upon a naked rock than upon a bed of down? In both instances our fall is arrested, and we sustain a contrary and equal reaction; yet in the one case we might suffer hardly any injury, while in the other we should be bruised to death. The reason of the difference is this: when we fall on a bed of down, the resistance is applied gradually; when we fall on a rock, it is applied instantaneously. We do not strike the bed with the same force that we do the rock; we move along with the bed, and of course do not lose our motion at once, and we receive in the opposite direction merely what we lose. A violent blow, if equally diffused over the human body, may be sustained without injury. Thus, if an anvil be laid on the breast, a man may receive on it a heavy blow from a great hammer with impunity.*

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* Lardner's Mechanics, p. 47.

There are many instances where action and reaction mutually destroy each other, and no motion results. Thus, when a child stands in a boat and pulls by a rope attached to the stern, he labors in vain to make the boat advance. Dr. Arnott tells us of a man who attached a large bellows to the hinder part of his boat, with the view of manufacturing a breeze for himself, being ignorant that the reaction would carry the boat backward as much as the impulse of the artificial wind carried it forward.† A force which begins and ends within a machine has no power to move it.

221. *Variable motion.*—When a moving body is subjected to the energy of a force which acts on it without interruption, but in a different manner at each instant, the motion is called, in general, *variable motion.* We have instances of variable motions in the unbending of springs, in the action of the wind on the sails of a ship, and in the action of gunpowder on a ball while it is passing through the barrel of the gun. In each of these cases, the velocity of the moving body is constantly augmented, yet the degree of augmentation is diminishing until it finally ceases.

When a moving body receives, each successive instant, the same increase of velocity, it is said to be *uniformly accelerated.* If a small wheel were revolving without resistance, and at the end of every second I should apply a given impulse, the wheel would be uniformly accelerated; for, by its own inertia, it would retain all its previous motion, and, by the second law of motion, the repetition of the same force, at equal intervals, would increase its velocity at a uniform rate. If the intervals at which this force was repeated, were indefinitely diminished, the same kind of effect would take place; and the same would evidently be the case, were the force to operate without cessation. Such a force is that of *gravity.*

222. It has already been stated (Art. 14, 15), that gravity is a quality which belongs alike to all matter in proportion to its quantity; and that at different distances from the center of the earth, it varies inversely as the square of the distance. The manner in which this force decreases as the distance increases, will be seen at one view by the following table, beginning with the distance of the surface from the center:

Distance,	1	2	3	4	5	10	20	60
Attraction,	1	$\frac{1}{4}$	$\frac{1}{9}$	$\frac{1}{16}$	$\frac{1}{25}$	$\frac{1}{100}$	$\frac{1}{400}$	$\frac{1}{3600}$

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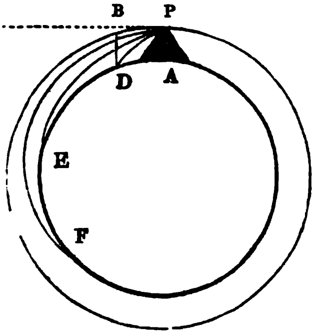
* Arnott's El. Phys., p. 104.

† Ibid., p. 107.

Hence it appears that a body placed 20 times as far from the center of the earth as the surface is from the center, is attracted only $\frac{1}{400}$th part as much; and at the distance of 60 times the radius of the earth, the same force is diminished 3600 times.[*] At this distance therefore it would take 60 seconds, or one minute, for a body to fall through the space it falls at the surface of the earth in one second; that is, through $16\frac{1}{12}$ feet. But all distances within a few hundred feet of the earth bear so small a ratio to the earth's radius, that the force of gravity may be considered as the same unvarying force, in relation both to the weights of bodies and to the velocities with which they fall. (See Art. 15.)

223. *Path of a projectile.*—It is not alone by the direct fall of bodies that the gravitation of the earth is manifested. The curvilinear motion of bodies projected in directions different from the perpendicular, is a combination of the effects of the uniform velocity that has been given to the projectile by the impulse which it has received, and the accelerated or retarded velocity which it receives from the earth's attraction. Suppose a body placed at P (Fig. 135), at any distance above the earth. If left to the earth's attraction alone, it would move with accelerated velocity toward the center. By horizontal projection alone, it would describe the line PB uniformly. By the two combined, it will describe the arc of an ellipse, PD, PE, or PF, according to the magnitude of the projecting force. And that force might be increased, until the body would go entirely round in a circle, or an ellipse, to P again. Though the path of a projectile is considered a parabola, it is strictly an ellipse of very great eccentricity, whose remote focus is the center of the earth; and could the body move unobstructed through the earth, and be governed by the same law of gravity as prevails outside of it, or if the whole earth were collected into a point at its center, the body would actually describe a narrow ellipse, passing close around the center with immense velocity, and returning to P, however small the projecting force. The law of gravity *within* the earth is different from that without, namely, *directly as the*

FIG. 135.



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[*] This last is nearly the distance of the moon from the earth; and it is found by calculation that the moon is actually drawn toward the earth, away from the straight line in which she tends to move, by exactly this force.

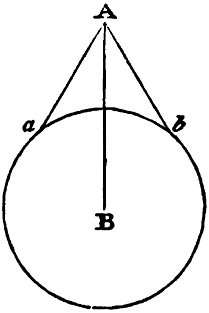
distance (Art. 17). Even according to *this* law, Newton has proved, that the projected body, if it could pass through the earth unobstructed, would return again to P, having described an ellipse, whose *center*, not *focus*, is at the center of the earth. On either supposition, therefore, the track of a missile is a part of an orbit described about the center of the earth; and the velocity of projection at P can be computed, which would be requisite for the description of any one of all these orbits.

224. *Orbits.*—The phenomenon of the revolution of the moon round the earth, is nothing more than the combined effects of the earth's attraction, and the tendency it has to move forward in a straight line which is a tangent to its orbit. And were any of the heavenly bodies to explode, we may conceive that the fragments would proceed in a rectilineal direction until, approaching, severally, within the sphere of influence of some large body, whose attraction would combine with their projectile force, they would forever afterward continue to revolve around that body, as the satellites revolve around their primaries.

225. *Attraction of small masses.*—But the attraction of gravitation is manifested by comparatively small masses of matter. The effect of a high mountain is perceptible upon a plumb-line, causing it to deviate sensibly from a perpendicular, so that the same star in the zenith would change its apparent place when viewed on opposite sides of the mountain. This was observed by two French astronomers, near Mount Chimborazo in South America, as early as the year 1738; and the experiment was repeated in 1772, with all possible accuracy, by Dr. Maskelyne, astronomer royal of Great Britain, at the base of the mountain Schehallien, in the eastern part of Scotland. Mr. Cavendish, a distinguished English philosopher of the last century, rendered sensible even the attraction of a sphere of lead, by bringing it near a small bullet, suspended from one arm of an exceedingly delicate torsion balance. The sphere when brought near the bullet disturbed its equilibrium.

226. By gravity, bodies are directed toward the center of the earth. We are not to infer from this fact that there is any peculiar force (like that of a large magnet, for example) residing at the center, but merely that the effect of the earth, taken as a whole, is the same as though its matter were condensed into the center. The line of attraction passes through the center because such a line is the *resultant* of the separate attractions of all the particles composing a sphere. Thus if AB (Fig. 136) be a line passing through the centre of the sphere,

FIG. 186.



and A be any body, and *a* and *b* two particles of matter in the sphere equally distant from A, it is evident that their combined actions would be expressed by the diagonal which would coincide in direction with AB. The same is true of any other particles taken equally distant from A in opposite hemispheres. At different parts of the earth, therefore, the directions of falling bodies are not *parallel*, but form converging lines. But on account of the great magnitude of the earth, two places 100 feet distant will not vary *one second*, and when a mile asunder, they will not differ *one minute* from perfect parallelism. For all the purposes of machinery, therefore, as well as for experiment, the direction of the lines of gravity may be considered as parallel.

227. Since bodies in falling toward the earth are uniformly accelerated, the *velocity* acquired must be proportioned to the time the body has been falling: at the end of ten seconds it has acquired ten times the velocity which it had at the end of one second. And in Art. 23, it has been shown that the *spaces* described are proportioned to the squares of the times; so that the space described during 100 seconds, is not merely 100 times as great as that described in one second, but it is the square of 100, or 10,000 times as great. This conclusion was arrived at mathematically, long before it was established by actual experiment. There were two difficulties which stood in the way of such a verification, viz., the little time occupied in descending through such perpendicular heights as the experimenter can command, and the resistance of the air, which, when the velocity becomes great, acts as a powerfully retarding force. We can rarely command a perpendicular eminence of more than four hundred feet, and yet the time occupied in the whole descent is only about five seconds, a period too short to enable us to mark distinctly the respective rates at which the successive intervals are described. Atwood's machine (Fig. 133, Art. 205) affords the means of obviating both these difficulties, and verifying the laws of falling bodies with great accuracy.

228. *Laws of fall shown by Atwood's machine.*—The object of the machine, so far as respects experiments on falling bodies, is to render the descent of bodies so *gradual*, that the relation between the times and spaces can be accurately observed. By recurrence to the figure, and to the descriptions given in Art. 205, we shall readily see how this object may be accomplished. Suppose the poises *m* and *n* to be each 31½ ounces, then the quantity of matter in both is 63 ounces. Now, since one of these rises as the other descends, the force of gravity retards the one as much as it accelerates the other, and they are in effect the same as though they were entirely destitute of gravity. If a small weight, as one ounce, were let fall *freely* from the top of the machine, it would fall through so small a space almost in an instant, and we should be unable to mark the rate at which it would pass over the successive portions of the graduated scale FB; but if it be laid on the weight *n*, it must carry *n* along with it; that is, it must make *n* descend and *m* ascend, and therefore the motion belonging to one ounce will be distributed throughout 64 ounces, and its velocity will be retarded in the same ratio. Consequently the weight *n* will descend only $\frac{1}{64}$th part as fast as a body falling freely; and as a body falling freely descends in one second about 16 feet, or 192 inches, the weight *n* will descend $\frac{192}{64} = 3$ inches in one second. The comparative progress of this weight, and of a body falling freely for several successive seconds, will be seen in the following table:

Time,	1	2	3	4	5	6
Body falling freely, in feet,	$16\frac{1}{12}$	$64\frac{1}{3}$	$144\frac{3}{4}$	$257\frac{1}{3}$	$402\frac{1}{12}$	579
Do. in Atwood's Machine, in inches,	3	12	27	48	75	108

Hence it appears that in six seconds, while a body would fall freely through 579 feet, it may in the same time descend only nine feet in Atwood's machine. But the latter is a uniformly accelerated velocity, and subject to the same laws as the former, and it may therefore be employed to investigate the laws of falling bodies. The results correspond remarkably with theory, so that when the instrument is well constructed, and managed skillfully, the descending weight clicks upon the stage or brass plate *p*, at the very instant required.

229. We see in nature the law of acceleration of falling bodies indicated by the impetuosity with which bodies fall from any considerable height upon the earth. Meteoric stones, falling from the sky, sometimes bury themselves deep in the ground. Aeronauts that have fallen from balloons have been dashed in pieces. It is, however, a rare occurrence to see a body falling from any great height *perpendicularly:* most instances of accelerated motion, which come under our observation, are in bodies falling down *inclined planes*, where the same law of acceleration prevails. (Art. 123.) A fragment of rock descending from the side of a mountain, has its speed augmented as it goes, until its momentum becomes irresistible, and large trees are prostrated before it.

Some years since, a remarkable example of the acceleration of bodies descending an inclined plane, was to be seen at the *slide of Alpnach*, in Switzerland. On Mount Pilatus, near Lake Luzerne, was a valuable growth of fir-trees, which, on account of the inaccessible nature of the mountain, had remained for ages undisturbed, until a German engineer contrived to construct a trough in the form of an inclined plane, by which these trees were made to descend by their own weight, through a space of eight or nine miles, from the side of the mountain to the margin of the lake. Although the average declivity was no more than about one foot in seventeen, and the route often circuitous and sometimes horizontal, yet so great was the acceleration, that a tree descended the whole distance in the short space of six minutes. To a spectator standing by the side of the trough, at first was heard, on the approach of the tree, a roaring noise, becoming louder and louder; the tree came in sight at the distance of half a mile, and in an instant afterward shot past with the noise of thunder and the rapidity of lightning. When a tree happened to "bolt" from the trough, it cut the standing trees quite off.*

230. *Composition and resolution of motion.*—*Simple* motion is that which arises from the action of a *single* force; *Compound* motion is that which is produced by *several* forces acting in different directions. Strictly speaking, we have no example of a simple motion, since in the absolute motion of all bodies, their own proper motion is combined with that of the earth in its diurnal and annual revolutions, and we know not with how many others. (Art. 198.) In an enlarged sense, therefore, all motions are compound. But in the foregoing distinctions we have reference only to relative motions, as those which take place among bodies on the earth. In accordance with the second law of motion (Art. 214), a force striking upon a body in motion, will produce the same change of motion as though the body had been at rest when the force struck it. This may at first view appear inconsistent with experience, especially in regard to opposite motions. Let us, therefore, consider the principle in its application to several different cases. Conceive the ice of a frozen river to be first stationary, and afterward to float down with the current. Standing on the bank, I roll a ball directly across the river. Will it pass in the same direction in both cases? It will not; for in the first case it will pass across *perpendicularly* to the banks, and in the second case it will go across *diagonally*. But now let me stand upon the ice and roll the ball. Since I float along with the ice, I am at rest with respect to that motion, and the ball, though moving diagonally, as before, appears to me to go directly across the stream.* If the ball was rolled not directly across but obliquely, making any angle with the bank, if I stood upon the floating ice, and was therefore at rest with respect to one of the motions of the ball, I should see the other motion in the same manner as though I had stood on the shore and the ice had been at rest. But if, when a ball is rolling toward me, I strike it in a direction exactly opposite to its course, but do not stop it, can the blow be said to produce the same change of motion as though the body had been at rest when the blow was applied? —*Ans.* If I had been moving in the same manner as the ball before the blow, then stopping a part of the motion of the ball would have given it a relative motion in the opposite direction; since having none of my own motion stopped, I should leave it behind. This is what takes place when a cannon-ball is fired in a direction contrary to that in which the earth is revolving about the sun. The cannon moves onward and leaves the ball behind. (Art. 198.)

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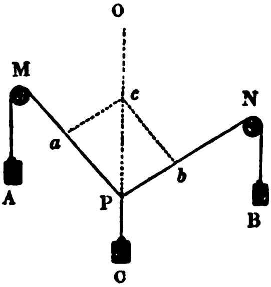
* Playfair's Works, i, 96.

FIG. 137.



231. The laws respecting the composition and resolution of motion which are demonstrated in Chap. III, admit of being satisfactorily confirmed by experiment. Let two small wheels M, N (Fig. 137), be attached to a wall or board. Let a thread be passed over them, having weights A and B, hooked upon loops at its extremities. From any part P of the thread, let a weight C be suspended, in such a manner as to be in equilibrio with A and B. The weight C, therefore, is the resultant of the forces A and B; and since its direction is that of gravity, it will be represented by a line drawn directly upward from P. From P, on the line PO, take P*c*, having as many inches as there are ounces in C; and from *c* draw *ca* parallel to NP, and *cb* parallel to PM. If the sides P*a* and P*b* of the parallelogram thus formed be measured, it will be found that P*a* will consist of as many inches as there are ounces in A, and P*b* of as many inches as there are ounces in B; consequently, the lines P*c*, P*a*, and P*b*, have the same ratio to each other as the forces or weights C, A, and B. But P*c* is the diagonal, and P*a* and P*b* are the sides of a parallelogram. Hence the diagonal of a parallelogram represents a force equivalent to the two forces represented by the sides.*

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* In the same manner two persons sitting in a boat under sail, toss a ball from one to the other in the same manner as though they were at rest on land.

This experiment is an illustration of the composition of *forces* rather than of the composition of *motions;* but a simple experiment will show that the same law holds good with respect to a body actually set in motion by two different forces. A ball is placed at one of the corners of a smooth table. To the same corner are attached two springs, respectively in the line of the two sides of the table, and capable of giving a simultaneous impulse to the ball. The springs moreover are so proportioned to each other, that one will drive the ball across one side of the table, in the same time that the other will drive the ball across the other side. Now on letting go both springs at once, the ball will pass, in the same time, across the diagonal of the table to the opposite corner.†

232. We daily observe examples strikingly illustrative of these laws. In crossing a river, the boatman heads up the stream, and so combines the direction of the boat with that of the current, as to move directly across in a line which is the diagonal between the two. Rowing, swimming, and flying, are severally instances of motion in the diagonal between two forces. In feats of horsemanship, when the rider leaps up from his saddle, we are surprised not to see the horse pass from under him; but he retains the motion he has in common with the horse, and does not in fact ascend perpendicularly, but obliquely, rising in one diagonal, and falling in another. In the common feats of jumping through a hoop, and alighting again on the saddle, an inexperienced rider would be likely to project his body forward in the same manner as he would do in leaping through the same hoop from the ground. In such a case, instead of alighting on the saddle, he would alight either before the horse or upon his head or neck. All that is requisite in order to execute this feat, is to leap directly upward from the saddle to a sufficient height to clear the lower part of the hoop with the feet. By the speed which the rider has in common with the horse, his body will, without any exertion on his part, pass through the hoop, and he will alight again on the saddle, on the other side, in his descent.

233. *Resolution of force in the sailing of a ship.*—The *sailing of a ship* affords an instructive illustration of the principles of the composition and resolution of motion. When a ship sails in the same direction as the wind, she is said to be *scudding*, or sailing *before the wind*, and if she had but one sail, it would act with the greatest advantage when perpendicular or

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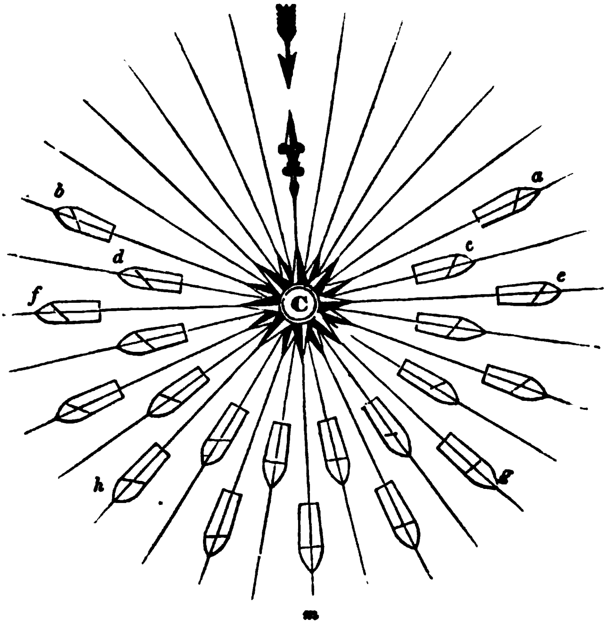
*** Lardner's Mech., p. 51.** **† Lib. Useful Knowledge, Mechanics, p. 5.**

nearly so to the wind. When a ship advances against the wind, and endeavors to proceed in the nearest direction possible to the point of compass from which the wind blows, she is said to be *close hauled.* A large ship will sail against the wind with her keel at an angle of six points with the direction of the wind; and sloops and smaller vessels may sail much nearer. When a ship is neither sailing before the wind, nor is close hauled, she is said to be *sailing large.* In this case, her sails are set in an oblique position, between the direction of the wind and that of the intended course; as represented in the various plans of vessels in Fig. 138, where the direction of the wind is represented by the arrow, and the position of the yards and sails, which are necessary for proceeding on the various points of the compass, are shown by the transverse line on each plan. The relation of the wind to the course of the vessel, is determined by the number of points of the compass between the course she is steering, and the course she would be steering if close hauled. In Fig. 138, the ships *a* and *b* are close hauled, and the ships *c* and *d* (the former steering east by north, and the latter west by north) have the wind one point large. The ships *e* and *f*, one steering east and the other west, have the wind two points large. In this case, the wind is at right angles with the keel, and is said to be *upon the beam.* The ships *g* and

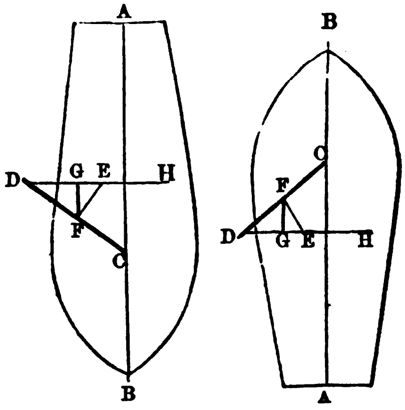
FIG. 138.



h, steering southeast and southwest, have the wind six points large, or, as it is commonly termed, *upon the quarter*, and this is considered as a very favorable manner of sailing, because all the sails co-operate to increase the ship's velocity; whereas when the wind is directly aft, as in the vessel *m*, it is partly intercepted by the after sails, and prevented from striking with its full force on those which are forward.*

234. *Opposite directions by the same wind.*—To one who has never studied the doctrine of the composition and resolution of forces, it is apt to appear mysterious that a ship is able to sail with a wind partly ahead; and still more, that two ships are able to sail in exactly opposite directions by the same wind. It is proposed to explain these phenomena. Let AB (Fig. 139,) represent the keel of a ship, and CD the sail; and let the wind come in from the side, in the direction of HD. Let ED represent the whole force of the wind, and resolve ED into two forces, viz. into EF perpendicular, and FD parallel to the sail DC. Then it is manifest that EF alone represents the effective force of the wind upon the sail. But EF is not wholly employed in urging the ship forward, since it is oblique to her course; therefore, again resolve EF into GF parallel with the course, and EG at right angles with it. The latter force is lost by the lateral resistance of the water, while GF is employed in propelling the ship on her way.

Fig. 139.



By inspecting Fig. 139, it will readily be seen that another ship may sail in the opposite direction by the same wind. When the wind strikes the sail at right angles, or in the direction EF, then only one resolution is necessary; for if EF represents the whole force of the wind, GF will represent the force that propels the ship forward, while EG will represent the part which is lost by the lateral resistance of the water.

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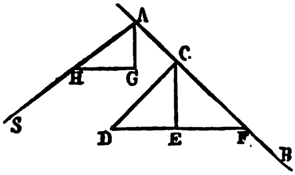
* Bigelow's Elements of Technology, p. 218.

Since, resolving the force of the wind after the foregoing manner, the effective part of the force, viz. GF, will not wholly disappear until the wind is directly ahead, it might seem possible to sail much nearer the wind than is found to be actually practicable. But, though, on account of the peculiar shape of vessels, the forward resistance is much less than the lateral, yet it is *something*, and therefore requires more or less of the force that acts parallel with the keel to overcome it.

235. The doctrine of the composition and resolution of motion, by reducing a great number of complicated motions to one, or by enabling us to estimate the precise influence of forces that act obliquely, has greatly simplified inquiries in Mechanics, and proportionally advanced the science. It is only by such means that the complex motions of the heavenly bodies, and the equally diverse forces that control them, could ever have been understood. The subject has also an extensive and important application in estimating the powers of machinery. Since, for example, any one side of a triangle or polygon, is always less than the sum of all the remaining sides, it follows, other things being equal, that a mechanical effect will always be more economically produced by a single force acting in the proper direction, than by a number of forces acting in different directions.

Fig. 140.



236. *Resolution of forces in the flying of a kite.*—Let AB (Fig. 140) be a kite, held by the string AS. Let DF represent the force of the wind blowing horizontally, and resolve it into two forces, viz., DC perpendicular, and CF parallel to the kite. Then DC will be the only effective part of the wind, since that part which acts parallel to the kite can have no influence on its motions. Again, resolve DC into two forces, namely, EC perpendicular, and DE parallel to the horizon. Then EC will represent the upward force of the wind, and DE its force in a horizontal direction. Now when the string AS makes such an angle with the kite that its downward force AG, added to the weight of the kite, shall equal EC, and its horizontal force GH equal DE, the kite will be at rest.

237. When two motions which are not in the same straight line are combined, one of which is uniform, and the other accelerated, the moving body describes a *curve*. (Art. 49.) If two accelerating forces act, in which the rate of acceleration is the same, the motion is *rectilinear;* but if the rate of acceleration is different, the motion is still *curvilinear.* The nature of the curve will depend upon the proportion between the impulsive and accelerating forces.

CHAPTER III.

THE CENTER OF GRAVITY.

238. The principles which have been discovered respecting the composition and resolution of forces, and respecting the center of gravity, have alike contributed greatly to simplify the doctrines of Mechanics. It is characteristic of a great and penetrating mind, to devise means of divesting intricate subjects of their complexity, and thus to bring easily within the grasp of the mind, subjects otherwise too much involved to be within its comprehension. By the rule of simple multiplication, we easily multiply any number by one thousand: indeed, it is nothing more than to annex three ciphers to the number itself; but how tedious would be this process, were the rule of multiplication undiscovered, and we were unacquainted with any other method of arriving at the result, except to add the given number to itself one thousand times! In like manner, by means of the rules for the composition of motion, we are enabled to reduce a thousand different motions to one; and by the doctrine of the center of gravity, we are taught how we may make a force, situated at one single point, equivalent to an infinite number of forces, situated in as many different points. And, instead of pursuing the endless diversities of motions to which the different parts of a complicated system of bodies may be subject, we are taught how to follow merely the motions of a single individual point. By the earth's attraction, all the particles which compose the mass of a body, are solicited by equal forces in parallel directions downward. If these component particles were placed in mere juxtaposition, without any mechanical connection, the force impressed on any one of them could in no wise affect the others, and the mass would, in such a case, be contemplated as an aggregation of small particles of matter, each urged by an independent force. Then, according to Art. 56, the *resultant* constitutes another force parallel to the others. But the bodies which are the subjects of investigation in mechanical science, are not found in this state. Solid bodies are coherent masses, the particles of which are firmly bound together, so that any force which affects one, being modified according to circumstances, will be transmitted through the whole body.

239. As all bodies which are subjects of mechanical inquiry, on the surface of the earth, must be continually influenced by terrestrial gravity, it is desirable to obtain some easy and summary method of estimating the effects of this force. To consider it, as is unavoidable in the first instance, the combined action of an infinite number of equal and parallel forces, soliciting the elementary molecules downward, would be attended with manifest inconvenience. An infinite number of forces, and an infinite subdivision of the mass, would form parts of every mechanical problem. To overcome this difficulty, and to obtain all the ease and simplicity which can be desired in elementary investigations, it is only necessary to determine some force, whose single effect shall be equivalent to the combined effect of the gravitation of all the molecules of the body. Such a force is obtained by supposing all the action to be concentrated in the center of gravity. We have already defined it thus: *The center of gravity of a body, is that point about which, if supported, all the parts of a body (acted upon only by the force of gravity) would balance each other in any position.* (Art. 70.)

240. *To find the center of gravity by experiment.*—We will first suppose the body to be in the shape of a piece of board of uniform thickness. Suspend it by one corner, and from the same corner let fall a plumb-line, and mark its line of direction on the surface of the board. Suspend the board from any other point, and mark the line of direction of the plumb-line as before, and the point where these lines intersect each other must obviously be the center of gravity, since that center is in both of the lines.

But when the body is not of uniform thickness, but is any irregular solid, suspend the body by a thread, and let a small hole be bored through it, in the exact direction of the thread, so that if the thread were continued below the point where it is attached to the body, it would pass through this hole. The body being successively suspended by several different points in its surface, let as many small holes be bored through it in the same manner. If the body be then cut through, so as to discover the directions which the several holes have taken, they will all be found to cross each other at one point within the body. Or the same fact may be discovered thus: a thin wire which nearly fills the holes, being passed through any one of them, will be found to intercept the passage of a similar wire through any other.*

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* Lardner's Mechanics, p. 110.

A convenient method of finding the center of gravity of a body is *to balance it in different positions across a thin edge*, as the edge of a knife or a prism. The same thing may be effected, when the shape of the body will admit of it, by laying it on the edge of a table, and letting so much of it project over the edge, that the slightest disturbance will cause it to fall. The center of gravity is the point in which the several lines marked on the body, where the edge cuts it, intersect one another. From some or all of the foregoing trials, the center of gravity of bodies may be nearly ascertained; but in order to find it with absolute exactness, we are frequently obliged to resort to intricate mathematical processes.

241. *Equilibrium.*—By whatever method the center of gravity of a body has been ascertained, we shall find that when that is supported, the body will remain at rest in every position. Thus a globe of uniform density will rest on a horizontal plane, with any point of its surface for a base, since the center is directly above that point. An irregular body of any form, with a delicate axis passing precisely through its center of gravity, will rest in any position. But when a body is suspended on an axis *not* in the center of gravity, it will rest only when the center is directly above or directly below the axis of support (Art. 85); in every other position, one component of gravity will give it circular motion round the axis. If the center is above the support, the equilibrium is *unstable*, for the center cannot return, if disturbed; but if the center is below, the equilibrium is *stable*, since the center always returns to the same position. Again, let a curved body, as an ellipsoid, lie on a horizontal plane; it will have a stable or an unstable equilibrium, according as its center of gravity *rises* or *falls*, when it is rolled either way. A prolate spheroid, standing on its *end*, has an unstable equilibrium; lying on its *side*, its equilibrium is stable. A gate, hung so that its center rises as it is opened, shuts spontaneously. In none of the cases mentioned in this article, however, is there any *stability* in the body, since the base of support is only a point or a line, and the least force will move the center out of the vertical, passing through the pivot. For *stability*, there must be some breadth of base.

242. *Stability.*—The stability of a body not only requires that the center of gravity should be low, but that *the line of direction* (or the line which is drawn through the center of gravity perpendicular to the horizon) *should fall within the base.* (Art. 84.) The further it falls from the extremity of the base, the more stable is the position. Hence the stability of a pyramid, when standing on its broad base, and its instability when inverted. For the same reason, all broad vessels, as steamboats, are difficult to upset, while vehicles with narrow bases are easily overturned. When a load is so situated as to raise the center of gravity, it increases the liability to upset, because it increases the facility with which the line of direction is thrown without the base. Thus carts loaded with hay, or bales of cotton, are very liable to be overturned. The same is true of stages carrying passengers or baggage on the top. On the other hand, a large ship well supplied with ballast, is capsized with great difficulty, since the center of gravity of all parts of the ship is so low, as to render it difficult to throw the line of direction without the base. Yet if the center of gravity is very low, a ship will rock excessively in a rough sea, since the upper parts, near the deck, move over a greater space in proportion as their distance from the center of gravity is greater.

243. There are many remarkable structures which *lean* or incline a little; but so long as the line of direction falls within the base, and the parts of the mass have sufficient tenacity among themselves to hold together, the structure will stand. The famous tower of Pisa was built intentionally inclining, to frighten and surprise: with a height of one hundred and thirty feet, it overhangs its base sixteen feet.* This circumstance greatly enhances the emotion of the spectator from its summit. Many ancient spires, and other tall structures, are found to have lost something of their perpendicularity.

Rocking stones are rocks which are sometimes found so exactly poised upon a point beneath their center of gravity, that a very small force is sufficient to put them in motion. The rocking of a balloon when it begins to ascend, affords an illustration of the tendency of bodies to vibrate round the center of gravity.

244. The *motions of animals* are regulated in conformity with the doctrines of the center of gravity. A body is seen tottering in proportion as it has great altitude and a narrow base; but it is a peculiarity in man to be able to support his figure with great firmness on a very narrow base, and under constant changes of attitude. This faculty is acquired slowly, because of the difficulty. The great facility with which the young of quadrupeds walk, is ascribed in part to their broad supporting base. Many of our most common motions and attitudes, depend for their ease and gracefulness upon a proper adjustment of the center of gravity. The erect posture of a man carrying a load upon his head—leaning to one side when a heavy weight is carried in the opposite hand—leaning forward when a weight is on the back—or backward when the weight is in the arms; these are severally examples in point. When a man rises from his chair, he brings one foot back, and leans the body forward, in order to bring the center of gravity over the base; and without adjusting it in this manner, it is hardly possible to rise. A man standing with his heels close to a perpendicular wall, can not bend forward sufficiently to pick up any object that lies on the ground near him without himself falling forward.

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* Some travelers, however, are of opinion that the tower of Pisa has settled from a perpendicular to an inclined position.

The art of rope or wire dancing, depends in a great degree upon a skillful adjustment of the center of gravity. The rope-dancer frequently carries in his hand a stick loaded with lead, which he so manages as to counterbalance the inclinations of his body, which would throw the line of direction out of the base. Upon a similar principle, the equestrian balances himself on one foot on a galloping horse.

The *vegetable creation* is subject also to these general laws of nature. Trees, by the weight and height of their tops, would seem peculiarly liable to fall; but their roots afford a corresponding breadth of base, while their perpendicular trunks, and the symmetrical disposition of the branches, conspire to increase their stability.

245. The position of the center of gravity of any number of separate bodies, is never altered by the *mutual action* of those bodies on each other. If, for example, two bodies, by mutual attraction, approach each other, the center of gravity remains at rest, until finally the bodies meet in this point. If, by their mutual action, they contribute to make each other revolve in orbits, it is around their common center of gravity. Thus the earth and moon revolve around a common center of gravity: the same is true of the sun and all the bodies that compose the solar system. Were the projectile force to be suspended, and the bodies abandoned to the mutual action of each other, they would all meet in their common center of gravity. (Art. 91.) This naturally results from the principle that the momenta on opposite sides of the center of gravity are equal, and that bodies by their mutual action produce equal momenta in each other.

246. *Solution of practical questions.*—The doctrines of the center of gravity suggest the readiest method of solving a great number of *practical problems.*

Suppose three persons were carrying a stick of timber (A by himself supporting one end, and B and C by a handspike lifting together toward the other end), and it were required to determine at what distance from the end of the stick the handspike must be placed in order that the three persons might bear equally. A stick of timber being a body of regular shape and uniform density, has its center of gravity coincident with the center of magnitude. We may therefore proceed on the supposition that the entire weight is collected in the center. Now in order that B and C may together lift twice as much as A, they must be twice as near the center. But the distance of A from the center is one-half the length of the stick; therefore the distance of the required point from the center is one-fourth the length of the stick, and consequently it is one-fourth the same length from the end of the stick.

The result thus obtained from theory, may be easily submitted to the test of experiment. For if we take the weight of the stick of timber with a pair of steelyards, and then, resting one end on some support, attach a cord at the distance of one fourth of the length of the stick from the other end, and thus connect the stick with the steelyards, we shall find the weight equal to two-thirds of the whole.

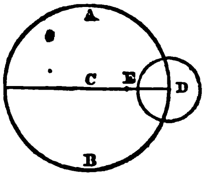
247. *Centrobaric mensuration.*—The method of finding *the areas of the surfaces and the solid contents of bodies*, particularly of such as are of unusual figure, may frequently be very much simplified, by applying the principles of the center of gravity. This is sometimes called the *centrobaric* method. It was discovered by Pappus, an ancient mathematician of Alexandria, but was more completely discussed and illustrated by Guldinus, professor of mathematics at Rome, about the year 1640.* This remarkable property of the center of gravity is expressed in the following propositions:

1. *If any line whatsoever revolves about a fixed point, the* SURFACE *which it generates is equal to the product of the given line into the circumference described by its center of gravity.*

2. *If a surface having any figure whatsoever, revolves about an axis, the* SOLID *generated, is equal to the product of that surface into the circumference described by its center of gravity.*

Thus the straight line CD (Fig. 141), revolving about the center C, describes a circle whose surface is equal to CD into the circumference of the circle described by its center of gravity, E. This is evident also from the consideration that, since E is the center of the line CD, the circumference described by it will be half the length of the circumference ADB; and the area of a circle is equal to the product of the radius into half the circumference.

FIG. 141.



Again, the small circle, having its center coincident with the extremity of the line D, and revolving round on the circumference of the circle described by CD, being everywhere perpendicular to the plane of the circle, would describe a solid figure like the ring of an anchor; and the line described by the center of gravity, that is, the circumference of the circle, multiplied into the area of the revolving figure, would give the solidity of the ring. In like manner, in a cone, the solidity is equal to the *area* of the generating triangle, multiplied into the circumference of the circle, formed by the revolution of the center of gravity of the triangle; and the surface is equal to the product of the *perimeter* of the same triangle, multiplied by the circumference described by the center of gravity of the same perimeter.

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* Hence these properties of the center of gravity are sometimes called *Guldinus' Properties.*

CHAPTER IV.

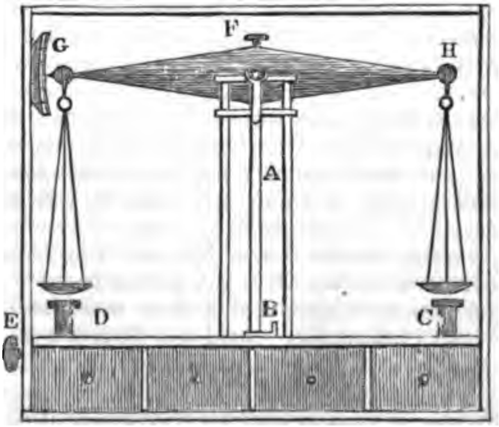
MACHINERY.

248. The organs employed in communicating motion, are tools, machines, and engines. *Tools* are the simplest instruments of art: these, when complicated in their structure, become *machines;* and machines, when they act with great power, take the name of *engines.* Among the ancients, machines were confined chiefly to the purposes of *architecture* and *war;* and they were moved almost exclusively by the *strength of animals.* Thus, in building one of the great pyramids of Egypt, vast masses of stone were raised to a great height, amounting together to 10,400,000 tons. In this labor were employed 100,000 men for 20 years. The advantage which man has gained by pressing into his service the great powers of nature, instead of depending on his own feeble arm, is evinced by the fact, that by the aid of the steam-engine, one man can now accomplish as much labor as 27,000 Egyptians, working at the rate at which they built the pyramids. In war also, while the use of gunpowder was unknown, engines of great power were invented for throwing stones and javelins, and for demolishing fortifications. Such were the catapulta, the balista, and the battering-ram, of the Romans. Yet it is remarkable, that during many ages, while such powerful auxiliaries were employed in architecture and in war, the ancients should have made so little use as they did of machinery in the ordinary processes of the arts. Nor did the philosophers of antiquity cultivate science with any reference to the improvement of the arts, an object which they considered below the dignity of true philosophy; but Lord Bacon was the first to show that "a principle in science is a rule in art."

249. *Simple machines.*—These, which constitute the chief elements of all machinery, have often been called, inappropriately, the "mechanical powers." They have already been considered *theoretically;* we are now to consider them *practically.* It will be recollected that, when two forces act on one another by means of any machine, that which gives motion is called the *power;* that which receives motion, the *weight.* The weight includes not only the proper weight of the body or bodies moved, but also every kind of resistance opposed to the action of the power, whether it arises from the quantity of matter in the body moved, or from the inertia of the machine itself, or from the air, or from friction. It will be further recollected, that an equilibrium is produced between two forces, when they, or the *momenta* which measure them, are equal. (Art. 11.) Now, since momentum is compounded of quantity of matter and velocity, a given momentum may be produced, either by giving a great velocity to a small weight, or a small velocity to a great weight. The consideration of this subject will be resumed hereafter.

250. *The balance.*—The principle of the lever has a most extensive application in the arts, and the forms under which it occurs are very various. We may contemplate it as having equal or unequal arms. The *balance* affords the most common example of a lever with equal arms. The necessity of arriving at the weight of bodies with the greatest degree of accuracy in pecuniary transactions, and more especially in delicate scientific researches, as those of chemical analysis, has induced men of science, and artists, to bestow great and united attention upon the construction of this instrument, until they have brought it to an astonishing degree of perfection.

FIG. 142.



The principal parts of the balance are the beam GH (Fig. 142), the points of suspension G and H, and the fulcrum F. In order to construct a perfect balance, the most important particulars to be attended to, are the length of the arms, that is, of the beam; the situation of the center of gravity of the whole instrument, with respect to the fulcrum or center of motion; and the position of the points of suspension.

1. The sensibility of the balance is increased by increasing the *lengths of the arms;* but unless the arms, when long, are at the same time of considerable weight, they will not have the requisite strength, but will be liable to bend; and an increase of weight adds to the amount of friction on the center of motion. It is not common, therefore, to make the arms of a very delicate balance more than nine inches in length; and, for the purpose of uniting lightness with strength, the beam is composed of two hollow cones placed base to base, as in Fig. 142.

2. The *center of gravity* of the instrument must be a little below the center of motion. For if the beam is balanced on its center of gravity, it will remain at rest in every position (Art. 108), whereas it must be at rest only when in a horizontal position. If the center of gravity is above the center of motion, the position is unstable (Art. 85), and upon the least disturbance of the equilibrium, the beam will be liable to upset. Finally, if the center of gravity is too far below the center of motion, the equilibrium will be too stable. Hence, in very delicate balances, the center of motion is placed a little above the center of gravity.

3. The *points of suspension* must be in the same right line with the center of motion. For since when weights are added to the scales, the effect is the same as though they were concentrated in the points of suspension, were those points above the center of motion, the center of gravity would be liable to be shifted above the center of motion, when the beam would upset; and if the same points were below the center of motion, the center of gravity would be too low, and the equilibrium too stable.

In order to prevent friction as much as possible, the *fulcrum* is made of hardened steel, and shaped into a triangular prism, or knife-edge, smoothly rounded, and turning on a plane of agate or steel, or some other very hard and polished substance.

251. It is only by a nice attention to all these particulars, that artists have been able to give to the balance so great a sensibility. Some have been made to turn with the 1000th part of a grain.* By loading the beam, the sensibility of the instrument is diminished; it is customary, therefore, to estimate its power, by finding what part of the weight with which it is loaded it takes to turn it. Thus, if when loaded with 7000 grains, it will turn with one grain, its power is $\frac{1}{7000}$. A balance constructed by Ramsden, a celebrated English artist, for the Royal Society, turned with the *ten millionth* part of the weight.* Delicate balances are usually covered with a glass case to prevent agitation from the air, and to secure them from injury. Fig. 142 represents an instrument of this kind made for the Royal Institution of Great Britain.†

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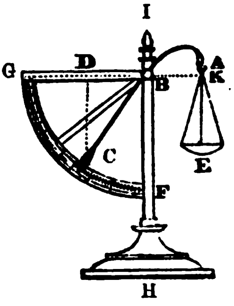
* Kater, in Lardner's Mechanics.

252. It is essential to an accurate balance, that the two arms should be precisely equal in length. The *false balance*, which is sometimes used with a design to defraud, has its arms unequal. The dealer turns such an instrument to his account, both in buying and selling. In buying he puts his weights on the longer side, for then it takes more than an equivalent to balance them; and, in selling, he puts his weights on the shorter side, because less than an equivalent will produce an equilibrium. The fraud may be detected by making the weights and the merchandise change places. The true weight may be determined from such a balance by the rule given in Art. 111; or more conveniently, by putting the article whose weight is to be determined into one scale, and counterpoising it with sand, shot, or any convenient substance, in the other scale, and then removing the article, and finding the exact weight of the counterpoise. It is evident that the weight of the merchandise will be the same as that of the weights employed to balance its counterpoise.

FIG. 143.



253. *The bent-lever balance.*—Fig. 143 represents an instrument so called. The arm BA inclines 135° degrees from BC, so that when C passes over the quadrant FG, from vertical to horizontal, A passes over a quadrant lying one-half above and one-half below the horizontal position. Hence the acting distance of C varies from zero to radius, that of A only from cos 45° to radius, about one-third as much. The instrument, therefore, somewhat resembles the steelyard, in which the poise gains moment only by increasing the *acting distance*, and the article weighed, only by increasing its *quantity*. The graduation on the quadrant FG will obviously be unequal.

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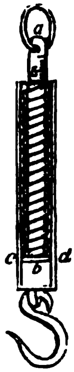
* Young's Lectures on Nat. Phil., i. 125.

† For a full account of the most accurate balances, see Kater on "Balances and Pendulums," in Lardner's Mechanics, p. 278.

254. *The steelyard.*—This is a lever having unequal arms, in which the same body is made to indicate different weights, by placing it at different distances from the fulcrum. A pair of steelyards has usually two graduated sides for determining smaller or greater weights. It will be seen that on the greater side, the weight is placed nearer the fulcrum. Consequently, the weight indicated by the counterpoise, when at a given distance from the fulcrum, will be proportionally greater. This instrument is very convenient, because it requires but one weight. The pressure on the fulcrum, excepting that of the apparatus itself, is only that of the article weighed, whereas in the balance, the fulcrum sustains a double weight. But the balance is susceptible of more sensibility than the steelyard, because the subdivisions of its weights can be effected with a greater degree of precision than the subdivision of the arms of a steelyard.

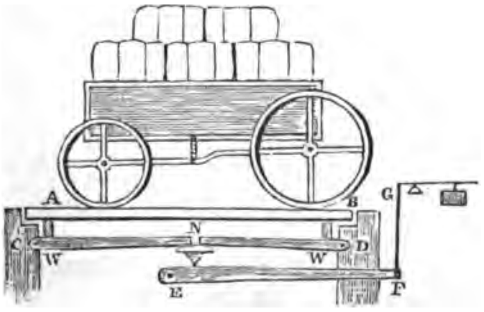
255. The *spring steelyard* is a very convenient instrument for weighing, in cases where the subdivisions of the weights are large. It depends on the elasticity of a spiral steel-spring, to compress or extend which requires a force proportioned to the degree of compression or extension. The manner of applying it will be easily understood from the representation in Fig. 144. After continued use, especially when loaded with heavy weights, the elasticity of the spring is liable to be impaired, and the accuracy of the instrument diminished. When made, however, in the best manner, spring steelyards retain their accuracy for a long time.

FIG. 144.



256. *Hay-scales.*—In Fig. 145 is represented a vertical section of a large *weighing machine*, such as is used for loads of hay, cotton, or other heavy merchandise.

FIG. 145.



AB, a section of the platform, resting loosely on a frame.

CN, DN, levers of the second kind, having their fulcrums at C, D, and resting on a bar at N.

W, W, pins which press upward against the platform when the levers are raised.

EF, a lever likewise of the second kind, having its fulcrum at E, and connected by a perpendicular arm with the beam of a pair of steelyards at G.

Four levers are usually employed, proceeding from the four corners of an immovable frame, or having their fulcrums firmly set in masonry. The levers all rest on the common support at N.

Suppose a load of merchandise is placed on the platform to be weighed. By the steelyards, we ascertain that the weight exerted at G is 100 pounds, which force is that exerted at F to raise the lever EF. Supposing, for convenience of computation, that the levers have their longer ten times the length of their shorter arms, then 100 pounds at F balances a force of 1000 pounds at N. This force is still further multiplied by the four levers so as to become 10,000 pounds, which is the weight of the load including that of the platform. If the platform rested on a single lever, this would of course sustain a weight of 10,000 pounds; but as each lever transmits to N a tenth part of the pressure at W, the whole at N will be a tenth of the whole upon the platform, whether the load rests on one, two, or all of the levers.

257. When a weight is supported by a lever which rests on two props, the pressure upon both fulcrums is equal to the whole weight. This principle is sometimes applied in ascertaining the weight of a body too heavy for the steelyards. The body is suspended immovably near the center of a pole, and the steelyards are applied to each end of the pole separately, the other end meanwhile resting on its fulcrum. The two weights being added together, make the entire weight of the body. If the body is suspended exactly in the centre of the pole, it will be sufficient to obtain the weight of one end and double it. The weight of the lever should, in both cases, be subtracted from the entire weight.

Since when a weight is sustained between two props, the *part sustained by each prop is inversely as the distance of the weight from it*, it follows that a load borne on a pole, between two bearers, is distributed in this ratio. As the effort of the bearers, and the direction of the weight, are always parallel, it makes no difference whether the pole is parallel to the horizon or inclined to it. Whether the bearers ascend or descend, or move on a level plane, the weight will be shared between them in the same constant ratio. (See Fig. 76.)

258. *Implements.—Handspikes* and *crowbars* are familiar examples of levers of the first kind. The handle and claw of a *hammer* form a *bent lever;* and shears, pliers, nutcrackers, and all similar instruments, are *double levers;* that is, they consist of two levers united. A pair of shears, with long handles, like those used by tinners, exhibit very strikingly the increase of power gained by bringing the weight or substance acted on nearer to the fulcrum. The jaws of animals exhibit a similar property. An oar, applied to a boat rowed by hand, a wheelbarrow, and a door shut by the hand applied to the edge remote from the hinges, severally furnish instances of levers of the second kind, where the weight is between the fulcrum and the power.

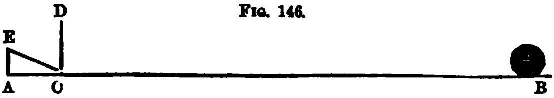
The *crane* is a lever of the second kind, which is much used when great weights are transported for a short distance, as heavy boxes of merchandise from a vessel to the wharf, or great masses of stone from the quarry to a car or boat. An example of the crane, on a small scale, is seen in the apparatus of a kitchen fireplace.

259. When one raises a *ladder* from the ground by one of the lower rounds, the ladder becomes a lever of the third kind, the power being applied between the weight and the prop. Since in all the mechanical powers, the power and weight have equal momenta, and since, in the third kind of lever, the weight has more velocity than the power, the power is as much greater than the weight as the velocity with which it moves is less. The difficulty experienced in raising a ladder from the ground by taking hold of the lowest round, or of shutting a door by applying the hand to the side next to the hinges, shows the mechanical disadvantage under which a lever of this kind acts. Yet it is very useful in cases when it is required to give great *velocity* to the body moved. *Sheep-shears* consist of two levers of this kind united. Here the whole force required is so small that to save it is of no consequence, while so soft and flexible a substance as wool requires the shears to be moved with considerable velocity. A pair of tongs is composed in the same manner; and therefore it is only a small weight that we can lift with them, especially when the legs are long.

260. *Bones of animals.*—One of the most remarkable applications of the third kind of lever, is in the *bones of animals.* These are levers, the joints are the fulcrums, and the muscles are the powers. The muscles are endowed with a strong power of contraction, by which they are made to pull upon a tendon or cord, which is inserted in the bone near the fulcrum. Thus, the fore-arm moves on the joint near the elbow as a fulcrum, a little below which is inserted a tendon, connected with a muscle between the elbow and the shoulder, which gives it motion.

The arrangement may be well represented by attaching a small cord to one of the legs of a pair of tongs, near the joint. It will require a considerable force to lift the leg by pulling at the string, especially if the string be pulled in a direction nearly parallel with the leg, as it ought to be, since the tendon which lifts the fore-arm acts in such a direction with respect to the arm. The muscles therefore act, in moving the bones, under a double mechanical disadvantage, their force being applied both obliquely and very near the fulcrum. The force which the muscles exert in raising a weight held in the palm of the hand is enormous, as will be comprehended from the following illustration. Let AB (Fig. 146) represent the fore-arm moving on the elbow joint at A, and having the tendon inserted at C, which we will suppose to be ten times nearer to A than B is to A. Consequently, a weight of 10 pounds at B, would require a force at C, acting directly upward, of 100 pounds. But the force of the tendon does not act *directly upward* in the direction of CD, but very obliquely, as in the direction of CE, of which the part EA only can contribute to support the weight. Suppose this part to equal $\frac{1}{10}$th of the whole force CE, and it follows that the muscular force exerted to raise a weight of ten pounds in the palm of the hand, would, were it to act without any mechanical disadvantage, be sufficient to raise a weight of 1000 pounds. Yet Dr. Young informs us, that a few years ago there was a person at Oxford, who could hold his arm extended for half a minute, with half a hundred weight hanging to his little finger.

Fig. 146.

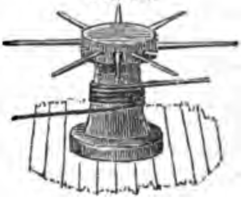


But by giving to the muscle the position it has, the greatest possible *compactness of structure* is obtained; while, by making it act so near the fulcrum, what is lost in force, is gained in *velocity;* and while the power acts through a small space, the hands are moved quickly through a great distance. In consequence of the dominion which man can gain over the stronger animals, and especially over the great powers of nature, he has little occasion to exert great strength with his naked hands: the celerity of their movements is to him a far more important endowment.

261. *The wheel and axle.*—When a lever is applied to raise a weight, or to overcome a resistance, the space through which it acts at one time is small, and the work must be accomplished by a succession of short and intermitting efforts. The common lever is, therefore, used only in cases where weights are required to be raised through small spaces. When a continuous motion is required, as in raising ore from the mine, or in weighing the anchor of a vessel, some contrivance must be adopted to remove the intermitting action of the lever and render it continual. The *wheel and axle*, in its various forms, fully answers this purpose. It may be considered as a revolving lever.

In numerous forms of the wheel and axle, the weight is applied by a rope coiled upon the axle; but the manner in which the power is applied is very various, and not always by means of a rope. The circumference of a wheel sometimes carries projecting pins, to which the hand is applied to turn the machine. An instance of this occurs in the wheel used in the steerage of a vessel. In the common *windlass*, the power is applied by means of a *winch*, which corresponds to the radius of a wheel. (See Fig. 103.) The axis is sometimes placed in a vertical position, and turned by levers moved horizontally. The *capstan* of a ship (Fig. 147) is an example of this. Levers answering to the radii of a wheel are inserted in holes mortised in the axis, and turned by several men working together. In some cases, as in the *tread-mill*, the wheel is turned by the weight of animals walking on the circumference, with a motion like that of ascending a steep hill.

FIG. 147.



262. *The double axle.*—The power of the wheel and axle being expressed by the number of times the diameter of the axle is contained in that of the wheel, there are obviously two ways by which this power may be increased; either by increasing the diameter of the wheel, or by diminishing that of the axle. In cases where great power is required, each of these methods is attended with practical inconvenience and difficulty. If the diameter of the wheel is considerably enlarged, the machine will become unwieldy, and the power will work through an unmanageable space. If, on the other hand, the power of the machine is increased by reducing the thickness of the axis, the strength of the axis will become insufficient for the support of that weight, the magnitude of which had rendered the increase of the power of the machine necessary. To combine the requisite strength with moderate dimensions and great mechanical power, is therefore impracticable, in the ordinary form of the wheel and axle. This has, however, been accomplished by giving different thicknesses to different parts of the axle, and carrying a rope, which is coiled on the thinner part, through a wheel attached to the weight, and coiling it in the opposite direction on the thicker part, as in Fig. 148. To investigate the proportion of the power to the weight in this case, let Fig. 149 represent a section of the apparatus at right angles to the axis. The weight is equally suspended by the two parts of the rope S and S′, and therefore each part is stretched by a force equal to half the weight. The moment of the force which stretches the rope S, equals half the weight multiplied by the radius of the thinner part of the axis. This force being on the same side of the center with the power, co-operates with it in supporting the force which stretches S′, and which acts on the other side of the center. Now the moments of P and S together, must be equal to the moment of S′ (Art. 107), and therefore if P be multiplied by the radius of the wheel, and added to half the weight multiplied by the radius of the thinner part of the axis, we shall obtain a sum equal to half the weight, multiplied by the radius of the thicker part of the axis. Hence the power multiplied by the radius of the wheel, is equal to half the weight multiplied by the difference of the radii of the thicker and thinner parts of the axis.

FIG. 148.

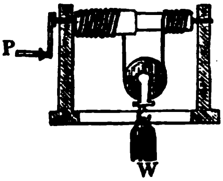
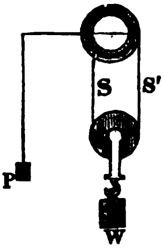


FIG. 149.

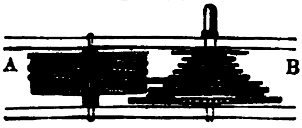


263. A wheel and axle constructed in this manner, is equivalent to an ordinary one, in which the wheel has the same radius, and whose axis has a radius equal to half the difference of the radii of the thicker and thinner parts. The power of the machine is expressed by the ratio which the radius of the wheel bears to half the difference of these radii; and therefore this power, when the diameter of the wheel is given, does not, as in the ordinary wheel and axle, depend on the smallness of the axle, but on the smallness of the *difference* of the thinner and thicker parts of it. The axle may, therefore, be constructed of such a thickness as to give it all the requisite strength, and yet the difference of the diameters of its different parts may be so small as to give it all the requisite power.

We see here strikingly exemplified the principle, that the weight sustained by a given power may be increased as its velocity is diminished. By inspecting Fig. 149, it will be seen that the string connected with the thinner part of the axle *unwinds*, while that connected with the thicker part *winds up*, by which means the ascent of the weight may be rendered slow in any degree, and a proportionally greater quantity of matter may be added to balance the constant moment of the power. The chief inconvenience of this contrivance arises from the fact that the axle must be of great length, as well as the rope which winds upon it.

264. *The fusee.*—A power, which varies according to some known law, may be made to produce a constant effect. We find an example in the mainspring and fusee of a watch. The mainspring in the barrel A (Fig. 150), has its greatest energy when first wound up; but this force constantly diminishes as the spring uncoils. The chain, therefore, at first passes from the barrel to the *smallest part* of the fusee B, where it has a small acting distance. As the spring uncoils, the chain runs upon the *enlarging* spiral of the fusee, so that the acting distance is made to increase just as fast as the intensity of the spring diminishes, and thus an equable force is communicated to the wheel-work of the watch.

FIG. 150.



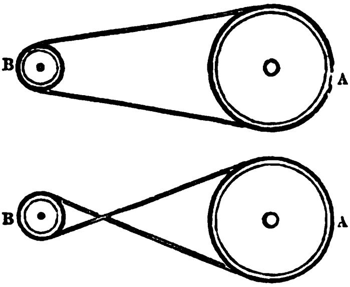
265. *Wheel-work.*—Motion may be transmitted by means of wheel-work in several different methods, the principal of which are, the friction of the circumference of one wheel upon that of another—the friction of a band—and the action of teeth.

One wheel is sometimes made to turn another, by the mere *friction of the two circumferences.* If the surfaces of both were perfectly smooth, so that all friction was removed, it is obvious that either would slide over the surface of the other, without communicating motion to it. But, on the other hand, if there were any asperities, however small, upon their surfaces, they would become mutually inserted among each other, and neither the wheel nor axle could move without causing the asperities on its edge to encounter those which project from the surface of the other; and thus both wheel and axle would move at the same time. Hence, if the surfaces of the wheel and axle are by any means made rough, and pressed together with sufficient force, the motion of either will turn the other, provided the load or resistance be not greater than the force necessary to break off these small projections which produce friction.

In some cases where great power is not required, motion is communicated in this way through a train of wheel-work, by rendering the surfaces of the wheel and axle rough, either by facing them with buff leather, or with wood cut across the grain. The communication of motion between wheels and axles by friction has the advantage of great smoothness and evenness, and of proceeding with little noise; but this method can be used only in cases where the resistance is not very considerable, and therefore it is seldom adopted in works on a large scale. Dr. Gregory mentions an instance of a saw-mill at Southampton, where the wheels act upon each other by the contact of the end grain of the wood. The machinery makes very little noise and wears well, having been used not less than twenty years.[*]

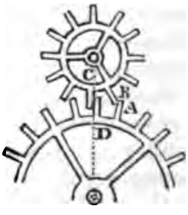
266. *Band-wheels.*—Wheel-work is extensively moved by the *friction of a band.* When a round cord is used, any degree of friction may be produced, by letting the cord run in a sharp groove at the edge of the wheel. When a strap or flat band is used, its friction may be increased by increasing its width. The surface at the circumference of a wheel which carries a flat band, should not be exactly cylindrical, but a little convex, in which case, if the band inclines to slip off at either side, it returns again by the tightening of its inner edge, as may be seen in a turner's lathe. When wheels are connected in the shortest manner by a band, they move in the same direction; if the band be crossed, they will move in opposite directions. (Fig. 151.) Wheels are sometimes turned by *chains* instead of straps or bands, and are then called *rag-wheels.* The chains lay hold upon pins, or enter into notches in the circumference of the wheels, so as to cause them to turn simultaneously. They are used where it is necessary that the velocities should be uniform, and where great resistance is to be overcome, as in locomotive steam-engines, chain water-wheels, &c.

Fig. 151.



267. *Toothed wheels.*—But the most common mode of moving wheel-work, is by means of *teeth* cut in the circumference of the wheels. The wheels of necessity turn in opposite directions. The connection of one toothed-wheel with another is called *gearing.* In the formation of teeth, very minute attention must be given to their figure, in order that motion may be communicated from one wheel to another, without rubbing or jarring. If the teeth are ill matched, as in Fig. 152, when the tooth A comes in contact with B, it acts obliquely upon it, and as it moves, the corner of B slides upon the plane surface of A in such a manner as to produce much friction, and to grind

Fig. 152.



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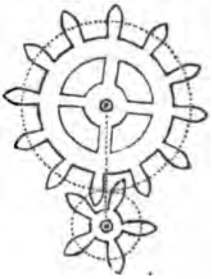
[*] Gregory's Mech., ii, 537.—Lardner's El. Mech., 183.

away the side of A, and the end of B. As they approach the position CD, they sustain a jolt the moment their surfaces come into full contact; and after passing the position CD, the same scraping and grinding effect is produced in the opposite direction, until by the revolution of the wheels the teeth become disengaged. To avoid these evils, the surfaces of the teeth are frequently *curved* so as to roll on each other with as little friction, and with as uniform force and velocity as possible. (Fig. 153.) Much pains and skill have been bestowed on this subject by mathematicians, with the view of ascertaining the kinds of curves which fulfill these purposes best.*

FIG. 153.



268. *Regulation of velocity by wheel-work.*—Wheel-work serves the purpose, not only of forming a convenient communication of motion between the power and the weight, but also of *regulating its velocity.* Thus, when the connection is formed by means of a band, as in Fig. 151, the speed of revolution of the wheel B, that carries the weight or sustains the pressure, may be altered at pleasure, by altering the ratio between the diameters of the two wheels. If the diameters are equal, the wheels will revolve at the same rate; if A remains the same, while the diameter of B is diminished, the rate of B will be increased in the same ratio; or if B remains the same, while the diameter of A is changed, the rate of B will be changed in the same manner. At least, this would be true, if there was no slipping of the band; it generally happens that the driven wheel has a little less velocity of circumference than the wheel that drives it, because of the slipping of the band. In the spinning-wheel a band passes round a large wheel and a small one called a spool, having the spindle for its axis; and in consequence of the great disparity in the size of the wheels, a great speed of revolution is given to the spindle by a comparatively slow revolution of the wheel. In a turner's apparatus, machinery for spinning cotton, and the like, a large hollow cylinder, or *drum*, is fixed horizontally, which is kept revolving by the moving power, and from which, motion is conveyed by bands to lathes, spindles, &c., to which any required rate of revolution is given, by altering the diameter of the small wheel that is connected with them and turns them. Sometimes a change is effected by making the drum of a conical shape, and then the rate imparted to the lathe or the spindle will be greater or less, according as the band proceeds from the larger or smaller part of the drum.

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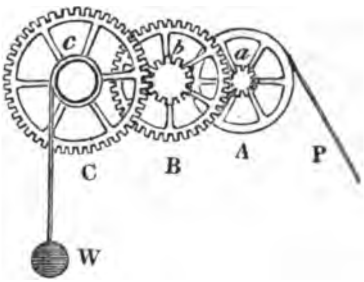
* See Blake on the form of the teeth of cog-wheels, Am. Journ. Sci., viii., 86.

269. *Wheels and pinions.*—A more exact way of regulating the rate of revolution, is by means of *wheels and pinions.* A small wheel on the same axis with a larger one is called a pinion, and its teeth are called *leaves.*

Fig. 154.

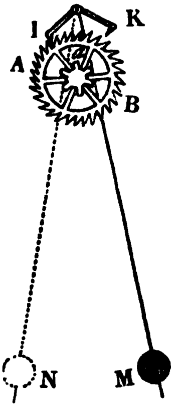


An example of this kind is seen in Fig. 154, where A, B, C, are three wheels, and *a*, *b*, *c*, are the corresponding pinions. As the leaves of the pinions successively pass between the teeth of the wheel, the divisions of the two circumferences must correspond to each other, and the number of teeth in the wheel will be as much greater than in the pinion, as the circumference of the wheel is greater than that of the pinion. Therefore it follows, that the number of teeth in a wheel, and of leaves in the pinion that acts upon it, expresses the ratio of the circumference or radius of the wheel to that of the pinion. Hence, in an equilibrium, the power multiplied by the product of the numbers of teeth in the several wheels, is equal to the weight multiplied by the product of the numbers denoting the leaves in each of the pinions. (Art. 114.)

It is further evident that the number of revolutions of the wheel, and that of the pinion connected with its circumference, will be inversely as the *number of teeth* in each. Thus in Fig. 154, if the pinion *a* has 10 teeth, and the wheel B has 100, *a* will make ten revolutions while B makes one. For the same reason *b* will make ten, while C makes one; so that if the circumference of A and *c* are also as 10 to 1, the power moves with 1000 times the velocity of the weight. By varying the ratio between the number of leaves in the pinion, and the number of teeth in the wheel with which it is connected, we may vary the rate of revolution of any wheel at pleasure.

270. A familiar instance of this is afforded in the mechanism of a common clock. A pendulum by falling gains a quantity of motion sufficient to carry it, on the other side, to the same height as that from which it fell; and were it not for the resistance of the air and the impediments, a pendulum when once set in motion would continue to vibrate by its own inertia and the force of gravity, and would thus afford, without the aid of any machinery, an exact measure of time. But, in order to continue its vibrations, some small force must be applied to it to compensate for the loss of motion from friction and resistance. This force is applied to the pendulums of clocks by the *weight*, and an analogous force is supplied to the *balance-wheel* of watches and chronometers by *springs*. In Fig. 155 let AB be a wheel having 30 teeth, and let N, M, be a pendulum, connected with the wheel by the *pallets* I, K; and to the axis *a* let a weight be hung. It is evident that this weight, were there nothing to arrest it, would descend by the force of gravity with accelerated velocity. It *endeavors* thus to descend, and hence exerts the required force on the pallets of the pendulum. For every time the pendulum performs a vibration, a tooth of the wheel escapes,* and the wheel runs down until the next tooth strikes upon the pallet, and thus gives it the impulse which is necessary to keep up the vibrations.

Fig. 155.



It would seem therefore that, for beating seconds, only a single wheel is necessary; nor would any more be absolutely indispensable; but in this case the weight would descend so fast, as soon to reach the floor, and the clock would require to be wound up again every few minutes. Hence a series of wheels is interposed between the pendulum and the weight, by which the descent of the latter is retarded upon the principle explained in Art. 269, and the descent of the weight is slower in proportion as the series is more extensive. In cheap clocks, as some of those made with wooden wheels, the series is short, or the number of wheels employed for retarding the descent of the weight is small, and such clocks require frequent winding up; but in clocks of finer workmanship, a greater number of wheels is interposed, and such clocks require to be wound up less frequently. Many go eight days, and some are made to go a whole year without winding.

271. *Wheel-carriages.*—When a loaded carriage is moving on a horizontal plane, free of obstacles, the resistance to be overcome does not consist of the weight of the load, directly, but of the friction occasioned by the weight. For, since the weight acts in a direction perpendicular to the plane, it cannot oppose the motion of the carriage in a direction parallel to the plane. Nor would increasing the weight to any extent, make any difference, were it not that we should thus increase the friction, which (as will be explained more fully hereafter) is proportioned to the weight.

When a carriage-wheel is made to *slide* on the ground (as when a wheel is *locked*), the whole amount of the friction is encountered without bringing in to our aid any mechanical advantage; but when a wheel turns on its axle, the friction is transferred from the ground to the axle, and each spoke of the wheel successively becomes a lever, turning on the ground as a fulcrum, while the power or force of the team is exerted on the end next to the axle. By thus transferring the friction from the ground to the axle, each spoke, in its turn, is made to aid in overcoming that friction. Thus, in Fig. 156, let C be the axle, CP the line of draught, and R the point where the wheel touches the plane. The force applied in the direction CP, acts on CR at C, and turns it on its fulcrum at R. This is the force by which the wheel is made to advance. But the friction on the axle at C, reacts in the opposite direction, having a leverage equal only to the radius of the axle, while the power which overcomes this, has a leverage equal to the radius of the wheel. Hence, in the wheel, there is a mechanical advantage gained in overcoming the friction, in the ratio of the radius of the wheel to the radius of the axle. Moreover, the axle may be made of such materials, and lubricated with such substances, as to render the actual amount of friction much less than it would be were the wheel made to slide on the ground.

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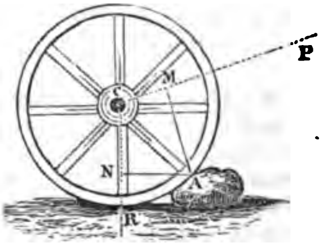
* Hence this connection is called the '*scapement.*

FIG. 156.



272. But wheels have another important advantage, namely, *in overcoming obstacles;* in which case they act on the principle of the bent lever. Thus, let A be an obstacle, on which the wheel must turn; from A draw the perpendiculars AN and AM, upon the lines of the forces; then the force of draught, F, multiplied by AM, is the moment of that force; and gravity, G, multiplied into AN, is the moment of the opposing force. The larger AM is, and the smaller AN, the more easily will the wheel be drawn over the obstacle. It is obvious that *large* wheels have on this account the greatest advantage. But in practice very large wheels can not be employed, since they would be either weak or too heavy, and the increased height of the axle would carry the center of gravity too high, and enhance the danger of upsetting. The difficulty of turning might also render unusually large wheels ineligible; and the axle might be raised so high, as to make the horse draw obliquely downward and increase the pressure on the ground, whereas the line of draught ought to be so adjusted as to lighten that pressure, especially where the road is soft and yielding.

When a wheel sinks below the surface, the force is rendered strikingly inefficacious from several causes. The fulcrum on which each spoke successively turns gives way, and diminishes greatly the mechanical advantage otherwise gained by transferring the friction from the ground to the axle, as before explained. Likewise, the mud or sand into which the wheel has sunk, opposes in front of the wheel an obstacle like that represented at A in Fig. 156, while the fulcrum on which the bent lever turns in the effort to lift the wheel over the obstacle gives way as in the other case, and a great part of the mechanical advantage is lost. From these considerations, it is easy to understand the reason of the superior advantages of hard and smooth roads.

273. The *line of draught* should not be horizontal, but inclined upward toward the breast of the horse, in an angle not less than 15 degrees with the horizon. This brings the strain nearly at right angles with the collar, whereas a horizontal draught lifts the collar upward, by which the force is wasted and the animal is choked.* The angle of draught, however, should be less than the above when the road is very smooth. The general rule is, that the angle should be the same as the inclination of a hill, down which the carriage would roll spontaneously. Consequently, in smooth Macadamized roads, the line of draught should be at a small angle, and on railways nearly horizontal.†

The effect of suspending a carriage on *springs*, is to equalize the motion by causing every change to be more gradually communicated to it, and to obviate shocks. Springs are not only useful for the convenience of passengers, but they also diminish the labor of draught; for whenever a wheel strikes a stone, it rises against the pressure of the spring, in many cases without materially disturbing the load, whereas without the spring, the load, or a part of it, must rise with every jolt of the wheel, and will resist the change of place with a degree of inertia proportionate to the weight, and the suddenness of the percussion. Hence springs are highly useful in baggage-wagons, and other vehicles used for heavy transportation.‡

A pair of horses draw more advantageously abreast than when one is harnessed before the other. In the latter case, the forward horse, being attached to the ends of the shafts, draws in a line nearly horizontal; consequently he does not act with his whole force upon the load, and moreover expends a part of his force in a vertical pressure on the back of the other horse.

274. *The pulley.*—Pulleys are divided into *fixed* and *mova-*

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* Fuller on Wheel-Carriages.

† Moseley's Mechanics applied to the Arts.

‡ Bigelow, El. Tech., p. 200.

ble. In the *fixed* pulley, as has been demonstrated in Art. 117, no mechanical advantage is gained, but its use consists in furnishing a convenient mode of *changing the direction* of the power. Thus, it is far more convenient to raise a bucket from a well by drawing downward, as is the case when the rope passes over a fixed pulley above the head, than by drawing upward leaning over the well. By means of the pulley, great facilities are afforded for managing the rigging of a ship. The sails at mast-head can be easily raised while the hands stand upon the deck; whereas, without the aid of ropes and pulleys, the same force removed to mast-head would operate under very great disadvantages. Similar facilities are afforded by this kind of apparatus for raising heavy weights, as boxes of merchandise, or heavy blocks of stone in building. If the system of pulleys is suspended from a *crane* or *derrick*, then after the weight is raised, it may be swung round, and deposited where it is to be used.

Fire escapes sometimes consist merely of a pulley fixed near the window of the apartment, around which a rope may be easily placed, having a basket attached to the end. The man seats himself in the basket, grasping, at the same moment, the rope on the other side of the pulley, and thus he lets himself gradually down.

275. The *movable* pulley, by distributing the weight into separate parts, so that it is supported at several different points at once, is attended by a mechanical advantage, proportioned to the number of such points of support. Movable pulleys may be arranged according to several different systems, which increase the efficacy of a given power in different ratios. It will be observed, however, that the ascent of the weight is in all cases retarded in proportion as the efficacy of a given power is increased. It must be further observed, that in using any system of movable pulleys, the whole weight of the pulleys themselves, together with the resistance occasioned by the rigidity and friction of the rope, all act against the power, and so far lessen the weight which it is capable of raising. In the more complex systems of pulleys, it is estimated that at least two-thirds of the power is expended on the machinery itself. On account, therefore, of the slowness of the motion which the weight receives, and the loss of power from the resistance of the ropes and blocks, such systems of pulleys are seldom employed. It is only in raising vast weights, such as large ships, or great masses of stone from a quarry, that they are ever used. For managing the rigging of a ship, the combination usually employed consists of not more than two or three movable pulleys. From its portable form, however, its cheapness, and the facility with which it can be applied, especially in changing or modifying the direction of motion, the pulley is one of the most convenient and useful of the simple machines.

276. *The inclined plane.*—This machine operates on the principle of oblique action. Thus the power has to encounter only a *portion* of the force of gravity at a time,—a portion which is greater or less, according as the plane is more or less elevated. When a plane is perfectly horizontal, it sustains the entire pressure of a body that rests on it; that is, the pressure on the plane is equal to the whole force of gravity acting on the body. As one end of the plane is elevated, this force is resolved into two, one of which is parallel and the other perpendicular to the plane. In proportion as the plane is more elevated, the part of the force which acts parallel with the plane is increased, until, when the plane becomes perpendicular to the horizon, it no longer sustains any portion of the weight, and the latter descends with the whole force of gravity.

277. The simplest example we have of the application of the inclined plane, is that of a plank raised at the hinder end of a cart for the purpose of rolling in heavy articles, as barrels or hogsheads. The force required to roll the body on the plank, setting aside friction, is as much less than that required to lift it perpendicularly, as the height of the plane above the ground is less than its length. Every one knows how much the facility of moving heavy loads is increased by such means, and how the force required to move them is diminished, by increasing the length of the plane while the height remains the same. Long inclined planes, constructed of plank, are frequently employed in building, especially where high walls are built of large masses of stone, the materials being trundled upon the plane on wheel-barrows or transported on heavy rollers. It is even supposed, that in building the pyramids of Egypt, the huge masses of stones were elevated on inclined planes. *Roads* also, except when they are perfectly level, afford examples of this mechanical power. When a horse is drawing a heavy load on a perfectly horizontal plane, what is it that occasions such an expenditure of force? It is not the weight of the load, except so far as that increases the friction; for gravity, acting in a direction perpendicular to the horizon, can oppose no resistance in the direction in which the load is moving. The answer is, that the force of the horse is expended chiefly in overcoming friction, and the resistance of the air. But when a horse is drawing a load up a hill, he has not only these impediments to encounter, but has also to overcome more or less of the force of gravity; that is, he *lifts* such a part of the load as bears to the whole load the same ratio, that the perpendicular height of the hill bears to its length. If the rise is one foot in twenty, he lifts one-twentieth of the load, and therefore encounters so much resistance in addition to those which he had to overcome on the horizontal plane. If the ascent were one foot in four, and the load were a ton, the additional force required above what would be necessary on level ground, would be 560 pounds.

278. *Railways* afford another striking exemplification of the principles of the inclined plane. By means of them the irregular surface of a country, however hilly and uneven, is reduced to horizontal levels and inclined planes. These are sometimes inclined at so slight an angle, that the tendency of the cars down the plane is only just sufficient to balance their friction, and they would remain at rest of themselves in any part of the plane, while a small force would move them either way. In other places the inclined planes are very steep for a short distance; and the cars ascending them are sometimes drawn up by means of a power (a steam-engine, for example) stationed on the summit; and sometimes cars descending on one side are made to draw up others on the other side, the two being connected by a chain or rope which passes round a pulley on the summit.

279. When railways first came into use, the power of one horse was considered only equal to a load of 10 tons; but it is now estimated that, upon a level railroad of the best construction, with carriages of the most perfect finish, a horse-power is equivalent to a load of 22½ tons, although an average load is considered to be about 16 tons. The resistances to be overcome have thus been reduced to only $\frac{1}{280}$ of the weight, and may be safely taken at $\frac{1}{200}$, while upon the best common roads it is never less than $\frac{1}{40}$, and is, in most cases, as great as $\frac{1}{20}$. The advantage of a good railroad over a common turnpike, when horses are employed, is therefore in a tenfold proportion. But railroads derive their greatest value from the employment of steam as a moving power, as exhibited in the locomotive. Horses tire at a moderate speed, but steam never tires, and is therefore peculiarly adapted to the transportation of passengers, where great expedition is required. By means of this force, a speed of 20 miles an hour is easily gained, and in some extreme cases, it has been pushed as high as 60 miles an hour. But such immense loads moving with such great velocities, acquire a momentum that is truly formidable, and involves inevitable danger.

280. In slow motions, canals have some advantages over railways. A horse easily draws on a canal a load of 30 tons; and it is said that the employment of several horses to a boat is advantageous, since the weight drawn increases in a higher ratio than the number of horses. When, therefore, heavy loads, with slow motions, are to be transported, canals have an advantage over railroads of two to one.* But canal-boats can not be made to move with much speed without injury to the canal, and great loss of force by the increased resistance of the water, this being augmented in proportion to the square of the velocity. In railways, on the other hand, the resistance arising from the friction of the wheels on the road, is diminished as the velocity is increased, while the resistance of the air, although increased by an increase of velocity like any other fluid, is so small as to occasion no serious impediment, being at a velocity of 14 miles per hour, only about 1 lb. on every square foot of the front of the leading carriage.

281. *The screw.*—When a road, instead of ascending a hill directly, winds round it to the summit, so as to lengthen the inclined plane, and thus aid the moving force, the inclined plane becomes a screw. In the same manner a pair of stairs, winding around the sides of a cylindrical tower, either within or without, affords an instance of an inclined plane so modified as to become a screw. These examples show the strong analogy which subsists between these two machines; or rather, they show that the screw is a mere modification of the inclined plane.

The screw is generally employed when severe pressure is to be exerted through small spaces, and is therefore the agent in most presses. Being subject to great loss from friction (upon which however its chief utility depends, as will be shown hereafter), it usually exerts but a small power of itself, but derives its principal efficacy from the lever, or from wheel-work, with which it is very easily combined. Thus, in Fig. 157, were the power applied directly to the screw, the mechanical advantage gained would hardly more than compensate for the loss by friction; but by means of the lever (which may be lengthened or shortened at pleasure), the power is greatly increased. Also, by means of the *endless screw* (Fig. 103), combined with the wheel and axle, a very powerful force may be exerted; and as the mechanical power of the screw depends upon the relative magnitude of the circumference through which the power revolves, and the distance between the threads (Art. 130), it is evident that, to increase the efficacy of the machine, we must either increase the length of the lever by which the power acts, or diminish the distance between the threads. Although, in theory, there is no limit to the increase of the mechanical efficacy by these means, yet practical inconvenience arises from the great space over which a very long lever traverses. If, on the other hand, the power of the machine is increased by diminishing the distance between the threads, and of course their size, the thread will become too slender to bear a great resistance. The cases in which it is necessary to increase the power of the machine, being those in which the greatest resistances are to be overcome, the object will evidently be defeated, if the means chosen to increase that power, deprives the machine of the strength which is necessary to sustain the force to which it is to be submitted.*

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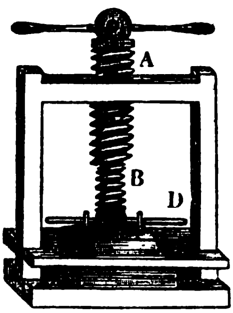
* Renwick's Practical Mechanics.

282. *Hunter's screw.*—These inconveniences are remedied by *Hunter's screw*, which, while it gives to the machine all the requisite strength and compactness, allows it to have an almost unlimited degree of mechanical efficacy. This contrivance, which is represented in Fig. 157, acts upon a principle similar to that of the double axle, as represented in Fig. 148, where the efficacy of the power is increased by diminishing the velocity of the weight, which is accomplished by making the rope unwind on one side while it winds up, with somewhat greater speed, on the other side. In the case before us, the screw is likewise composed of a smaller and a larger thread, the former turning *upward* while the latter turns *downward* with a little greater velocity, and consequently the screw, on the whole, advances with the *difference* between the larger and the smaller threads; and since this difference may be small to any extent, so the efficacy of the power may be increased indefinitely. It will be seen, however, that the motion of such a screw is exceedingly slow. Thus in Fig. 157, A descends, while B, playing in a concave screw in A, ascends; but the distance between the threads of A being greater than the distance between those of B, the screw, on the whole, advances with the difference. Suppose that A has 20 threads in an inch, and B 21; then during one revolution, A will *descend* through the 20th, while B *ascends* through the 21st part of an inch. The compound screw, therefore, will advance through a space equal to the difference: that is, through a space equal to $\frac{1}{20} - \frac{1}{21} = \frac{1}{420}$th of an inch. This small space is therefore, in effect, the distance between two contiguous threads; and the power of the machine is, as usual, expressed by the number of times their distance is contained in the circumference described in one revolution of the power. For example, let the circumference of the circle be one foot; then $12 \div \frac{1}{420} = 5040 =$

FIG. 157.



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* Lardner's El. Mech., p. 220.

the weight or resistance, the power being 1; or, in other words, the efficacy of the power is increased 5040 times.

283. It is obvious, however, from principles already explained, that the power will in this case move over 5040 times as great a space as the weight. It is on this principle that the screw affords the means of measuring very minute spaces, and hence is derived the *micrometer screw.* The very slow motion which may be imparted to the end of a screw, while the power moves over a space vastly greater, renders it peculiarly adapted to this purpose. For example, suppose a screw to be so cut as to have 50 threads in an inch; then each revolution of the screw will advance its point through the 50th part of an inch, and if that point acted against a thread or wire, it would move it over a graduated space only that distance in a whole revolution of the screw. Now suppose the head of the screw to be a circle an inch in diameter, and of course something more than three inches in circumference. This circumference may easily be divided into a hundred equal parts, distinctly visible; and if a fixed index be applied to it, the hundredth part of a revolution of the screw may be observed, by noting the passage of one division of the head under the index. But the hundredth part of a revolution carries the point of the screw only through the $\frac{1}{100}$ of $\frac{1}{50} = \frac{1}{5000}$th part of an inch. Such an apparatus is frequently attached to the limbs of graduated instruments, for the purposes of astronomical and other observations; by which means, a portion of the graduated arc no greater than the 10th part of a second, can be estimated.

In like manner, any other small space may be measured by the aid of the micrometer screw. Thus, any aliquot part of a pound or an ounce, in the steelyards, may be found by adapting the screw to the counterpoise so as to move it slowly over the space between two notches, and at the same time point out, by an index on its head, the exact portion of the space over which it passes.

284. *The wedge.*—If instead of moving a load on an inclined plane, the plane itself is moved beneath the load, it then becomes a wedge. Thus, if a perpendicular beam have one end resting upon an inclined plane (the beam being so secured as to be capable of moving only up and down), and the plane be drawn under it, the beam will be elevated; and the power required to effect this will be to that required to raise the beam when applied directly to it, *as the height of the plane to its length:*—or, considering the plane as a half wedge, the proportion will be, *as half the back of the wedge to its length.* (Art. 135.)

285. In the arts and manufactures, wedges are used where an enormous force is to be exerted through a very small space.

Thus it is resorted to for splitting masses of timber or stone. Ships are raised in docks by wedges driven under their keels. The wedge is the principal agent in the oil-mill. The seeds from which the oil is to be extracted are introduced into hair bags, and placed between planes of hard wood. Wedges inserted between the bags are driven, by allowing heavy beams to fall on them. The pressure thus excited is so intense, that the seeds in the bags are formed into a mass nearly as solid as wood. Instances have occurred in which the wedge has been used to restore a tottering edifice to its perpendicular position. All cutting and piercing instruments, such as knives, razors, scissors, chisels, nails, pins, needles, awls, &c., are wedges. The angle of the wedge, in these cases, is more or less acute, according to the purpose to which it is applied. In determining this, two things are to be considered—the mechanical power, which is increased by diminishing the angle of the wedge, and the strength of the tool, which is always diminished by the same cause. There is, therefore, a practical limit to the increase of the power, and that degree of sharpness only is to be given to the tool, which is consistent with the strength requisite for the purpose to which it is to be applied. In tools intended for cutting wood, the angle is generally about 30°; for iron it is from 50° to 60°; and for brass, from 80° to 90°. Tools which act by pressure may be made more acute than those which are driven by a blow; and, in general, the softer and more yielding the substance to be divided is, and the less the power required to act upon it, the more acute the wedge may be constructed.*

286. In many cases, the utility of the wedge depends on that which is entirely omitted in the theory, viz., the *friction* which arises between its surface and the substance which it divides. This is the case when pins, bolts, or nails, are used for binding the parts of structures together; in which case, were it not for the friction, they would recoil from their places, and fail to produce the desired effect. Even when the wedge is used as a mechanical engine, the presence of friction is absolutely indispensable to its practical utility. The power generally acts by successive blows, and is therefore subject to constant intermission; and but for the friction, the wedge would recoil in the intervals between the blows with as much force as it had been driven forward, and the object of the labor would be continually frustrated.

287. *The knee-joint.*—This is a contrivance by which a constantly increasing resistance is overcome by a force which acts nearly uniformly. In Fig. 158 we have a representation of it as exemplified in printing presses. Here, when the *platen* P first descends upon the sheet, the resistance is very slight, but as the lever by which the press is worked is pulled still further, the resistance rapidly increases, until, without mechanical aid, it would require a laborious and exhausting effort to render the contact sufficiently close to give a perfect impression. This difficulty is completely obviated by means of a combination of levers resembling the knee-joint, where the efficacy of the power is continually increased as the levers approach nearer to the same straight line; so that, without any additional effort on the part of the workman, the pressure is augmented a thousandfold. The action of this joint is exemplified in opening a pair of compasses, or a Gunter's scale which opens on a joint at the center. At first, the *thrust* exerted by the ends is slight; but as the two parts approach the direction of a straight line with each other, the thrust rapidly increases, until it becomes immensely great.*

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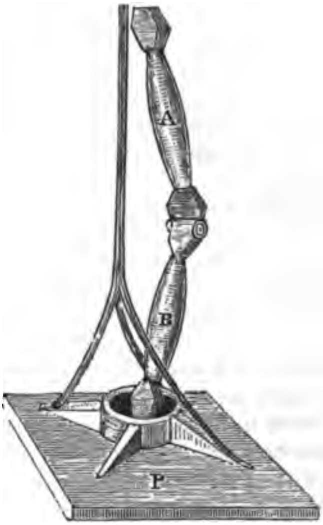
* Lardner.

FIG. 158.



288. *Machinery*.—Archimedes is said to have boasted to King Hiero, that "if he would give him a place to fix his machine (a *που στῶ*), he would move the world." Yet there can be no machine by the aid of which Archimedes could move the world, in any other way than by moving himself over as much more space than that over which he moved the earth, as his weight was less than that of the whole earth. If Archimedes had received the place he desired, and had also employed, what was equally desirable, a machine which operated free of all resistance, he must have moved with the velocity of a cannon-ball, to have shifted the earth only the twenty-seven millionth part of an inch in a million of years.

289. Machines are divided into two classes, those intended simply to *sustain* a weight and those intended to *move* it. In machines of the first class, estimating the effect by the weight sustained, it is evident that the efficacy of the power is increased. By means of a lever, for example, a man may sustain a weight ten times as great as he could by his unaided strength. We may perceive, however, on closer examination, that he does not in fact bear the whole weight, but only one tenth part of it. Let it be a lever of the second kind, where the weight is ten times nearer the fulcrum than the end is to which the power is applied. Now the hand that supports this end performs the same office as the second fulcrum in a lever of the first kind; and since (Art. 106) the pressure on each fulcrum is inversely as its distance from the weight, therefore, in the present case, nine parts out of the ten are borne by the prop, and only one by the power. If (says Carnot) Archimedes had obtained his "fixed point," it would not, in reality, have been Archimedes, but the fixed point that would have sustained the earth.

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* See an able article on this subject by Prof. Fisher, *American Journal of Science*, iii, p. 311.

In machines of the second class, the effect is not estimated simply by the weight moved, but we must take into the account the time occupied in moving it a certain distance, that is, the *velocity*. Hence, the effect of moving powers is estimated by the product of the weight moved multiplied by the velocity, or it is measured by the *momentum* produced. Moreover, in the former case, all resistance from friction, the rigidity of ropes, and so on, conspire with the power in sustaining the weight; but in machines of the second class, all such resistances oppose the action of the power, and require a greater power for a given weight than would be necessary, if the power were applied directly to the weight.*

290. Hence it will be inferred, that no *momentum*, or effective force, is gained by any machine. If a man, with his naked hands, can lift to a given height, as one foot, only 150 pounds in one second, it is impossible for him to perform any more labor than this by any mechanical contrivances.† On the contrary, when the structure of the machine is complicated, there is a loss of force, by employing the machine instead of the naked hands, proportioned to the resistance of the parts of the machine itself. It is to be remarked, however, that this doctrine proceeds on the supposition that the *useful effect* produced is estimated from the joint product of the *force*, *velocity*, and *time*. Thus, $F \times T \times V = \frac{1}{2}F \times 2T \times V = 2F \times \frac{1}{2}T \times V = F \times \frac{1}{2}T \times 2V$, &c.‡ A convenient method of estimating different forces is to draw a heavy weight out of a well, by a rope passing horizontally over a fixed pulley, near the top of the well. Suppose that a man can draw up a rock weighing 100 pounds, through the space of 50 feet in one minute. He would, of course, be able to draw up ten such masses in ten minutes, weighing in all 1000 pounds. Now by passing the rope over five pulleys (allowing nothing for the friction of the pulleys), he might with the same force lift the whole 1000 pounds at once; but it would rise ten times as slowly as the 100 pounds did before, and consequently would be ten minutes in reaching the top. Therefore, in a given time, it appears that the man would raise the same weight through a given space, with or without the aid of machinery. In the former case, the 100 pounds might have been raised during the ten minutes through the space of 500 instead of 50 feet; but $100 \times 500 \times 10 = 1000 \times 50 \times 10$: so that the labor performed would have been the same in both cases. Let us suppose that P is a power amounting to an ounce, and that W is a weight amounting to 50 ounces, and that P elevates W by means of a machine. In virtue of the property already stated, it follows, that while P moves through 50 feet, W will be moved through 1 foot; but in moving P through 50 feet, *fifty distinct efforts* are made, by each of which, if applied directly, 1 ounce would be moved through 1 foot.*

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* Venturoli's Mechanics, p. 164.

† Emerson's Mechanics, p. 150; Cavallo, Nat. Phil., i. 251.

‡ Gregory, i, 343.

291. *Use of machinery.*—What then, it may be asked, are the advantages gained by machinery? The advantages still are very great, for the following reasons:

1. By the aid of machinery, *we can frequently apply our force to much better purpose.* Thus in lifting a weight out of a well, or in raising ore out of a mine, it is obvious with how much more effect a man can work at the arm of a windlass, than he could draw directly upon the rope by stooping over the well. So in raising a rock from its bed by means of a handspike or crowbar, we can easily see how much more effectually we can bring our force to bear upon it than we could do with our naked hands.

2. By the aid of machinery, a man may be able to perform works to which his naked strength would be wholly incompetent. Thus, as in the preceding example, one might be able to lift a rock from its bed with a handspike, upon which he could make no impression with his naked hands; or, by means of pulleys he might raise a box of merchandise from the hold of a ship, which he could not start at all with his unassisted force. In each of these cases, *if the weight could be divided into small parcels*, and if the force could be as advantageously applied without machinery as with it, the labor would be performed as easily in a given time in one way as in the other. But it might not be possible or at least convenient thus to di-

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* **Lardner's Mechanics, p. 162.**

vide it. Or if, instead of dividing it into a number of parcels, the *same number of men* could act directly upon the weight at once, the amount of labor which they would all exert in raising the weight without machinery, would be the same as that which the single man before supposed would exert with his machinery. But it might not be convenient to assemble so many hands at a time; or perhaps such a number could not work advantageously together. A farmer has many occasions for lifting or removing great weights when his laborers are not more in number than two or three in all. These must therefore perform the labor of 50 times as many men by being 50 times as long about it. Thus, in Ques. 3, Art. 132, where a single man turning on a winch, with the force of 100 pounds exerts a force on the ship amounting to 161¼ tons, the ship would move as much slower than the hand as 100 pounds is less than 161¼ tons; and consequently a great length of time would be required for an individual to perform this labor, even supposing no resistance to be encountered from the machinery itself.

3. Machinery frequently enables a man to exert his *whole force* in circumstances where, without such aid, he could employ but a part of it. Thus, in winding silk or thread, to turn a single reel might not require one-fiftieth part of the force which the laborer is capable of exerting. Suitable machinery would enable him to turn fifty spools at once.

4. But the most striking advantage of machinery is not found in the facilities which it lends to the personal strength of man. It lies in this, that it affords the means of calling in to his assistance the superior powers of the horse and the ox, of water, of wind, and especially of steam. Here we find the excellence of mechanical contrivances fully exhibited; and nowhere else has the inventive genius of man displayed itself to so great advantage. But here, as in all other cases, the various combinations of simple machines *produce* no force: they only *apply* it. They form the communication between the moving power and the body moved, and while the power itself may be incapable of acting except in one direction, we are able, by means of cranks, levers, and toothed wheels, to direct and modify that force to suit our convenience or necessities. Every one may see examples of this in the construction of the most common saw-mill or flour-mill, turned by water. In a mill for grinding wheat, the stone is required to move horizontally, while the action of the waterfall is perpendicular. We therefore receive the whole force on the circumference of a wheel, and transmit it through several intermediate wheels to the revolving stone, where the grinding is performed. So in a saw-mill, the water first communicates a *rotary* motion to the wheel, and this motion is converted, by means of a crank, into what is called a *reciprocating* motion, as that of the saw in its ascent

and descent. By means of wheel-work the *velocity* of the moving body is increased or diminished at pleasure.

In short, machines enable us to form a convenient communication between the power and the weight; to give to the weight any required direction or velocity; to apply force to the best advantage; to vary the circumstances of velocity and time as the amount of our force may require; and to bring to our aid the great moving powers that exist in nature. Our next object, therefore, will be to see by what particular methods these several purposes are accomplished.

CHAPTER V.

REGULATION OF MACHINERY, AND CONTRIVANCES FOR MODIFYING MOTION.

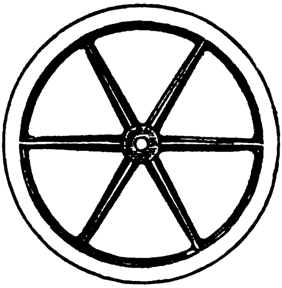
292. It is highly important to the successful operation of any machine, that its motion should be regular and uniform. Jolts and irregular movements waste the power, wear upon the machine, and perform the work unevenly. The sources of irregularity are various, but they are chiefly the three following, viz.: variations in the power, variations in the weight or resistance, and changes of velocity in parts of the machine itself. Thus, in the steam-engine, the fire may burn with more or less intensity, and produce corresponding quantities of the moving power; the load to be carried (as that of a steamboat) may be much greater at one time than at another, and be subject to sudden changes; and the motion of the piston, which carries the machinery, ceases altogether at the highest and lowest points, and would move a machine by *hitches* or separate impulses, were there no contrivance connected with it for keeping up a uniform motion.

293. *Regulators.*—The kinds of apparatus employed to obviate these difficulties, and to secure uniform movements to machines, are, in general, called *regulators*. Large machines or engines themselves, in consequence of their inertia, acquire and maintain to a considerable extent, uniformity of motion. A flour-mill carried by water, when it has acquired a certain rate of going, will not suddenly change that rate by any alteration in the force of the stream; and a ship sailing between the opposite forces arising from the impulse of the wind and the resistance of the water, will move steadily along, notwithstanding the breeze that carries it may fluctuate continually. We can see this principle sometimes operating on a smaller scale.

A grindstone turned by a winch moves steadily, although the force applied at one part of the revolution is much greater than at another. Large grindstones exhibit the advantage of this principle much more than small ones. But in many instances, this natural tendency toward uniform motion is not sufficient, and artificial contrivances are introduced expressly for this purpose. As examples of regulators we may especially notice three, the *pendulum*, the *fly-wheel*, and the *governor*.

294. *The fly-wheel, or balance-wheel.*—This regulator affords the most common and effectual method of equalizing motion, especially in heavy kinds of machinery. It consists of a heavy wheel (Fig. 159), affording as much weight as possible under as small a surface, in order that the inertia may be great, while the resistance from the air is small. It is therefore usually a heavy ring of iron with thick bars of the same metal. The fly is balanced on its axis, and so connected with the machinery as to turn rapidly around with it, and receiving a constant impulse from the moving power, it becomes a magazine or repository of motion. Consequently, by its inertia, it is ready to supply any deficiency of power that may arise from the sudden diminution of the moving force, or to check any sudden impulse which may result from an accidental excess of that force. Suppose, for example, the handle of a pump to be connected with a water-wheel, and to be carried by it. Here the power, namely the waterfall, is constant, while the weight is subject to continual alternations, amounting to a heavy load as the piston is ascending, but opposing scarcely any resistance while the piston is descending. The motion, therefore, would vary between nothing and a highly accelerated velocity, and the machinery would be subject to constant strains and jolts. A fly prevents these alternations, and renders the ascent and descent of the piston nearly uniform. In pile engines or stamping-mills a team of horses is sometimes employed to raise a heavy weight, which, when at a certain elevation, is suddenly disengaged, and falls with great force. As the disengagement is instantaneous, the horses would instantly tumble down, were not their motion checked by some contrivance which should prevent the machinery from receiving any sudden increase of velocity. This purpose is completely answered by the fly.*

FIG. 159.



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* Gregory's Mech., ii, 14.

295 Besides the use of the fly-wheel in regulating the action of machinery, it is employed for the purpose of *accumulating* successive exertions of a power, so as to produce a much more forcible effect by their aggregation than could possibly be done by their separate actions. If a small force is repeatedly applied in giving rotation to a fly-wheel, and is continued until the wheel has acquired a very considerable velocity, such a quantity of force will be at length accumulated in its circumference, as to overcome resistance and produce effects utterly disproportionate to the immediate action of the original force. Thus it would be very easy in a few seconds, by the mere action of a man's arm, to impart to the circumference of a fly-wheel, a force which would give an impulse to a musket ball equal to that which it receives from a full charge of powder.*

The same principle explains the force with which a stone may be projected from a sling. The thong is swung several times around by the arm until a considerable portion of force is accumulated, and then it is projected with all the collected force. If a heavy leaden ball is attached to the end of a strong piece of cane or whalebone, it may easily be driven through a board: by taking the end of the rod remote from the ball in the hand, and striking the board a smart blow with the end bearing the ball, such a velocity may easily be given to the ball as will drive it through the board.†

296. The astonishing effects of a fly-wheel, as an accumulator of force, have led some into the error of supposing that such an apparatus *increases the actual force of a machine.* So far from this, since a fly cannot act without friction and resistance from the air, a portion of the actual moving force must unavoidably be lost by the use of this appendage. In cases, however, where a fly is properly adjusted and applied, this loss of power is inconsiderable, compared with the advantageous distribution of what remains. As an accumulator of force, a fly can never have more force than has been applied to put it in motion. In this respect it is analogous to an elastic spring. In bending a spring, a gradual expenditure of power is necessary. On the recoil, this power is exerted in a much shorter time than that consumed in its production, but its total amount is not altered. In this way the fly-wheel is used. Thus, in mills for rolling metal, the water-wheel or other moving power, is allowed for some time to act upon the fly alone, no load being placed upon the machine. A force is thus gained which is sufficient to roll a large piece of metal, to which, without such means, the mill would be quite inadequate. In the same manner, a force may be gained by the arm of a man acting on a fly for a few seconds, sufficient to impress an image on a piece of metal by an instantaneous stroke. The fly has, therefore, been much used in coining-presses. Its power may be transmitted to the working point by means of a screw. At the extremities of the cross arm AB (Fig. 160), which works the screw, two heavy balls of metal are placed. When the arm AB is whirled round, those masses of metal acquire a momentum, by which the screw, being driven forward, urges the die with immense force against the substance destined to receive the impression. Some engines used in coining have flies with arms four feet long, bearing one hundred weight at each of their extremities. By turning such an arm at the rate of one entire circumference in a second, the die will be driven against the metal with the same force as that with which 7500 pounds weight would fall from the height of 16 feet; an enormous power, if the simplicity and compactness of the machine is considered. By the action of the fly, working in this manner, is produced the open work of fenders, fire grates, and sometimes ornamental articles wrought in metal. The cutting tool, shaped according to the pattern to be executed, is attached to the end of a screw; and the metal being held in a proper position beneath it, the fly is made to urge the tool downward with such force as to stamp out pieces of the required figure. When the pattern is complicated, and it is necessary to preserve with exactness the relative situation of its different parts, a number of punches are impelled together, so as to strike the entire piece of metal at the same instant, and in this manner the most elaborate work is executed by a single stroke of the hand.*

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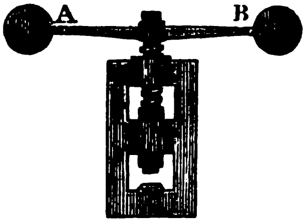
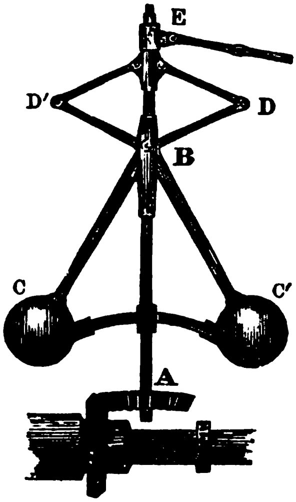
* Library of Useful Knowledge. † Ib.

FIG. 160.



297. To maintain a uniform velocity with a varying resistance, one of the most beautiful contrivances ever used is the *governor* (Fig. 161), an instrument used in mill-work, but the application of which is most conspicuous in the steam-engine, when that machine is applied to manufacturing purposes. The prin-

FIG. 161.



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* Lardner.

ciple on which the efficacy of this instrument depends, is easily explained. Let AB be a vertical axis which is made to revolve by the wheel A, acted on by the other parts of the machinery, and so that it always revolves with a velocity proportional to that of the fly-wheel. Two heavy balls C, C′ are attached to bent metal rods CBD, C′BD′, which work on a pivot at B, so that they are capable of receding from the axis AB. As the balls recede from the axis, the joints D, D′ recede from one another, and the joint E is drawn down. This joint E is connected with the end of a lever, or a system of levers, the action of which we shall presently explain.

Now, by the revolution of the spindle or axis AB, the balls C, C′, acquire an obvious tendency to fly off from the axis, and this tendency is resisted by their weight; so that, when the instrument is revolving with a certain velocity, the balls remain suspended, and neither move to or from the axis. A greater velocity, by giving a greater centrifugal force, would cause the balls to fly further off (Art. 209), and a less velocity would cause them to fall toward the axis. This is strictly true only when the range of balls is small, compared with the length of the rods to which they are attached, which, however, is always the case in practice. If, therefore, the action of the levers with which the joint E is connected, is directed upon the first mover in such a manner, that its energy is diminished when E is depressed, and increased when E is elevated, it is plain that the uniformity of velocity which is sought may be obtained. Let us suppose that the levers on which E works communicate with a valve which admits steam to the piston of a steam-engine, to which this governor is applied; and suppose that when E is raised, and the balls C, C′, rest in their seats, the valve is fully open, so as to allow the steam to flow in a full stream to the piston; but that, according as E is depressed, the levers gradually close the valve, so as to admit the steam in a constantly diminished quantity. Now, suppose that the engine has been working twenty printing-presses, and that the action of ten of them is suddenly suspended. The engine thus loses half its load, and would, if the same power of steam continued to be admitted, move with about twice its former velocity. But the moment an increased velocity is perceived in the machine, the balls C, C′, recede from the axis, draw down the joint E, partially close the valve, and check the supply of steam to the cylinder. The impelling power is thus diminished; and if it be diminished in exactly the same degree as the load, the machine will move with its former velocity; but if it should, at first, be more diminished, the velocity will be less than its former velocity, and the balls will again move toward the axis and open the valve, and will, at length, settle into that position in which the steam admitted to the cylinder is exactly proportioned to the load on the machine, and the proper velocity will thus be restored.*

298. *Contrivances for modifying motion.*—In Chapter III, we have already explained the mode in which motion is communicated, and its velocity regulated by *wheel-work.* We proceed now to consider a few examples of the more special contrivances by which motion is modified to suit particular purposes, recommending it to the student of mechanics to make himself acquainted with other contrivances of the same nature, by the actual inspection of machinery as opportunity may offer.

The motion required for a particular purpose may be *rectilinear*, as that of a carriage, or bucket drawn out of a well; or *rotary*, as in ordinary wheel-work; or *reciprocating*, as in a saw-mill or a pendulum.

The simplest mode of producing rectilinear motion is by means of a rope or chain, instances of which are familiar to every one. The simplest mode of *changing the direction* is by means of pulleys; but toothed wheels are also extensively employed for the same purpose.† The connection of one toothed wheel with another is called *gearing.* (Art. 267.) When both wheels with their teeth are in the direction of the same plane, it is called *spur gearing* (Figs. 152, 153, and 154); if the teeth, instead of being cut on the circumference in a direction parallel to the axis, are cut obliquely, so that, if continued, they would pass round the axis like a screw, it is called *spiral gearing* (Fig. 162); and when wheels are not situated in the same or parallel planes, but form an angle with each other, the wheels themselves are sometimes shaped like frustums of cones, having their teeth cut obliquely, and converging toward the point where the apex of the cone would be situated, and it is then called *bevel gearing.*‡ (Fig. 163.)

FIG. 162.

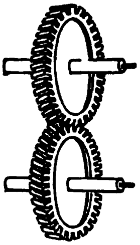


FIG. 163.

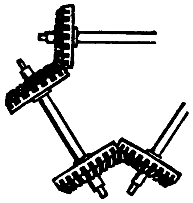
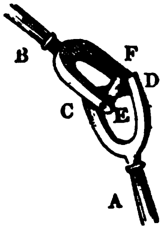


FIG. 164.



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* Library of Useful Knowledge.

† Emerson's Mechanics, Prop. CX.

‡ Bigelow's Tech., 232.

299. The *universal joint* consists of two shafts or arms, each terminating in a semicircle, and connected together by means of a cross upon which each semicircle is hinged. (Fig. 164.) When one shaft is revolved, either to the right or left, the other shaft revolves in the same direction, even when the shafts are inclined to each other.

FIG. 165.

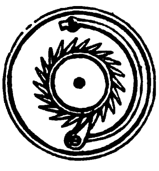
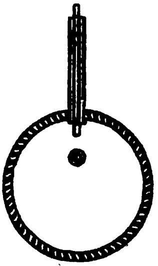


FIG. 166.

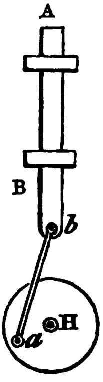


The *ratchet wheel* (Fig. 165) is used to prevent motion in one direction while it permits it in the opposite. The teeth are cut with their faces inclining as in the figure, and a *catch* is so placed as to stop the wheel in one direction, while it slides over the teeth without obstruction in the opposite direction.

The *eccentric wheel* (Fig. 166) revolves about an axis which is more or less removed from the center, and, consequently, the different portions of the circumference move with different degrees of velocity. Hence, if this wheel is made to act upon a shaft or pinion, as in the figure, it will carry it with a corresponding movement. In orreries, such wheels are employed for indicating the variable velocities of the heavenly bodies, as they revolve about their centers of motion.

300. *Reciprocating motion* is produced in various ways. The most common method is by means of the *crank*. In Fig. 167, a shaft AB is urged backward or forward (either vertically or horizontally), by means of the crank *ab*, moving on a wheel H, which may be turned by water or any other power acting at H. By considering the different positions of the crank during the revolution of the wheel, it will be readily seen that the shaft will move up and down like the saw in a saw-mill, or backward and forward, a use to which it is applied in polishing plane surfaces, as marble.

FIG. 167.



The motion produced by cranks is easy and gradual, being most rapid in the middle of the stroke, and gradually retarded toward the extremes; so that shocks and jolts in the moving machinery are diminished, or wholly prevented by their use.*

The steam-engine, as seen in steamboats, furnishes to the student of mechanics a valuable opportunity of observing various contrivances for producing, regulating, and modifying motion. Levers and wheels of various kinds and variously connected; fly-wheels and cranks; circular and reciprocating motions; and numerous other particulars which appertain to the "elements of machinery," are there seen to the greatest advantage.

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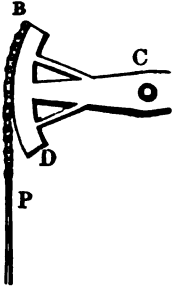
* Bigelow.

301. The *arch head* (Fig. 168) is a circular form given to the end of a lever, that moves on a gudgeon or pivot in a vertical plane (like the working beam of a steam-engine), by means of which the piston, or weight, whatever it be, is made to ascend and descend perpendicularly in a *straight* line, with a *uniform* motion, while the end of the lever itself works in the arc of a circle, and while the power, if fixed in the usual way at one point at the extremity of the lever, would lose a part of its efficacy, as its direction became oblique to that of the lever. In the figure, BD represents the arch head of a working beam, which turns on a gudgeon at C. At the top B is applied a flexible chain resting loosely upon the arch, and always maintaining a *vertical position* in every situation of the beam; and, being a tangent to the circle at the point of contact, always acting at right angles to the lever, and consequently imparting to the weight a *uniform* motion. In cases where force is to be exerted only in one direction, as in working a pump, the power is applied while the piston is descending, and drags up the other end of the lever. Here, in the descent of the working end, so small a resistance is encountered, that it is sufficient to give a slight preponderance to that end, and it will drag up the piston after the moving force is withdrawn which carried it down. But in cases where a continued force is required, both in the ascent and descent of the piston (as in most kinds of manufacturing operations), then the flexible chain, being incapable of forcing the weight upward, is inapplicable, but the object is attained by arming the arch head with *teeth* that act on *rack-work* in the upper end of the piston rod.

FIG. 168.



302. *Practical rules relating to machinery.*—If the given power is not able to overcome the given resistance when *directly* applied, then we must employ a longer time by making the power move over a greater space than the weight, in order to make the momentum of the power exceed that of the weight. This is effected by means of machinery. We must consider that the power must have a momentum which is equal to the amount of all the resistances to be overcome, in order just to *sustain* or counterbalance the weight; and to put the machine *in motion*, a greater power must be allowed, more or less, according to the velocity with which the machine is required to move. The ratio between the power and the weight in a given machine, when in equilibrium, may be ascertained by observing the *comparative spaces* over which they move in a given time; the power being as much less than the weight, as its space is greater. A due proportion between the two must also be observed; for if the machine or engine is competent to overcome the resistance, and perform its work in a convenient time, it is sufficient for the end proposed; and to increase the power any further, must not only be a needless expense, but the engine would lose time in working. If a weight is to be moved but a little way, the lever is the most simple, easy, and ready instrument. If the weight be very great, the common screw is preferable. But if the weight is to be moved over a great space, the wheel and axle, the pulley, or a system of pulleys, or the endless screw (Fig. 103), is to be employed.

303. But most machines are combinations of some or all of those elements, which are called the *simple machines*. Thus the lever is combined with the screw in a common press; the wheel and axle with pulleys in various ways, and with the endless screw; pulleys are combined with pulleys, and wheels with wheels. The wedge is the only one among the mechanical powers, that does not admit of combination with others. In wheels with teeth, the number of teeth that play together in two wheels ought to be *prime* to each other, that the same teeth may not meet at every revolution, but as seldom as possible. The strength of every part of a machine ought to be made proportional to the stress it is to bear; and no part must be stronger or heavier than is necessary, for all superfluous matter is nothing but a dead weight upon the machine, and serves for nothing but to clog its motion. The accomplished mechanic contrives all the parts to last equally well, so that when the machine fails every part shall be worn out.

304. Every machine ought to be made of as *few parts*, and as simple as possible, to answer its purpose; not only because the expense of making and repairing will be less, but it will be less liable to get out of order. Any useless motions also waste some portion of the power. Uniformity or steadiness of motion is carefully to be preserved. All these advantages are more easily attained in large than in small machines. All mechanical errors have a less ratio to the motion of the machine in great machines than in small ones, and these will therefore work with more uniformity and exactness, although, being proportionally weaker, they are less able to resist any violent shocks.*

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* Emerson's Mechanics, 4to, p. 175.

CHAPTER VI.

FRICTION.

305. The term *friction*, in its usual acceptation, being generally understood, we have already employed it in the foregoing pages, but we proceed now to inquire more particularly respecting its nature, the laws of its action, and its effects upon machines.

In investigating the mathematical principles of mechanics, we first proceed on the supposition that the forces in question act without any impediments; that the surfaces which move in contact are perfectly polished and suffer no friction; that axes and pivots are mathematical lines and points; that ropes are perfectly flexible; and, in short, that the power is transmitted through the machine to the working point without sustaining the least loss or diminution. Great simplicity is attained by first bringing the subject to this ideal standard of perfection, and afterward making suitable allowances for all those causes which operate in any given case to prevent the perfect action of a machine.

306. *Nature of friction.*—Surfaces meet with a certain degree of resistance in moving on each other, in consequence of *the mutual cohesion of the parts*, a principle which has the greater influence in any given case in proportion as the surfaces are smooth. But a much greater resistance arises from the asperities which the surfaces of all bodies have, though in very different degrees, according to their different degrees of smoothness. An extreme case is that of two brushes moving on each other, the hairs of which become interlaced (especially when the brushes are pressed together), and oppose a great resistance. Even bodies apparently very smooth, as polished metals, exhibit under the microscope numerous inequalities. Under the solar microscope, the finest needle exhibits a surface as rough as the coarsest iron tools do when viewed by the naked eye. To these inequalities of surface, is principally ascribed the friction of bodies, when closely in contact; the prominent parts interlock with one another, or meet, and must be broken down before the surfaces can move. Hence, friction is diminished by processes which level these inequalities, either by polishing the surface, or by coating it with some lubricating substance which fills up the cavities.

307. Forces of this nature, which act by the resistance they occasion to motion, are called *passive* forces. They produce very different effects in machines when in a state of equilibrium, and in a state of motion. In the one case they assist the power; in the other case they oppose it. Thus, a weight placed on an inclined plane, will require a less power to *support* it in consequence of the friction of the plane; and a weight suspended by a rope passing over a pulley will require a less weight to *balance* it, on account of the friction of the axle. But the same passive forces operate in just the contrary way when a machine is to be put in motion; for then a power must be applied, which is sufficient not only to overcome the weight itself, but also the amount of all the resistances. For example, in order to draw a load up an inclined plane, we have to overcome not only the force of gravity by which the load endeavors to descend down the plane, but also the amount of the friction and all the other resistances which impede its motion, although the load would be kept from *descending*, that is, in a state of equilibrium, by a less force in consequence of these resistances. The principle is most strikingly observed in the wedge, where the difficulty of making the wedge *advance*, is greatly increased by friction, but the same cause operates to prevent it from *recoiling*.

Two philosophers of great eminence have severally performed an extensive series of experiments on friction, namely, M. Coulomb, member of the Academy of Sciences at Paris, and Professor Vince, of the University of Cambridge in England; and upon their investigations is founded a great part of all that is known with precision respecting the laws of friction.

308. *Kinds of friction.*—The forms under which this sort of resistance presents itself, are chiefly of two kinds, namely, that of bodies *sliding*, and of bodies *rolling* on each other. To the former of these let us first attend. Experiments on the friction of sliding bodies may be made, either by placing them on a table, and observing the weights which they respectively require to drag them along the table, or by placing them on an inclined plane, and observing at what angle the plane must be elevated in order that the body may *begin* to slide. In the former case, the table is prepared by attaching a vertical pulley to one edge over which a string is passed, one end being connected to the body in question, and the other end to a pan, like that of a balance for containing weights. From this simple arrangement, a great variety of particulars may be ascertained respecting the friction of sliding surfaces. A body shaped like a brick, with a broader and a narrower side, may be tried on each of its sides separately, and thus it may be seen whether, in a given weight, the *extent of surface of contact* makes any difference; the body may be loaded with different weights, and hence may be learned the *influence of pressure* upon friction; the body may be tried as soon as it is laid on the table, and after remaining on it for a longer or shorter time, in order to learn whether this circumstance alters the friction ; different kinds of bodies may be tried, and the *influence of different materials* ascertained ; and finally, by dragging the body off the table with different degrees of velocity, the *relation of friction to velocity* may be investigated.

309. *Laws of sliding friction.*—From experiments like the foregoing, endlessly varied, the following conclusions were established:

1. In a given body, *extent of surface* makes no difference in regard to friction; a brick set on its end, or laid on its edge, meets with the same resistance from this cause as when laid on its side.

2. Friction is proportionate to the *pressure.* If the pressure of the brick is doubled or trebled by laying weights upon it, the amount of friction will be increased in the same ratio.

3. Friction is increased by bodies *remaining for some time in contact with each other.* In some cases it does not reach its maximum under four or five days. This principle therefore affects slow motions much more than such as are rapid. In the mutual contact of metals, the friction attains its maximum almost instantaneously. But when metal rubs against wood, or one piece of wood against another, the friction is always increased by resting. Two pieces of wood acquire the utmost friction in an hour or two; while iron running on oak will have its friction augmenting for five or six days. The application of a coat of tallow seems to protract the limit of friction. This limit is attained by the greased surfaces of iron and copper in four minutes; while pieces of wood, treated in the same way, will have their friction gradually augmented during nine or ten days.*

4. The friction is less between surfaces of *different* kinds of matter, than between those of the *same* kind. Copper slides on copper, or brass on brass, with greater difficulty than copper on brass; and it is a general rule never to let two substances of the same hardness move upon each other. To this rule cast-steel is said to form the only exception; in other cases, pivots revolve with less resistance on either harder or softer substances, than upon those of the same material with themselves.† When between the surfaces of wood newly planed, the friction would be equal to one-half the pressure, and when between two metallic surfaces it would be equal to one-fourth, between the wood and metal it would amount to only one-fifth the pressure.‡

5. Friction is much greater at the first moving of a load, than after it is brought freely into motion. In many instances, it is

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* Leslie, El. Nat. Phil., i, 225.

† Allen's Mechanics, p. 137.

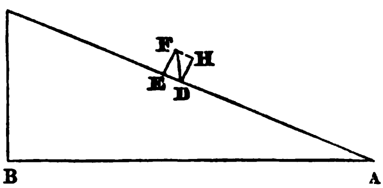
‡ Leslie.

reduced, when a body has attained its final velocity, to less than one-half of what it was at first.* With regard to different degrees of velocity over a given space, it is a *general* principle, that *the friction is the same for all velocities;* that a carriage, for example, in traveling from one place to another, would encounter the same resistance from friction, whether it performed the journey in one hour or in ten. The amount of friction, however, is augmented in very slow motions, and greatly diminished in those that are very swift. In this instance, the increase in the one case and the diminution in the other, appears to have some relation to the principle, that the friction of bodies is increased by their remaining in contact. From some observations of Professor Playfair, made at the slide of Alpnach, where large fir-trees were carried with great velocity down an inclined plane eight miles in length, it would appear that, in the case of very great velocities, friction is not, according to the common doctrine, either proportioned to the pressure, or independent of the velocity; but that the ratio to the pressure is greatly diminished, and the actual resistance is far less than at common velocities. Thus, none but large trees could descend the plane at all; and when a tree broke into two pieces, the larger part would proceed while the smaller would stop; and the trees acquired in their descent a rapidity of motion, incompatible with the supposition that "friction acts as a uniformly retarding force," which has been considered as an established principle.†

The foregoing considerations are in favor of rapid traveling, whether on common roads or on railways, since the amount of the resistance is so much less than in slow movements; and accordingly it is said that the great speed given to stage-coaches in England, amounting, in some instances, to 10 or 12 miles per hour, has not been attended with the degree of exhaustion to the teams that would have been anticipated.‡

310. *Angle of friction.*—The angle at which an inclined plane must be elevated, in order that a given body resting on it may be on the point of sliding, is called the *angle of friction*, or sometimes the *angle of repose.* Let ABC (Fig. 169) be the inclined plane, and let FD, parallel to BC, represent the whole

FIG. 169.



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* Leslie, Nat. Phil., i, 218.

† Playfair's Works, vol. i, or Ed. Phil. Jour., vi, 345. Also see Nicholson's Oper. Mech., ii, 225.

‡ Nich. Oper. Mech.

weight of the body. FD being resolved into FE perpendicular to the plane, and ED coincident with it, ED will be the force with which the body tends to slide down the plane, and, of course, that which is resisted by the friction, and hence it may be taken as the representative of the friction. But ED is to FE as BC to BA, or as the height of the plane to the base, or as the tangent of the angle of friction to radius. Consequently, putting F for the friction, P for the whole pressure on the plane, and α for the angle of friction, then,

$$F : P :: \tan\alpha : \text{rad} \; ; \; \therefore F = P \times \tan\alpha \, ;$$

that is, the friction is always such a part of the entire pressure, as is denoted by the product of that pressure into the tangent of the angle of friction. This angle often determines the figure which natural objects spontaneously assume. Hence, sand-hills are more sloping than eminences composed of ordinary mould, the movable particles arranging themselves in obedience to the foregoing law. The angle of friction of iron pressing on iron, is found to be 16 degrees. The angle given to the threads of a screw, is regulated on this principle.*

311. *Rolling friction.*—The laws of friction in *rolling* bodies were ascertained by Coulomb, by comparing the forces necessary to roll a cylinder upon a table under various circumstances; and by similar experiments, were found the modes in which friction takes place in bodies *revolving* on an axis. The comparative loss of power in these three cases is as follows:

Friction of the	sliding body equal to $1/4$ the pressure, or	25 per ct.
do	revolving do	15
do	rolling do	5

In the case of hollow cylinders revolving on an axis, the *leverage* of the wheel aids in overcoming friction, as has been already explained in Art. 271.

312. *Friction-wheels.*—This is a contrivance by which friction is diminished in the greatest degree possible, and owes its efficacy in part to the operation of the same principle.† Here the axis of a wheel, instead of revolving in a hollow cylinder, or instead of rubbing against a fixed surface, rests, at each of its extremities, on the circumference of two wheels placed close by the side of each other, with their circumferences intersecting. The axis rests at the point of intersection, and as it revolves, the wheels revolve with it with the same velocity of circumference, and thus all friction between them and the axis is prevented; and what remains in the machine in consequence

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[*] Leslie.

[†] A fine example of the application of friction-wheels is seen in Atwood's Machine.

of the weight of the wheels themselves is *transferred* to the axles, and therefore (Art. 265) is diminished in the ratio of the diameter of one of the wheels to that of its axis. This combination may be repeated by several pairs of friction-wheels. Eight wheels may reduce the friction to the thousandth part.[*]

313. Other more common methods of diminishing friction are, by rendering the surfaces smooth, by using rollers, and by lubricating the parts in contact. The amount of friction in the several simple machines is very different. In the lever it is very small, especially when the turning-edge is of hardened steel, and shaped like a knife or prism, and turns upon a hard and smooth basis. The wheel and axle acting upon the same principle as the lever, occasions but little friction. The stiffness of the cordage, however, and the friction of the gudgeons[†] of the axis, have an effect in most cases equal to about 8 or 10 per cent. of the entire resistance. The pulley is attended with great loss from this source. It is rarely less than 20 per cent., and often exceeds 60. The inclined plane involves but little friction when bodies simply roll on it; but when heavy bodies rest on axes, as in wheel-carriages, the resistance from friction takes place in the same manner as upon plane surfaces. The transportation on inclined planes, as railways, is usually by means of wheels, since the resistance to sliding movements is too great to permit the use of them. The screw is attended with a great amount of friction. Those with sharp threads have more than those with square threads, and the endless screw has most of all.[‡] In both the screw and the wedge, the friction evidently exceeds the resistance; otherwise they would not retain their position.

314. Friction is not, therefore, in all cases to be considered as unfavorable to the operation of machinery. It is, in many instances, a highly useful force. Many structures, as those of brick and stone, owe no small part of their stability to the roughness of the materials of which they are composed; without this resistance, the screw and the wedge would lose their efficacy, and wheels could not advance, nor could animals walk on the ground; and nails would lose their power of binding separate parts together. The art of polishing surfaces depends on the same cause, and the edges of most cutting instruments are saws, the teeth of which are more or less fine, and act on a similar principle. Even in certain rotary motions, friction, by a rope or otherwise, becomes a moving force, and urges a body in particular directions contrary to the force of gravity.*

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[*] Leslie, El. Nat. Phil., i. 238.

[†] Pivots when large take the name of *gudgeons*.

[‡] Emerson.

The resistance which moving bodies sustain from the air, and from water, will be considered hereafter.

CHAPTER VII.

PROJECTILES AND GUNNERY.

315. Early in the 17th century, Galileo, a celebrated Italian philosopher, established the mathematical theory of projectiles, as given in the former part of this work. Since missiles thrown by instruments of warfare are among the number of projectiles, these principles were supposed to constitute the foundation of the art of gunnery; and several of the kings of Europe held out munificent encouragement to experiments in this department of science.† Galileo had indeed intimated that the *resistance of the air* (which is not taken into the account in the mathematical theory of projectiles) might occasion some deviation in bodies from the curve of a parabola; but this fluid appears so light in comparison with the heavy metals of which balls are made, being 10,000 times lighter than lead, that for a long time afterward it was totally neglected.

About the year 1740, Mr. Robins, an eminent English mathematician, instituted a series of experiments on gunnery, which showed that the parabolic theory of projectiles is so modified by the resistance of the air, as to be wholly inapplicable to practice. Experiments on the same subject have since been performed by Count Rumford, and by Dr. Hutton of the Royal Military Academy at Woolwich, which have confirmed the results obtained by Mr. Robins. By the labors of these several gentlemen, the parabolic theory of projectiles has received its proper modifications.

316. *The path of a projectile.*—It is ascertained, in general, that projectiles, moving *slowly*, describe curves which are nearly parabolas; while such as move *swiftly*, deviate very far from this curve. Indeed, the curve which a body moving with great velocity in the air describes, is so complicated that the utmost resources of the calculus have hardly been able to investigate its nature;‡ and it is a remarkable fact, that we understand the motions of the heavenly bodies better than we do those of a cannon-ball, and can trace the path of a planet better than we can that of an arrow. The parabolic figure described in the case of projectiles moving slowly, may be observed in tracing the path of a small stone thrown into the air, and more especially in the curves described by jets of water, spouting upward, as in fountains. But when the jet of water is more rapid, and spouts at a high angle, as 45 degrees for example, we can plainly see that the curve deviates greatly from a parabola. The remote branch of the curve is seen to be much less sloping than the rising branch; and even in very great jets, which are to be seen in some great water-works, the falling branch is almost perpendicular at its remote extremity; and the highest point of the curve is far from being in the middle between the spout and the place where the water falls. This unequal division of the curve by its highest point may also be observed in the flight of an arrow or a bomb-shell.*

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* See Whewell's Bridgewater Treatise on Astronomy and General Physics, chap. xii.

† Robison's Mech. Phil., i, 167.

‡ Hutton's Tracts, No. 37.

317. The following facts also show the discordance between the parabolic theory of gunnery and experience. A cannon-ball, fired in such a direction and with such a velocity, that its random or horizontal range ought to be 24 miles, comes to the ground short of one mile. The times of rising and falling, if that theory held good, ought to be equal; but the time of rising is greater than that of falling at great elevations, and at small elevations, less than that of falling. According to the theory, the greatest random is at an angle of elevation of 45 degrees (Art. 166), but in practice it is found to be much below this. The greatest random of an arrow is when the elevation is about 36 or 38 degrees. Indeed, the angle for the greatest horizontal range may be at all degrees from 45° to 30°; the slowest motions and the largest shot being almost at 45°, but gradually more and more below that degree as the shot is smaller and the velocity is greater; till at length, with the most rapid motions and the smallest shot, the angle is little above 30 degrees.† The following experiments were made in France by Borda, with a 24-pounder, with the same charge of powder in each experiment:

Elevation.	Range.
15°	1950
30	2235
45	2108
60	1700
75	950

Whence it appears that at the elevation of 15 and 75, the randoms, instead of being the same (being equally distant from 45), were as the numbers 1950 and 950.

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* Robison's Mech. Phil., i, 185.

† Hutton, Tract 37.

318. All this discordance between theory and practice is owing to the resistance of the air, which, when the projectile moves with great velocity, becomes enormous. Nor will it be difficult, on a little reflection, to comprehend the reason why this resistance should be so great. The force with which a projectile strikes the air at rest, is the same as that with which the air, moving with equal velocity, would strike the body at rest. This, in the case of a cannon-ball, would greatly exceed the most violent hurricane. Again, as a ball moves through the air, it displaces, that is, gives motion to, great quantities of air; yet whatever motion it imparts to other bodies is extinguished in itself. The loss of motion, therefore, increases very fast with the velocity. It is said to be in general as the square of the velocity; so that a body moving through the air with ten times the velocity of another body, would encounter one hundred times as much resistance. In very swift motions the resistance was ascertained by Robins to be even much greater than in the ratio of the square of the velocity.

319. *The ballistic pendulum.*—The researches of Mr. Robins were made chiefly by the aid of an instrument of his own invention, called the *ballistic pendulum.* It consists of little more than a large block of wood, like a log, suspended after the manner of a pendulum. Now if a bullet be fired into the block, as the bullet will be stopped, and as it imparts to the block whatever motion it loses, consequently the momentum of the block and ball together, after the stroke, is precisely that of the ball before the stroke. Hence the weight of the block and that of the ball being known, and the velocity imparted to the block being readily determined by observation, it is easy to find the velocity of the ball; for the weight of the ball is to the weight of the block, plus the bullet, as the velocity of the block, plus the bullet, is to the velocity of the ball.

Example.—On firing an 8-pound cannon-shot into a ballistic pendulum that weighed 500 pounds, the pendulum was made to move over a space of 16 feet per second: what was the velocity of the ball? *Ans.* 1016 feet per second.

320. This simple apparatus is sufficient for ascertaining a great number of particulars relative to the art of gunnery. If the ball is fired nearly in contact with the block, we find with what velocity it leaves the gun; if at different distances from the block, we find how much the velocity is retarded by passing through the air, for those distances respectively. If at a given distance we vary the charge of powder, we find the respective changes which the velocity undergoes, and hence learn the ratio that ought to be observed between the powder and the ball, in order to produce the maximum effect. The effects resulting from variations in the length, shape, and bore of the gun, are also ascertained with equal facility.

The following are some of the practical results ascertained by the experiments of Mr. Robins, Count Rumford, and Dr. Hutton. A musket-ball, discharged with a common charge of powder, issues from the muzzle of the piece with a velocity between 1600 and 1700 feet in a second. The utmost velocity that can be given to a cannon-ball is a little more than 2000 feet per second, and this it has only at the moment of leaving the gun. In order to increase the velocity from 1600 to 2000 it requires half as much more powder, which involves a hazardous strain upon the gun, and the velocity will be reduced to 1300 before the ball has proceeded 500 yards.*

321. *Effect of a vacuum behind the ball.*—Mr. Robins, in the course of his experiments on the resistance of the air, made a curious observation. In very moderate velocities, the retardations were nearly as their squares. As the velocities were increased, the resistances increased at a somewhat greater rate, but with a certain observable regularity, till the velocity exceeded 1100 feet per second. But when the velocity is increased from 1100 to 1200, the increase of resistance is prodigious. After this, the resistance goes on increasing nearly with its former regularity. Mr. Robins accounts for this singular fact in the following manner. As the ball rushes through the air, the air falls in behind it, being pressed in by the weight of the surrounding air. But the ball may move so rapidly that the air can not instantaneously fill up the place left by the ball. In this case the ball is retarded, not only by the resistance of the air which it displaces, but also by its having the pressure of the atmosphere on one side and not on the other. It is found by calculation, that the velocity with which air rushes into a vacuum, is about 1300 feet per second; consequently, it is when the ball is moving at this rate, that it loses the entire pressure of the atmosphere behind it.† It is remarkable (says Dr. Robison) that this is also nearly the velocity of sound, and there is an observation of this kind, which seems to have some connection with the mechanical fact observed by Mr. Robins. If a person stand in such a direction from a cannon, when it is discharged, that the ball may pass him at no great distance, he will hear the noise made by the ball rushing through the air at the time of its flight, and as the ball approaches him, the noise should become more audible. But he will hear the noise loudest at the very first, immediately following the report of the gun; and after about two seconds, he may observe the sound change all at once, and not only become more faint, but even change its kind, after which the sound increases as the ball comes nearer. It seems highly probable that this abrupt alteration in the sound takes place just at the time when the resistance undergoes such a change; and that it is owing to the difference in the nature of the undulations, when there is a void behind the ball, and when there is not.*

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* Robison's Mech. Phil., i, 201.

† Robins, Tract on Gunnery, p. 181.

322. From the foregoing considerations it is inferred that great charges of powder are absolutely useless in the service of artillery, especially when the distance of the object is considerable, and that a velocity exceeding 1100 should not be aimed at. The maximum service charge is $\frac{2}{3}$ the weight of the ball. In close naval engagements great velocities are injurious, for the ball may then pass through both sides of the vessel without lodging, and the number of splinters produced by a ball in rapid motion, is much less than is caused by one moving more slowly. By reducing the charge we may also reduce the size and strength of the gun; and hence guns are made of smaller dimensions now than formerly, in order to do the same execution. The velocity with which a charge of powder expands itself at first, is estimated by Hutton as high as 5000 feet per second. As it expands, this velocity is of course constantly diminishing, but will exceed that of the ball while the latter is passing through the barrel of the gun, and will act as a constantly accelerating force. Long guns therefore give to balls a greater velocity than short ones; but the gain secured in this way after a moderate length is so small† (there being also some disadvantages peculiar to long guns), that cannon have of late years been much shortened. In the naval service, *carronades* have been introduced. These are a short kind of gun, with small bore, requiring for a charge of powder, only one-twelfth the weight of the ball. Their weight and thickness are proportionally reduced, yet in close action they produce effects superior to those of long guns.‡

323. *Windage.*—It has been found that no difference is caused in the velocity or range, by varying the weight of the gun, nor by the use of wads, nor by different degrees of ramming, nor by firing the charge of powder in different parts of it; but that a very great difference in the velocity arises from a small degree in the *windage.*[*] Indeed, with the usual established windage only, viz., about $\frac{1}{20}$ of the caliber, no less than between $\frac{1}{4}$ and $\frac{1}{3}$ of the powder escapes and is lost, and as the balls are often smaller than the regulated size, it frequently happens that half the powder is lost by unnecessary windage.[†] To this cause also, namely, too great windage, Dr. Hutton ascribes a great part of the sideways deviation of a ball; since, if in passing through the barrel of the gun, it is knocked from side to side, it will finally take the last direction which it happened to have at the muzzle of the gun.[‡] Another cause of this deviation from the line of direction, arises from a want of perfect sphericity in the ball, by which means the two sides do not meet with equal resistance. Rifles owe their superiority over common guns, chiefly to their obviating this deviation. They have a spiral groove cut in their bore, making about a turn and a half in the whole length of the barrel. The ball, which is made to fit close to avoid too great windage, has a corresponding motion impressed on it, which it retains after it leaves the gun, continuing to revolve around the line of direction. Whatever inequalities, therefore, may exist in the ball, their effects are neutralized, by their being first on one side and then on the other of this line.

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* Robison's Mech. Phil., i, 200.

† The random, according to Dr. Hutton, increases only as the fifth root of the length, which is so small an increase as to amount to only about a seventh part more range for a double length of gun.—(Hutton, Tract 37.)

‡ Renwick, Heads of Lectures, in Literary and Scientific Repos., iv, 281.

324. *Ricochet.*—When a ball is projected from a piece of ordnance at a small angle of elevation, and falls upon water, or on a plane of hard earth, its flight will not cease, but it will rise again and describe a second curve similar to the first, but less; and it will continue to rebound, until the whole of its projectile velocity is destroyed. This species of firing is called *ricochet.* It is applied with great advantage from sea-coast batteries upon shipping, and in the attack of fortresses. The pieces are fired with small charges of powder and elevated only from 3 to 6 degrees. The word signifies *duck and drake*, or rebounding; because the ball or shot thus discharged, goes bounding and rolling along, killing or destroying every thing in its way, like the bounding of a flat stone along the surface of water when thrown almost horizontally.

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[*] By windage is meant, the difference between the diameter of the ball and that of the bore of the gun.

[†] Hutton, Tract 3, 255.

[‡] Hutton, Tract 38, Prob. 4.

CHAPTER VIII.

APPLICATIONS OF THE PENDULUM.

325. THE pendulum has three most important and interesting uses, viz.: as affording a *measure of time*, the means of determining the *figure of the earth*, and a *standard of weights and measures.*

Time is any portion of indefinite duration, from which it may be considered as separated, as a given number of yards of cloth are measured off from a piece of unknown length. For the measure of time we may employ any instruments or marks that divide it into equal portions. The period occupied by a given quantity of sand in running through a funnel, as in the hour-glass—the successive intervals through which the surface of a column of water descends while discharging itself by an aperture in the bottom or side of the vessel—the beats of the hand, as in music, or even the pulsations of the wrist, are so many different modes of measuring time. But as the intervals measured by any of these modes are not perfectly uniform, and as they are incapable of that minute subdivision which is sometimes required, none of them is adapted to the purposes of the astronomer, who seeks by this method to estimate the motions of the heavenly bodies; or of the mariner, who depends on his time-piece for the means of ascertaining his longitude at sea.

326. *The pendulum as a measurer of time.*—The adaptation of the pendulum to the measuring of time, was first noticed by Galileo on observing the vibrations of a lamp suspended from the ceiling of a church. The theoretical doctrines of the pendulum revealed the fact, that all the vibrations of a pendulum of given length, are performed in nearly equal times, and, when made to move in the arc of a cycloid (Art. 160), in times that are exactly equal. Hence the pendulum becomes a measure of time. Galileo, indeed, used this instrument by itself, counting the number of its vibrations; but Huygens, an astronomer of Holland, first connected it with clock-work, by which its motions are indefinitely continued, more precisely regulated, and the number of its vibrations exactly registered. Pendulums are usually made of such a length as to beat seconds, or to indicate, at each vibration, the $\frac{1}{86400}$ part of a mean solar day; but as the period occupied by the passage of a star from the meridian round to the same meridian again (called a sidereal day) is less than that occupied by the sun, so the pendulum regulated to solar time, may, by being shortened a little, be made to indicate the corresponding aliquot parts of a sidereal day. Since, moreover, the times of vibration are as the square roots of the lengths (Art. 161), a pendulum may be made to beat half-seconds by being made only one-fourth the length of the seconds pendulum, or to beat once in two seconds by making it four times as long.

327. *The cycloid.*—As the vibrations of the pendulum in circular arcs are not exactly equal to each other, it has been attempted to make it vibrate in the arc of a cycloid (Art. 159); but the practical difficulties involved in making the pendulum rod adapt itself to the cheeks of a cycloid are so great, that this method has been abandoned in practice, and is at present regarded only as a theoretical curiosity. Where the arcs of vibration are very small, they will not differ sensibly from portions of a cycloid, and the times of vibration may without sensible error be considered as equal. In cases where extreme accuracy is required, the exact allowance, mathematically determined, may be applied to vibrations performed in circular arcs to reduce them to the corresponding vibrations in a cycloid; and a still further allowance may be made for the resistance of the air, so as to make the vibrations correspond to such as they would be if performed in a vacuum.

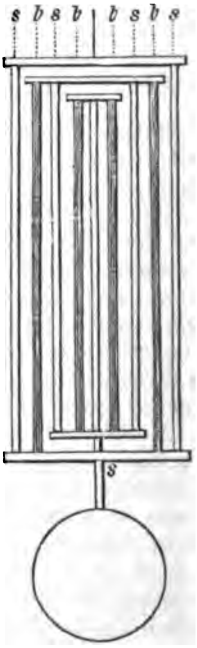
328. *The brachistochrone.*—The cycloid is the curve of *swiftest descent;* that is, a body will descend from one point to another, not in the same vertical, in less time on this than on any other line. This proposition, in general, is demonstrated by the aid of the calculus; but the cycloid may be readily compared with the inclined plane by experiment, two balls being started at the same instant, and allowed to roll down the plane and the curve respectively. The ball reaches the bottom soonest by way of the curve. The velocity acquired at first by a direction more nearly perpendicular, more than compensates for the greater length of the course.

Moreover, if both the balls be let off at any different heights on the cycloid, they will reach the lowest points at the same instant, since it is a property of this curve, that the accelerating force at any point in it is proportional to the distance of that point from the lowest point; consequently, the upper ball being urged by a force as much greater as its distance is greater, reaches the bottom at the same instant with the lower ball. On the same principle the pendulum performs all its vibrations in a cycloid in equal times.

329. *Compensation pendulum.*—The pendulum is liable to some sources of inaccuracy, which it has cost much labor and skill to obviate. The rod and bulb are both subject to expand by heat and contract by cold. In the former case the center of oscillation (Art. 150) is carried too far from the center of motion, that is, the pendulum becomes too long, and the clock goes too slow; in the latter case, the pendulum being shortened, the clock goes too fast. With a pendulum having an iron rod, a difference of temperature of 25 degrees would make a difference of six seconds in 24 hours in the rate of the clock.*

Pendulums so constructed as to remedy these defects arising from change of temperature, are called *compensation pendulums*. The general principle of these instruments is as follows: we connect with the pendulum rod some substances, which are made to expand or contract in a direction opposite to that in which the rod itself contracts or expands, and thus maintain the center of oscillation at a uniform distance from the center of suspension. The length of the theoretical or simple pendulum, it may be remarked, depends wholly on the distance between these two points, and not upon the length of the pendulum rod. If then we make the pendulum rod of one kind of metal, as steel, and connect with the bulb another kind of metal, as brass, which is expanded more by the same amount of heat, a less length of the latter expanding upward, will compensate for a greater length of the former expanding downward, and the center of oscillation will be kept at the same constant distance from the center of suspension. Now it is found by experiment that the lengths of these substances, which are equivalent to each other, are as 1 : .6091, or as 100 : 61 nearly. Therefore, if we connect 61 inches of brass with 100 inches of steel, the former so arranged as to expand upward, while the latter expands downward, we shall have a compensation pendulum. The first instrument constructed on this principle is called the *gridiron pendulum*, a name which it derives from the fancied resemblance which the parallel bars of steel and brass have to the gridiron. In Fig. 170, the five bars marked *s* are of steel, and the four marked *b* are of brass. It will be seen by the figure that the rods of steel can elongate themselves only downward, while those of brass can expand only upward; and being combined in the ratio of 100 to 61, they will exactly compensate each other. A clock furnished

Fig. 170.



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* Kater.

with the gridiron pendulum is found capable of keeping very accurate time. Indeed a clock of this kind, constructed by Harrison, the inventor of the gridiron pendulum, gained the great premium offered, by the British Board of Longitude, for a time-keeper which would keep time for a given period to a certain degree of accuracy.

330. If brass expanded a little more than twice as fast as steel, then *one* expansion of brass upward would balance *two* expansions of steel downward; and therefore, three rods would suffice for compensation, though five would be used to give symmetry to the pendulum. But since brass expands only a little more than *three halves* as fast as steel, it requires *two* expansions of the former to counteract *three* of the latter; hence five rods, three of steel and two of brass, are absolutely necessary; and since all but the center one must act in pairs, for the sake of symmetry, *nine* rods are always used, as shown in the figure.

Another form of the compensation pendulum consists of a jar of mercury, suspended from the rod in the place of the bulb. By means of a kind of stirrup, the jar is so connected with the rod as to expand upward, while the rod expands downward. The mercury being a very expansible metal, the length of the mercurial column required for the compensation of a steel rod, of 42 inches (including the stirrup), is only 6.31 inches.* This compact and simple form of the compensation pendulum, makes this variety peculiarly eligible.

331. In stationary time-pieces, the pendulum affords the most eligible, as it is the most accurate, measure of time. But where the instrument is required to be portable, as in watches and chronometers, a spiral spring, called the main-spring, is the moving-power, instead of a weight; and a balance-wheel takes the place of the pendulum. The hair-spring attached to this maintains its vibrations, coiling up in one direction and uncoiling in the opposite, while the main-spring supplies the power in the same manner as the weight of a clock.

332. *Form of the earth ascertained by the pendulum.*—The use of the pendulum *in investigating the figure of the earth*, results from its power of measuring the intensity of the force of gravity in any given place. (Art. 161.) Now as this force varies inversely as the square of the distance from the center of the earth, when the force of gravity is ascertained at a great number of places remote from each other, a comparison of these observations indicates the respective distances of these places from the center of the earth, and of course, when the observations are sufficiently multiplied, they indicate, collectively, the figure of the earth.

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* **Kater, in Lardner's Mechanics, p. 381. Baily, Astron. Trans., 1824, p. 381.**

If, therefore, one were to start from the equator with a pendulum which there vibrated seconds, and should proceed with it to the north pole, and there count the number of vibrations it would make in an hour, he might thus ascertain the respective forces of gravity at those two points, and hence learn the ratio between the equatorial and polar diameters. For, according to Art. 161, the number of vibrations performed by a pendulum in any given time, varies as the square root of the force of gravity. Although it might be convenient, were it practicable, to derive the ratio between the equatorial and polar diameters of the earth directly from observations made at these two points, yet as that can not be done, we can only obtain the law of curvature, and hence derive the figure of the earth, by observations made in many different latitudes. Such observations have been made with the greatest accuracy by Kater, Sabine, and others; and, although the result shows that the figure of the earth is that of an oblate spheroid, yet the difference between the two diameters, or the *ellipticity* as it is technically called, is somewhat greater when determined by the pendulum, than when estimated by measuring the length of the respective degrees of the meridian from the equator to the pole. By a comparison of a great number of pendulum experiments, Baily, an English astronomer, makes the ellipticity of the earth $\frac{1}{285}$ of the equatorial diameter, while that derived from a similar comparison of measures of the meridian, is $\frac{1}{298}$. Hence, the pendulum would lead us to the conclusion that the diameter of the earth, when taken through the equator, is about 34 miles greater than when taken from pole to pole, while actual measurements would make the same excess about 26 miles.*

Measurements of arcs of the meridian, made for the express purpose of determining the true figure of the earth, have been executed with an astonishing degree of accuracy, in various countries, and in different latitudes, from the equatorial regions to points within the Arctic circle. By these means, the nature of the curve that encompasses the earth would be ascertained, and the figure of the earth determined. The combined result of all these measurements is as already stated.

333. *The pendulum as a standard of linear measures.*—In order to insure confidence in business transactions, it is very essential that the weights and measures employed should be and remain of a certain known amount or length—a condition which cannot be attained otherwise than by adapting them to a fixed and invariable standard. To fix on such a standard is a matter of more difficulty than would at first be imagined. All things on the face of the earth are more or less subject to change, not only the works of man, but even those of nature. The elements decompose or wear down, in time, the hardest rocks. In ancient times, the standards of weights and measures were derived from the parts of the human body; as, a *hand's breadth*, a *cubit*, a *digit*, &c. The English linear measures were, for a long time, referred to the length of a yard-stick, taken from the length of the arm of Henry the Seventh, and preserved in the tower of London.* Of the defectiveness of such a standard, it is sufficient to mention the impossibility of verifying it after the death of the king. The idea of standards involves two conditions that are indispensable—the constancy of the thing itself, and the power of verification, should any change in it even be suspected. These conditions are secured by connecting the standard with the immutable laws of nature. In the year 1790, the French government undertook to effect a complete change in the weights and measures in use throughout the world, and to derive an unalterable standard from the dimensions of the earth itself. For this purpose, they undertook the determination of the exact length of a quadrant of the meridian, extending from the equator to the north pole. A certain aliquot part (the ten millionth) of this they denominated a *metre*, which was to be the unit of all *linear* measures. The square of this furnished a measure of *surfaces*, and the cube a measure of *solids*. Measures of *weight* were derived from the weight of a certain volume of water.

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* Herschel's Astronomy.

334. We may exemplify the precautions necessary in order to make the pendulum an accurate standard of measures, by reviewing the conditions under which it is adopted as a standard in the State of New York.

The standard is the pendulum vibrating seconds, in a cycloidal arc, and in a vacuum, in Columbia College, in the city of New York. The vibrations are required to be in a *cycloidal* arc, because those performed in circular arcs are not absolutely uniform. But the vibrations performed in circular arcs may be *reduced* to the corresponding cycloidal arcs upon mathematical principles. As the resistance of the air might be unequal at different times, such an allowance must always be made for this, as will neutralize its effect. A particular spot is designated, because local causes have some influence upon the vibrations of the pendulum.

The unit of linear measures is the *yard*, which is of such magnitude as to bear to the pendulum the proportion of 1 to 1.086158. Some have proposed to make the seconds pendulum itself the unit of linear measures, and to make all other measures of length parts and multiples of this; but it is thought advisable to retain the common measures to which the habits of society are adapted, determining their exact amount by referring them to this invariable standard. The usual subdivisions of the yard into feet, inches, and so on, remain as they are. The standard *temperature* of the standard yard-stick, is that of melting ice. It is necessary to attend to this circumstance, because all bodies are expanded by heat and contracted by cold, so that the yard-stick is of a uniform length only at a given temperature.

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* Adams on Weights and Measures.

The unit of measures of *weight* is the avoirdupois pound, of such magnitude that a cubic foot of pure water, at its maximum density,* shall weigh 1000 ounces, or 62½ pounds.

The unit of dry measures of *capacity* is the gallon, a vessel of such magnitude as to hold exactly 10 pounds of pure water, at its maximum density. The bushel, therefore, holds 80 pounds. The unit of liquid measures is also a gallon, containing 8 pounds of distilled *water*, at its maximum density.

The government of the United States has adopted, for the different standards, the following bases:

Standard of Length. The yard of 3 feet or 36 inches, from the scale of Troughton, which is a brass scale of 82 inches, made by a celebrated English artist for the survey of our coast. Hence the yard adopted as the standard, is identical with the British imperial standard.

Standard of Weight. The troy pound, containing 5762.38 grains, used as a standard at the mint of the United States.

Standard of Dry Measure. The British Winchester bushel of 2150.4 cubic inches, equal to 77.6274 pounds of distilled water, at the maximum density.

Standard of Liquid Measure. The English wine gallon of 231 cubic inches, equal to 8.339 pounds avoirdupois of distilled water, at the maximum density.

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[*] **Water expands as its temperature rises above or falls below a certain point; about 40° of Fahrenheit. Hence the necessity of employing it as a standard at its *maximum density*.**

PART III.—HYDROSTATICS.

335. *General definitions and remarks.*—The principles of Mechanics demonstrated and explained in the foregoing pages, are *universal* in their application, extending alike to all bodies, whether solid or fluid. But in addition to those properties which fluids have in common with solids, and which bring them under the general laws of Mechanics, they have also properties peculiar to themselves, and which give rise to a distinct class of mechanical principles, not applicable to solid bodies. These are embraced under the heads of *Hydrostatics*, the mechanics of liquids, and *Pneumatics*, the mechanics of gases.*

In mechanics after having ascertained a few fundamental principles by experiment and observation, the superstructure is raised chiefly by mathematical reasoning, and thus the great body of truths in that science is established; but in hydrostatics, and the other subjects of Natural Philosophy which follow, we are much more dependent on *experiment*, which frequently affords us more satisfactory evidence, than we can obtain by the application of abstract mechanical principles.

336. A *fluid* is a body whose particles are easily moved among each other, and yield to the slightest force impressed.

Perhaps no fluids, except the gases, possess the *perfect* fluidity contemplated in the above definition of Sir Isaac Newton. And some, called semi-liquids, yield to slight forces very slowly. But it fully marks the distinction between the most imperfect fluid and a pulverized solid. For the latter can be separated into minute solid bodies, called *grains*, whose parts are bound together by cohesion; but the smallest portion of fluid which can be separated from the mass, is still fluid, having movable parts. Hence it is, that a fluid will settle to a horizontal level in time, if not instantly; while the most finely powdered solid, if piled up, will settle only to a certain inclination, greater or less, according to the smallness and smoothness of its particles.

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* The nomenclature of Mechanics is somewhat confused. As Mechanics has two senses; a general one, applicable to all matter, and a limited one, having reference only to solids, so Hydrostatics is much employed to denote the whole branch, *mechanics of liquids*, and a subdivision of the same, the *statics of liquids*. Hydrodynamics, used by many writers, is objectionable on the same ground. If the whole branch, *mechanics of liquids*, were called *Hydro-mechanics*, then its appropriate division would be, *Hydrostatics*, or the *statics of liquids*, and *Hydrodynamics*, or the *dynamics of liquids*.

Fluids are of two kinds; those which do not sensibly change their volume, when force is applied to them, and those which can be compressed and dilated beyond limit, as force is applied or removed. The former are called *liquids*, as water and mercury; the latter, *gases*, as the air.

337. It has been usual to denominate liquids and gases respectively *non-elastic* and *elastic* fluids, on the supposition that water and other liquids are nearly or quite incompressible. An experiment performed by the Florentine academicians, as long ago as 1650, seemed to prove that water is wholly incompressible. They filled a hollow ball of gold with water, and subjected it to a strong pressure. The water, not yielding to the compression, oozed through the pores of the gold. Considering the great density and compactness of this metal, the experiment was for a long time held as proving decisively that water is wholly incompressible. In this experiment, the compression was too slight to be perceived; but more recently Canton and Perkins subjected water to great pressures in the ocean, and found the diminution to be nearly *one-twelfth* of its volume for a pressure of 30,000 lbs. per square inch. An ingenious instrument, lately devised by Oersted, renders the compression, produced by a single atmosphere, visible to the naked eye. He makes the compression of water equal to 46 *millionths* of its volume per atmosphere. Still, for all the purposes of experiment or calculation, liquids may be regarded as *incompressible fluids;* and the gases, in distinction from them, may be called *compressible fluids*.

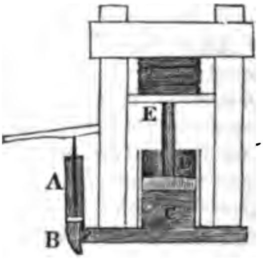
CHAPTER I.

LIQUIDS AT REST.

338. *Hydrostatics*, in its limited sense, is the name of that branch which treats of the mechanical properties of liquids at rest.

Transmitted pressure.—It is an observed property of fluids, that any force, which is applied to one part of a fluid, is transmitted undivided to every part of it. For instance, if a plug, occupying one square inch of the surface of a cask filled with water, receives a blow of 50 lbs., every square inch of the interior of the cask is instantly affected by a bursting pressure of 50 lbs. And so, if I attempt to force a cork into a vessel full of water, the pressure will be felt not merely by the portion of the water directly in the range of the cork, but by all parts of the mass alike; and the liability of the body to break, supposing it to be of uniform strength throughout, will be as great in one place as in another; and it will break at the point where it happens to be the weakest, however that point may be situated relatively to the place where the cork is applied; and the effect will be the same whether the stopper be inserted at the top, the bottom, or the side of the vessel.

Fig. 171.



339. *The hydraulic press.*—This principle is recognized in the construction of *Bramah's hydrostatic*, or *hydraulic press*, represented in Fig. 171. AB is a small cylinder in which moves the piston of a forcing pump, and CD is a large cylinder in which also moves a piston, having the upper end of its rod pressing against a movable plank E, between which and the large beam above is placed the substance to be subjected to pressure. By the action of the pump handle, water is raised into the small cylinder, and, on depressing the piston, it is forced through a valve at B into the larger cylinder and raises the piston D, which expends its whole force on the body at E. Now, since whatever force is applied to any one portion of the fluid, extends alike to every part, therefore the force which is exerted by the pump upon the smaller column, is transmitted unimpaired to every inch of the larger column, and therefore tends to raise E with a force as much greater, in the aggregate, than that impressed upon the surface of the smaller, as this surface itself is larger than that of the small piston. The power of such a machine may be made enormously great; for, supposing the hand to be applied at the end of the handle, with a force of only ten pounds, and that this handle or lever is so constructed as to multiply that force five times, the force with which the smaller piston will descend will be equal to 50 lbs.; and let us suppose that the head of the large piston contains the smaller 50 times, then the force exerted to raise the press-board will equal 2500 lbs. A man can indeed easily exert many times the force supposed, and can therefore exert a force upon the substance under pressure, equal to many tons.

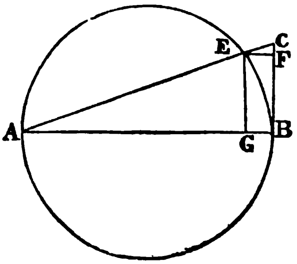
The hydrostatic press involves far less loss from friction than any other species of press, and it is said that the naked force of man is more effective when applied in this way than in any other. The relation of power to weight in the hydrostatic press is readily explained on the principle of virtual velocities (Art. 138). For it is obvious that, the same quantity of water being transferred from the small to the large cylinder, the velocity of the large piston will be less than that of the small one in the same ratio as its area is greater than the area of the small piston. Therefore the intensity of pressures on the two pistons being inversely as the velocities, will be directly as the areas.

This press is now much used for pressing paper or books, cotton, hay, and other substances, for uprooting trees, and testing the strength of ropes and building materials. The tubular bridge over the Menai Straits was constructed at the water level, and raised more than 100 feet to the top of the piers on which it rests, by means of two hydraulic presses. The weight of each portion lifted at once was more than 1800 tons.

340. *Level surface.*—When a fluid is at rest, its surface is level; that is, every part of it is perpendicular to the line of gravity. This results from the mobility of the particles; for if we suppose any part of the surface not horizontal, the force of gravity acting on a particle of that surface, may be resolved into two components, one perpendicular to the surface, the other parallel to it; the latter would meet with no resistance, and the particle would therefore move along the surface, contrary to the supposition, that the fluid is at rest. As the lines of gravity are not parallel, but convergent to points near the center of the earth, the surface of water, therefore, is not a plane, but of convex form, and nearly spherical. The water level deviates from a plane only about one second in 100 feet,* and hence for small distances may be regarded practically as a plane.

The amount of depression of level water below a tangent plane is estimated by the formula, $d = \frac{2}{3}L^2$, where L represents the distance in miles, and d, the depression in feet. Let FE (Fig. 172), or its equal, GB, be the depression for the distance BE. For moderate distances, this arc may be considered equal to its chord. Hence $GB(=d) : EB(=L) :: EB : AB$; or $d = \frac{L^2}{AB}$; i. e., reducing d to feet, while L and AB are kept in miles,

FIG. 172.



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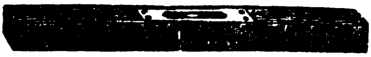
* $\frac{360 \times 60 \times 60 \times 100}{25000 \times 5280} = \frac{1296}{1320} = 0''.982.$

$$d=\frac{L^2\times 5280^2}{AB\times 5280}=\frac{L^2\times 5280}{7912}=\tfrac{2}{3}L^2\text{, nearly.}$$

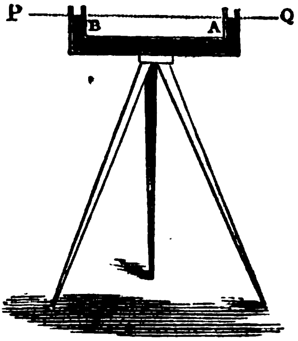
This gives, for one mile, $d=\tfrac{2}{3}\times 1^2=8$ inches; for 2 miles, $d=2$ ft. 8 in.; for 3, $d=6$ feet; for 100 miles, $d=6667$, &c. Thus, if a straight canal is 100 miles long on the same level, each end is more than a mile below the tangent to the water at the other end.

FIG. 173.



341. *The spirit-level.*—A practical application of this principle is made in the art of *leveling*. A level is sometimes made by merely cutting a groove or channel in a flat piece of board and filling it with water. When the board is brought into such a situation that the water in the groove remains stationary, the position is horizontal. But the *spirit-level* is the instrument more commonly employed for this purpose. This consists of a small bent cylindrical tube of glass, from two to six inches long, filled with spirits of wine or ether, except a small space which is occupied by a movable bubble of air. When such a tube is placed horizontally, the bubble of air will remain stationary in the center of the tube, at a fixed mark; but whenever the tube is inclined, in the least degree, the bubble will ascend toward the elevated end. Spirit-levels are much used for adjusting astronomical, surveying, and other delicate instruments. Figure 174 represents a leveling staff used in surveying and grading lands. The liquid in the two arms of the tube A and B being precisely on a level, any two objects, P and Q, may be brought accurately to the same horizontal level by *sighting* P with the eye at A, and then sighting Q with the eye at B.

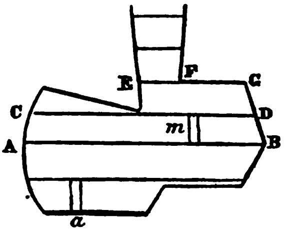
FIG. 174.



342. *Pressure as depth.*—From the principle of equal transmission of force in a fluid, it is proved, that if a liquid is uniformly dense, its pressure on a given area varies as the perpendicular depth, whatever the form or size of the reservoir. Let AB be a vessel of any form, and let us consider the pressure exerted on any part, as for example, one square inch a of the base. Suppose the water to be divided into strata by equidistant horizontal planes; and take a vertical column, one inch square, in any one of these strata, as m in the stratum, ABDC.

Since *m* presses by its own weight on a square inch of the surface AB, it transmits the same pressure to every square inch of the water below AB, both on the walls of the vessel, and upward on AB itself. Hence all the equal columns in ABDC are exactly upheld by this pressure, and exert no more force on the base, than *m* alone does. Thus *a* receives from *m*, or, which is the same thing, from the stratum ABDC, the same pressure as though *m* stood directly upon *a*. This reasoning applies to each stratum above or below ABDC, whatever its horizontal extent. Therefore, the pressure on *a* depends only on the *number* of strata, that is, on the perpendicular *depth* of the water.

Fig. 175.



Since the force is transmitted in every direction at each point of the fluid, therefore the square inch, *a*, sustains the same pressure, if at the same depth, whether it is horizontal, vertical, or oblique, and whether in a situation to receive a downward, or an upward pressure. Thus the upward pressure on each inch of FG is equal to the downward pressure on each inch of EF, since those areas are at the same depth.

If the area pressed upon is oblique or vertical, then the points of it are at unequal depths; in this case, the depth of the area is understood to be the *average* depth of all its parts, that is, the depth of its center of gravity.

If the fluid were compressible, it is obvious that the pressure increases at a faster rate than the depth; since strata of equal thickness are more dense, and therefore heavier, according as their depth is greater.

343. *Estimation of pressures.*—One cubic foot of water weighs 1000 ounces, or 62.5 pounds. Therefore, the pressure on *one square foot*, at the depth of *one foot*, is 62.5 lbs. From this, as the *unit* of hydrostatic pressure, it is easy to determine the pressures on all surfaces, at all depths; for it is obvious, that the pressure varies as the surface pressed upon, when the depth is the same, and it has been shown, that, on a given surface, the pressure varies as the depth of its center of gravity; it therefore varies in the compound ratio of the two. Let $p =$ pressure; $a =$ area pressed upon; and $d =$ the depth of its center of gravity; then $p = ad \times 62.5$.

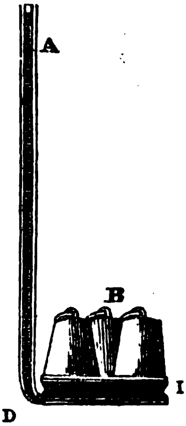
Depth.	Pounds per sq. ft.	Depth.	Pounds per sq. ft.
1 ft.	62.5	100 ft.	6,250
10	625	1 mile	330,000
16	1000	5 miles	1,650,000

We may hence infer the immense pressures exerted in the depths of the ocean, which are even greater than is indicated by the above numbers, since a cubic foot of sea-water weighs 64.37 lbs., instead of 62.5 lbs. It is said that the Greenland whale sometimes descends to the depth of a mile, but always comes up exhausted, and blowing out blood, showing that the pressure had so acted upon the vessels as to cause them to discharge a portion of their contents into the lungs. We may also apprehend the cause of the great difficulty experienced in confining a high column of water. The strength of banks, dams, flood-gates, and aqueduct pipes, must increase in the same ratio as the perpendicular depth from the surface of the water, without regard to its horizontal extent.

344. *Column of water whose weight equals the pressure.*—A convenient mode of conceiving readily of the amount of pressure on an area, in any given circumstances, is this; consider the area pressed upon to form the horizontal plane base of a hollow prism; let the height of the prism equal the average depth of the area; and then suppose it filled with water. The weight of this column of water is equal to the pressure. For the contents of the prism (whose base $= a$, and its height $= d$), $= ad$; and the weight of the same $= ad \times 62.5$ lbs.; which is the same expression as was obtained above for the pressure.

On the bottom of a cubical vessel full of water, the pressure equals the *weight* of the water; on each side of the same, the pressure is *one-half* the weight of the water; hence, on all the five sides, the pressure is *three times* the weight of the water; and, if the top were closed, the pressure on the six sides is the same, three times the weight of the water.

Fig. 176.



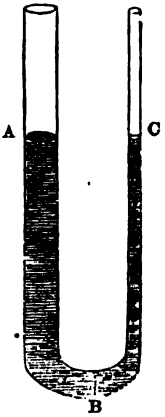
345. *Illustrations of hydrostatic pressure.*—A vessel may be formed, so that both its base and height shall be great, but its cubical contents small; in which case, a great pressure is produced by a small quantity of water. The hydrostatic bellows is an example. In Fig. 176, the weight which can be sustained by the column AD is equal to that of a prism or cylinder of water, whose base is DI, and its height, DA. It is immaterial how shallow is the stratum of water on the base, or how slender the tube AD.

In like manner, a cask, after being filled, may be burst by an additional pint of water; for, by screwing a long and slender pipe into the top of the cask, and filling it with water, the pressure is easily made greater than the strength of the cask can bear. The same cause is supposed sometimes to produce great effects in nature, such as splitting rocks, heaving up mountains, and other effects resembling earthquakes. For, suppose that in the interior of a mountain there were an empty space ten yards square, and only an inch deep, in which the water had lodged so as to fill it entirely; and suppose that a crevice in the earth should extend from this spot 200 feet above, which should also become filled with water by rain or otherwise: then we find that the whole pressure exerted on the interior of the cavity is 22½ millions of pounds.

The statement that a weight however large may be raised by a quantity of water however small, is frequently, though improperly, called the *hydrostatic paradox*. There is in this nothing more paradoxical than in the case of the lever, or any other machine, by means of which a small weight gives motion to one ever so great.

346. *The same level in connected vessels.*—In tubes or reservoirs, which communicate with each other, water will rest only when its surface is at the same level in them all. In Fig. 177, if water is poured into A, it will rise in the other branch of the tube, so as to stand at the same level, AC, in both. The pressure toward the right on any cross-section B, equals the area of B, multiplied by its depth below A. So the pressure on the same toward the left, equals its area multiplied by its depth below C; but these two pressures must be equal; therefore B is at equal depths below A and C; in other words, A and C are on the same level. Hence, if vessels and tubes very diverse in shape and capacity be connected with a reservoir, and water be poured into any one of them, it will rise to the same level in them all, since pressure depends only on perpendicular depth. Hence, also, water conveyed in aqueducts or running in natural channels, will rise just as high as its source. Between the place where the water of an aqueduct is delivered and the spring, the ground may rise into hills and descend into valleys, and the pipes which convey the water may follow all the undulations of the country, and the water will run freely, provided no pipe is laid higher than the level of the spring. Waters running in natural channels in the earth are governed by the same law.

Fig. 177.



The aqueducts constructed by the ancient Romans, were among the most costly monuments of their arts. Several of them were from thirty to one hundred miles in length, and consisted of vast covered canals built of stone. They were carried over valleys and level tracts of country upon arcades, which were sometimes of stupendous height and solidity.* From the fact that the ancients built aqueducts with so much labor, raising them to a great height in crossing valleys, instead of availing themselves of the principle under consideration, some have supposed that they were unacquainted with this principle. It appears nevertheless that they were acquainted with it, and even understood the use of pipes in conveying water; but probably the difficulty of making pipes strong enough to resist the pressure when laid at a considerable depth below the source, prevented their general use.

347. *Center of pressure.*—The center of pressure of any surface immersed in water, is that point through which passes the resultant of all the pressures on the surface. It is the point, therefore,.at which a single force must be applied, in order to counterbalance all the pressures exerted on the surface. If the surface be a plane, and horizontal, the center of pressure coincides with the center of gravity, because the pressures are equal on every part of it, just as the force of gravity is. But if the plane surface is vertical, the center of pressure is lower than the center of gravity, since the pressure increases with the depth. The place of this center is most easily determined by the calculus. A few results are merely stated here. If the vertical side of a vessel full of water is rectangular, the center is *one-third* of the distance up from the middle of the base to the middle of the upper side. If triangular, with its base horizontal, the center of pressure is *one-fourth* of the distance up from the middle of the base to the vertex. If triangular, with the top horizontal, the center of pressure is *half* way up on the bisecting line.

348. *The loss of weight in water.*—When a body is immersed in water, it suffers a pressure on every side, which is proportional to the depth. The lateral pressures, being at the same depth, balance each other; but this can not be true of the vertical pressures, since the top and bottom of the body are at unequal depths. It is this excess of the upward above the downward pressure which constitutes the *buoyant power* of a fluid. The *loss* of weight in a body immersed in water is *equal to the weight of water displaced.* For before the body was immersed, the water occupying the same space was exactly supported, being pressed upward more than downward by a force equal to its own weight. The weight of the *body*, therefore, is diminished by this same difference of pressures, that is, by the weight of the displaced water.

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* Bigelow's Elements of Technology, p. 308.

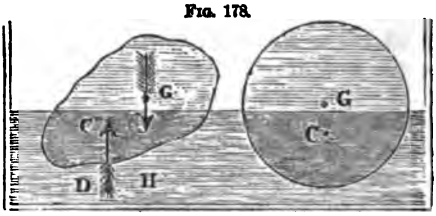
349. On the supposition of the complete incompressibility of water, this loss is the same at all depths, because the weight of displaced water is the same. As water, however, is slightly compressible, its buoyant power must increase a little at great depths. Calling the compression .000046 for one atmosphere (= 34 feet of water), the bulk of water at the depth of a mile is reduced by about $\frac{1}{140}$, and its specific gravity increased in the same ratio; so that, *possibly*, a body might sink near the surface, and float at great depths in the ocean. But this is not *probable* in any case, since the same compressing force may reduce the volume of the solid as much as that of the water. And, furthermore, the increase of condensation in water by increased depth is so slow, that even if solids were incompressible, most of those which sink at all would not find their floating place within the greatest depths of the ocean. For example, a stone twice as heavy as water must sink 100 miles before it could float.

350. *Equilibrium of floating bodies.*—If the body which is immersed has the same density as water, it simply loses its whole weight, and remains wherever it is placed. But if it is less dense than water, the excess of upward pressure is more than sufficient to support it; it is, therefore, raised to the surface, and comes to a state of equilibrium after partly emerging. In order that a floating body may have a stable equilibrium, the three following conditions must be fulfilled:

1. *It displaces an amount of water whose weight is equal to its own.*
2. *The center of gravity of the body is in the same vertical line with that of the displaced water.*
3. *The metacenter is higher than the center of gravity of the body.*

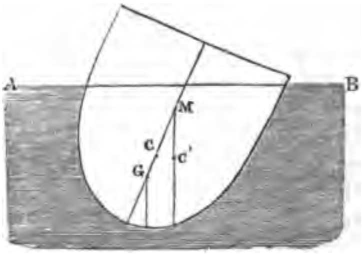
The reason for the *first* condition is obvious; for both the body and the water displaced by it are sustained by the same upward pressures, and therefore must be of equal weight. That the *second* is true, is proved as follows: In Fig. 178, let C be the center of gravity of the displaced water, while that of the entire body is at G. Now the fluid, previous to its removal, was sustained by an upward force equal to its own weight, acting in the vertical line DC; and the same upward force now acts upon the floating body at the point C. But the body, being free to move, is carried downward by a force acting in the direction of the vertical line GH. Were

FIG. 178.



these two forces exactly opposite and equal, they would keep the body at rest; but this is the case only when the points C and G are in the same vertical line: in every other position of these points, the two parallel forces tend to turn the body round. (Art. 57.)

FIG. 179.



351. *The Metacenter.*—To understand the *third* condition, the *metacenter* must be defined. When a floating body is slightly inclined from its state of equilibrium, as in Fig. 179, and a vertical is drawn through the new center of displaced water, C′; this vertical of course intersects the former vertical line, GC; the intersection, M, is called the metacenter. When this point is situated higher than the center of gravity G, as in the figure, it is clear that the upward force through C′, and the downward force through G, tend to restore the body to its position of equilibrium. But if M were lower than G, the rotation would be in the other direction, and the equilibrium is unstable. If the metacenter were at G, the equilibrium would be neutral, and the body would rest indifferently in any position; as, for example, in a sphere of uniform density. (Fig. 178.)

If only the first condition is fulfilled, there is *no* equilibrium; if only the first and second, the equilibrium is *unstable;* if all the three, the equilibrium is *stable.*

In accordance with the third condition, it is necessary to place the heaviest parts of a ship's cargo in the bottom of the vessel, and sometimes, if the cargo consists of light materials, to fill the bottom with stone or iron, called *ballast*, lest the masts and rigging should raise the center of gravity too high for stability. On the same principle, those articles which are prepared for life-preservers, in case of shipwreck, should be attached to the upper part of the body, that the head may be kept above water. The danger arising from several persons standing up in a small boat is quite apparent; for the center of gravity is elevated, and liable to become higher than the metacenter, thus producing an unstable equilibrium.

352. *Floating in a small quantity of water.*—As pressure on a given surface depends solely on the depth, and not at all on the extent or quantity of water, it follows, that a body will float as freely in a space slightly larger than itself, as on the open water of a lake. For instance, a ship may be floated by a few hogsheads of water in a dock whose form is adapted to it.

And it might at first seem absurd to say, in such a case, that the displaced water weighs as much as the vessel, when *all* the water in the dock may not weigh a hundredth part as much. The expression "displaced water" plainly refers to the amount which would fill the place occupied by the immersed portion of the body. An experiment illustrative of the above is, to float a tumbler within another, by means of a spoonful of water between.

353. *Floating of heavy substances.*—A body of the most dense material may float, if it has such a form given it as to exclude the water from the upper side, till the required amount is displaced. Ships are built of iron, and laden with substances of greater specific gravity than water, and yet ride safely on the ocean. A block of any heavy metal, as lead, may be sustained by the upward pressure beneath it, if only the water is excluded from the upper side by a tube.

354. *Specific gravity.*—The weight of a body compared with the weight of the same volume of the standard, is called its *specific gravity.*

Distilled water, at 39°, the temperature of its greatest density, is the standard for all solids and liquids, and common air, at 32°, for gases. Therefore the specific gravity of a solid or a liquid body, is the ratio of its weight to the weight of an equal volume of water; and the specific gravity of an aeriform body, is the ratio of its weight to the weight of an equal volume of air. Hence, to find the specific gravity of a solid or liquid, divide its weight by the weight of the same volume of water; but in the case of a gas, divide by the weight of the same volume of air.

355. *Methods of finding specific gravity.*—1. For a solid heavier than water, *divide its weight by its loss of weight in water.* The loss of weight in a submerged body is (Art. 348) equal to the weight of the displaced water, which of course has the same volume; therefore, dividing the weight of the body by its *loss* of weight in water, is the same as dividing by the weight of the same volume of water. 2. If the solid is lighter than water, it may be attached to a heavy body which is already weighed in water. It now not only loses all its own weight, but occasions loss in the heavy body. This loss, plus its own weight, is therefore equal to the weight of the water which it displaces. Hence, to find the specific gravity of a body lighter than water, *divide its weight by its weight added to the loss it occasions to a heavier body previously balanced in water.* 3. For a liquid, *weigh equal volumes of the liquid and of water, and divide the former weight by the latter;* or, *weigh* *a heavy body in each, and divide the loss in the liquid by the loss in water.* For the loss in each case equals the weight of the displaced fluid.

356. *The hydrometer, or areometer.*—In commerce and the arts, it is common to obtain the specific gravities of substances in a more direct way, by instruments constructed for the purpose. The general name for such instruments is the *hydrometer*, or *areometer*. But other names are given to such as are limited to particular uses; as, for example, the *alcoömeter*, for testing the purity of alcohol. The hydrometer, represented in Fig. 180, consists of a hollow ball, with a long slender stem; and since it will sink until it has displaced a quantity of the fluid equal in weight to itself, it will of course sink to a greater depth the lighter the fluid. From the depths to which it sinks, therefore, as indicated by the graduated stem, the corresponding specific gravities are estimated. *Nicholson's hydrometer* (Fig. 181) is the most useful of this class of instruments, since it may be applied to finding the specific gravities of solid bodies. In addition to the hollow ball of the common hydrometer, it is furnished with a dish AB for receiving weights, and a cavity beneath for holding the substance under trial. The instrument is so adjusted that when 1000 grains are placed in the upper dish, the whole will sink in distilled water at the temperature of 40° Fahr. to a mark m on the stem. If, therefore, in the case of different *fluids*, we add weights until m stands at the level of the fluids, the quantities displaced will be the same in all, and the weight of each quantity will be that of the instrument W together with the weight w added to the dish, which will constitute the *numerator* of the fraction expressing the specific gravity (Art. 354), while W + 1000 expresses the *denominator*, being the weight of an equal bulk of water; and since the weight of the instrument is known, the only thing to be determined in any case is the weight to be added, in order to sink the point m to the level of the fluid. Then $\frac{W+w}{W+1000}$ = the specific gravity.

FIG. 180.

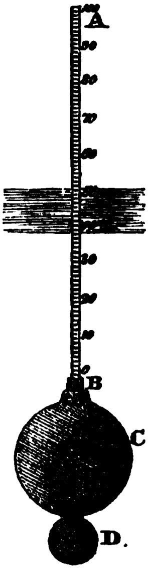
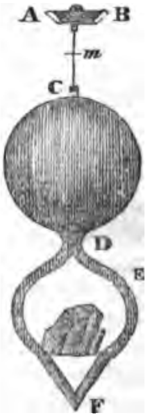


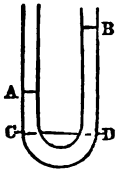
FIG. 181.



To determine the specific gravity of a *solid* body not exceeding 1000 grains in weight, we first place the instrument in water, put the body into the dish, and add weights to sink the point *m* to the level of the water. Then, since the weight of the body itself and the weight added, together equal 1000 grains, the weight of the body = 1000 — the weight added. Now we place the body below, and ascertain what weight must be added to the dish to sink it to the same level as before. This weight will show the loss of weight of the body in water; and the weight in air, divided by the loss of weight in water, gives the specific gravity. If the stem of the instrument is only one-fortieth of an inch in diameter, it will rise or fall nearly one inch by the subtraction or addition of one-tenth of a grain, and thus give the specific gravity correct to five places of decimals.

357. The specific gravity of liquids may be obtained conveniently, though not with great accuracy, by means of a recurved tube and scale. If, in Fig. 182, the columns AC of one fluid, and BD of another, are in equilibrio, their specific gravities are inversely as their heights. For, when the heights of two columns are equal, their pressures on a given area are obviously as their densities, or specific gravities; and if heights vary, we have seen that pressures are as those heights; hence, if the pressures are *equal*, the specific gravities must vary inversely as the heights. The heaviest liquid should be poured in first, till it stands somewhat above CD, the zero mark of the graduated scale; and then the lighter one should be poured slowly into one branch, till it presses the other liquid down to the zero line. The heights of both are reckoned upward from CD, since the heavy liquid below CD balances itself.

FIG. 182.



358. *Table of specific gravities.*—An accurate knowledge of the specific gravities of bodies is of great use for many purposes of science and the arts, and they have therefore been determined with the greatest possible precision. The heaviest of all known substances is platinum, whose specific gravity, in its state of greatest condensation, is 22, water being 1; and the lightest of all ponderable bodies is hydrogen gas, whose specific gravity is .073, common air being 1. Also, air is 818 times lighter than water. Hence, by calculation, it will be found that platina is about 247,000 times as heavy as hydrogen, and a wide range is allowed to the various bodies which lie between these extremes. The metals, as a class, are the heaviest bodies; next to these come the metallic ores; then the precious gems; and finally, minerals in general, animal, liquid, and vegetable substances, in order, according to the following table:

Metals (pure), not including the bases of the alkalies and earths, from	5—22

Platinum	22.0	Copper	8.90
Gold	19.25	Steel	7.84
Mercury	13.58	Iron	7.78
Lead	11.35	Tin	7.29
Silver	10.47	Zinc	7.00

Metallic ores, lighter than the pure metals, but usually above	4.00
Precious gems, as the ruby, sapphire, and diamond	3—4
Minerals, comprehending most stony bodies	2—3
Liquids, from ether highly rectified to sulphuric acid highly concentrated	¾—2
Acids in general, heavier than water.	
Oils " lighter; but the oils of cloves and cinnamon are heavier than water; the greater part lie between .9 and 1	.9—1
Milk	1.032
Alcohol (perfectly pure)	.797
" of commerce	.835
Proof spirit	.923
Wines; the specific gravity of the lighter wines, as Champagne and Burgundy, is a little less, and of the heavier wines, as Malaga, a little greater than that of water.	
Woods, cork being the lightest, and lignum vitæ the heaviest	.24—1.34

359. The human body, when the lungs are filled with air, is lighter than water, and but for the difficulty of keeping the lungs constantly inflated, it would naturally float. With a moderate degree of skill, therefore, swimming becomes a very easy process, especially in salt water. When, however, a man plunges, as divers sometimes do, to a great depth, the air in the lungs becomes compressed, and the body does not rise except by muscular effort. The bodies of drowned persons rise and float after a few days, in consequence of the inflation occasioned by putrefaction. Quadrupeds swim much more easily than man, because the motions of the limbs necessary to sustain themselves nearly coincide with their natural motions in walking, while the body maintains its usual posture.

Rocks and stones being only a little more than twice as heavy as water, of course nearly half their weight is sustained while they are immersed in water; and hence the increased weight which is felt when a large stone is lifted from the bed of a river, as soon as it reaches the surface. Large masses of rocks are transported with far greater facility by torrents, on account of their diminished weight. On the same principle, the limbs feel very heavy after lying for some time in a bath.

360. The *magnitude* of bodies may often be most conveniently and accurately estimated from the doctrine of specific gravities. Suppose we wish to ascertain the exact number of solid inches contained in a stone of rude and irregular shape, we should find great difficulty in applying to it any linear measurement; but if we ascertain its loss of weight in water, then we have the weight of an equal bulk of water, and since 1000 ounces contain 1728 cubic inches, we may easily find how many cubic inches correspond to the weight of water of equal magnitude with the body in question.

361.—*Practical questions in hydrostatics.*—

1. In a hydrostatic press (Fig. 171) the height of the small column AB on which the power acts is 2 feet above the bottom of the large piston CD; the diameter of the cylinder AB is 1 inch, and of the cylinder CD 1 foot. By means of a screw turned by a lever, a man can exert a force on A equal to 500 pounds; what amount of pressure can he apply with the aid of this press, at the beginning of each stroke, combining his own strength with the pressure of the column of water AB?

Ans. 72098.17 lbs.

2. A junk bottle, whose lateral surface contained 50 square inches, was let down into the sea 3000 feet; what pressure would the sides of the bottle sustain, no allowance being made for the increased specific gravity of the sea-water?

Ans. 65104.166 lbs.

3. A Greenland whale sometimes has a surface of 3600 square feet; what pressure would he bear at the depth of 800 fathoms?

Ans. 1080,000,000 lbs., or more than 482,142 tons.

4. A mill-dam, running perpendicularly across a river, slopes at an angle of 25 degrees with the horizon. The average depth of the stream is 12 feet, and its breadth 500 yards; required the amount of pressure on the dam?

Ans. 15,971,906 lbs., or 7130+ tons.

5. A mineral weighs 960 grains in air, and 739 grains in water; what is its specific gravity? *Ans.* 4.344.

6. What are the respective weights of two equal cubical masses of gold and cork, each measuring 2 feet on its linear edge? *Ans.* The gold weighs 9625 lbs. = 4.297 tons; the cork weighs 120 lbs.

7. On one of the peaks of the Alps, is a single mass of granite rock of a nearly globular shape, which is estimated by measure to contain 5949 cubic feet. The whole mass is so nicely balanced on its center of gravity, that a single man may give it a rocking motion. By trial made upon a small fragment, its specific gravity was found to be 2.6; what is its weight?

Ans. 431.568 tons.

8. An island of ice rises 30 feet out of water, and its upper surface is a circular plane, containing $\frac{3}{4}$ths of a square acre. On the supposition that the mass is cylindrical; required its

weight, and depth below the water, the specific gravity of sea-water being 1.0263, and that of ice .92?

Ans. Weight 242,900 tons; depth 259.64 feet.

9. Wishing to ascertain the exact number of cubic inches in a very irregular fragment of stone, I ascertained its loss of weight in water to be 5.346 ounces; required its dimensions?

Ans. 9.238 cubic inches.

10. Hiero, king of Syracuse, ordered his jeweller to make him a crown of gold containing 63 ounces. The artist attempted a fraud by substituting a certain portion of silver; which being suspected, the king appointed Archimedes to examine it. Archimedes, putting it into water, found it raised the fluid 8.2245 inches; and having found that the inch of gold weighs 10.36 ounces, and that of silver 5.85 ounces, he discovered what part of the king's gold had been purloined; it is required to repeat the process? *Ans.* 28.8 ounces.

11. The specific gravity of lead being 11.35; of cork, .24; of fir, .45; how much cork must be added to 60 lbs. of lead, that the united mass may weigh as much as an equal bulk of fir?

Ans. 65.8527 lbs.

12. A cone, whose height is 6 inches, is immersed in water, with its vertex downward. Its specific gravity being $\frac{1}{8}$th that of the water, what part of the axis will be immersed?

Ans. One-half.

13. What is the weight of a chain of pure gold, which raises the water 1 inch in height, in a cubical vessel whose side is 3 inches? and suppose a chain of the same weight were adulterated with $14\frac{1}{2}$ oz. of silver, how much higher would it raise the water in the vessel?

Ans. Weight = 91.35 oz.; height .133 in. more.

CHAPTER II.

LIQUIDS IN MOTION.

362. *Definitions and remarks.*—That branch of Natural Philosophy which treats of liquids *in motion*, is usually denominated *hydraulics*, from ὕδωρ, *water*, and αὐλὸς, *a torrent.* It embraces, therefore, the phenomena exhibited by water discharged from orifices in reservoirs—projected obliquely or perpendicularly—flowing in pipes, canals, and rivers—oscillating in waves—or opposing a resistance to the progress of solid bodies.

In this subject, the deductions of *theory alone* are of little value, and are in fact so discordant with experience, that little reliance can be placed on them, except as modified by experiment. When thus modified, the truths learned respecting the laws that govern the motions of liquids, have a high degree of practical utility.

The causes of this discordance between theory and experiment are such as the following: the different states in which the same fluid is found in consequence of changes of temperature, and different degrees of purity—the attraction existing between its particles and preventing that perfect fluidity which is contemplated by the definition, and upon which our reasonings are founded—the friction against the sides of the vessel—the resistance of the air—the size of the vessel in proportion to the aperture—the shape of the aperture itself—its position in relation to the sides of the vessel—and the vortices, or irregular motions which are communicated to the fluid by a variety of causes. The lines described by the particles of a fluid in motion are called *filaments*.

363. *Depth and velocity of discharge.*—From an aperture which is small compared with the reservoir, the *velocity* of discharge varies as the *square root of the depth*. For the *pressure* on a given area varies as the depth (Art. 342); if the area is removed, this pressure is a moving force to produce momentum; therefore *momentum* varies as the depth (D). But momentum varies as quantity (Q) into velocity (V); hence $QV \propto D$. But it is obvious, that Q and V vary alike, since the greater the velocity of discharge, the greater, in the same ratio, is the quantity discharged. $\therefore Q^2 \propto D$, or $Q \propto D^{\frac{1}{2}}$; also $V^2 \propto D$, or $V \propto D^{\frac{1}{2}}$.

It follows, that the velocity of discharge at any point is equal to that acquired by a body falling from the top of the fluid to the place of discharge. For, in falling bodies, the velocity varies as the square root of the distance; and when the depth of water is indefinitely small, it is obvious that the velocity of discharge is identical with that of a body *beginning* to fall. Hence, the two series *begin* with the same value, and *increase* by the same law; they are therefore equal throughout. A body having fallen $16\frac{1}{12}$ feet, and water discharged at the depth of $16\frac{1}{12}$ feet, each has a velocity of $32\frac{1}{6}$ feet per second. An orifice 16 inches below the surface will discharge *twice* as much water in a given time as one at the depth of 4 inches; and, in order to draw off *five* times as much, we must place the gate *twenty-five* times as deep. Since a body, when projected upward with a certain velocity, will rise to the same height as that from which it must have fallen to acquire that velocity, consequently, if a fluid issue from the side of a vessel through a spout bent upward, it would, were it not for the resistance of the air, and friction at the orifice, rise to the level of the fluid in the reservoir.

As the velocity of discharge at any depth is equal to that of a body which has fallen a distance equal to the depth, it is theoretically immaterial whether water is taken upon a wheel from a gate at the same level, or allowed to fall on the wheel from the top of the reservoir. In practice, however, the former is best, on account of the resistance which water meets with in falling through the air.

364. *Descent of surface.*—When water is discharged from the bottom of a cylindric or prismatic vessel, the surface descends with a *uniformly retarded* motion. For the velocity with which the surface descends varies as the velocity of the stream, and therefore as the square root of the depth (Art. 363). But this is a characteristic of uniformly retarded motion, that it varies as the square root of the distance from the point where the motion terminates, as we have seen in the case of a body ascending perpendicularly from the earth.

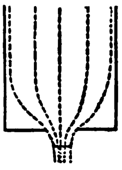
The descent of the surface of water in a prismatic vessel has been used for measuring time. The *clepsydra*, or water-clock of the Romans, was a time-keeper of this description. The graduation must increase upward, as the odd numbers 1, 3, 5, 7, &c.; since, by the law of this kind of motion, the spaces passed over in equal times are as those numbers.

If a prismatic vessel is kept full, it discharges *twice* as much water in the same time as if it is allowed to empty itself. For the velocity, in the first instance, is *uniform;* and in the second, it is *uniformly retarded*, till it becomes zero. We reason in this case, therefore, as in regard to bodies moving uniformly, and with motion uniformly accelerated from rest, or uniformly retarded till it ceases (Art. 25), that the former motion is *twice* as great as the latter.

365. *Discharge from orifices in different situations.*—Other circumstances besides *area* and *depth* of the aperture, are found to have great influence on the velocity of discharge. Observations on the directions of the filaments are made, by introducing into the water fragments of some opaque substance, having the same density as water, whose movements are visible. From such observations it appears that the particles of water descend in vertical lines, until they arrive within three or four inches of the aperture, when they gradually turn in a direction more or less oblique toward the place of discharge. This convergence of the filaments extends outside of the vessel, and causes the stream to diminish for a short distance, and then increase. The smallest section of the stream, called the *vena* *contracta*, is at a distance from the aperture, varying from *one-half* its diameter to the *whole*.

366. If water is discharged through a circular aperture in a thin plate in the bottom of the reservoir, and at a distance from the sides, as in Fig. 183, the filaments form the vena contracta at a distance beyond the aperture equal to *one-half* its diameter; the area of the section at the vena contracta is less than *two-thirds* (0.64) the area of the aperture; and the quantity discharged is about 0.62 of that obtained by calculation for the full size of the aperture.

Fig. 183.



If the form of the reservoir is adapted to the curvature of the filaments, as far as to the vena contracta, or a little beyond, as in Fig. 184, it is found the most favorable for free discharge, which in some cases reaches 0.98 of the theoretical discharge. The stream is smooth and pellucid like a rod of glass. The most unfavorable form, is where the pipe, instead of being external, as in the case just described, projects inward, as in Fig. 185; the filaments in this case reach the aperture, some ascending, others descending, and therefore interfere with each other; hence the stream is much roughened in its appearance, and the flow is only 0.53 of what is due to the size of the aperture and its depth.

Fig. 184.

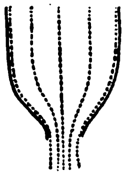
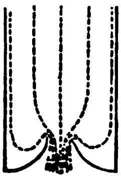
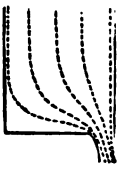


Fig. 185.



When the aperture is through a thin plate, the contraction of the stream, and the amount of discharge are both modified by the circumstance of being near one or more sides of the reservoir. There is little or no contraction on the side next the wall of the vessel, since the filaments have no obliquity on that side; and the quantity is on that account increased. The filaments from the opposite side also divert the stream a few degrees from the perpendicular.* (See Fig. 186.)

Fig. 186.



367. *Friction in pipes.*—As has just been stated, a pipe projecting a little beyond the vena contracta, and adapted to the form of the stream, very much increases the quantity discharged; but beyond that, the longer the pipe, the more does it impede the discharge by friction. The friction varies directly as the length of the pipe, and inversely as its diameter. In order, therefore, to convey water at a given rate through a long pipe, it is necessary either to increase the head of water, or to enlarge the pipe, so as to compensate for friction. If a given quantity of water is discharged per second by an aperture five inches in diameter, through the thin bottom of a reservoir, and we wish to discharge the same at the end of a horizontal pipe 150 feet long, of the same diameter as the orifice, it will require *ten* times the head of water to accomplish it; or it may be done by the same head of water, if we use a pipe 7.9 inches, instead of 5 inches, in diameter.

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* Weisbach's Mechanics and Engineering, vol. i.

An aqueduct should be as *straight* as possible, not only to avoid unnecessary increase of length, but because the force of the stream is diminished by all changes of direction. If there must be change, it should be a gradual curve, and not an abrupt bend. When a pipe changes its direction by an *angle*, instead of a curve, there is a useless expenditure of force; a change of 90° requires that the head of water should be increased by nearly the height due to the velocity of discharge. For instance, if the discharge is *eight* feet per second (which is the velocity due to one foot of fall), then a right angle in the pipe requires that the head of water should be increased by nearly *one* foot, in order to maintain that velocity.*

368. *Change of velocity in a pipe of variable size.*—If a pipe is kept full by a current which flows through it, then the *velocity varies inversely as the area of the section.* It is obvious that the quantity (Q) passing any section in a given time, varies as the area of the section (A), and also as the velocity (V); that is, $AV \propto Q$. But the same quantity must pass through every section in the same time; therefore, since Q is constant, AV is constant also. Hence, A varies inversely as V. When a pipe is more expanded in one part than another, the water, which enters the expanded part, moves as much slower as the section is greater. Though the momentum is not affected by this change, yet the *vis viva*, which is the measure of the *working power* of the water, is *diminished;* for the vis viva varies, not as AV, but as AV^2. (Art. 34.) Hence the economy of having an aqueduct uniform in size, as well as direct in its course.

369. *Jets.*—If water is discharged from an orifice in any other than a vertical direction, it describes a parabola, since each particle may be regarded as a projectile. (Art. 49.)

If a semicircle be described on the perpendicular side of a vessel as a diameter, and water spout horizontally from any point, its *range*, measured on the level of the base, equals *twice the ordinate* of that point. For, the velocity with which the fluid issues from the vessel, being that which is due to the height BG (Fig. 187), is such as if preserved would carry the

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* Weisbach.

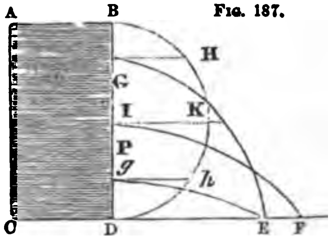


FIG. 187.

jet through a space equal to 2BG, in the time of the fall through BG; but after leaving the orifice, it arrives at the horizontal plane in the same time as that in which a body would fall freely through GD. And since, in falling bodies, the times are as the square roots of the spaces (Art. 23), therefore $\sqrt{BG} : \sqrt{GD} ::$ time down BG : time down GD, or to the plane DF. But in the time of describing BG, the jet would be carried uniformly and horizontally over a space equal to 2BG; therefore, to find how far it will proceed in a horizontal direction, while it is descending through GD, we have

$$\sqrt{BG} : \sqrt{GD} :: 2BG : \frac{2BG \times \sqrt{GD}}{\sqrt{BG}} = 2\sqrt{(BG \times GD)} = 2GH,$$

that is, double the ordinate to the point G. Let $DE = 2GH$, and the jet will reach the point E.

The greatest range occurs when the fluid spouts from the center, for then the ordinate (and of course twice the ordinate) is greatest; and the range at equal distances above and below the center, is the same.

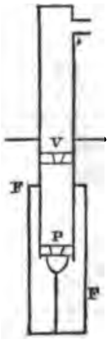
The remarks already made respecting pipes, apply to those which convey water to the jets of fire-engines and fountains. If the pipe or hose is very long, or narrow, or tortuous in its direction, or if the jet-pipe is not smoothly tapered from the full diameter of the hose to the aperture, much force is lost by friction and other resistances, especially in great velocities. If the length of hose is even *twenty* times as great as its diameter, 32 per cent. of height is lost in the jet, and more still, when the proportion of length to diameter is greater than this.

370. *Rivers.*—Friction and change of direction have great influence on the flow of rivers. A *dynamical equilibrium*, as it is called, exists between gravity, which causes the descent, and the resistances, which prevent acceleration, beyond a certain moderate limit; so that, in general, the water of a river moves uniformly. The velocity in all parts of the same section, however, is not the same; it is greatest at that part of the surface where the depth is greatest, and least in contact with the bed of the stream.

To find the mean *velocity* through a given section, it is necessary to float bodies at various places on the surface, and also below it, to the bottom, and to divide the sum of all the velocities thus obtained, by the number of observations. To obtain the *quantity* of water which flows through a given section of a river, having determined the velocity as above, find next the area of the section, by taking the depth at various points of it, and multiplying their mean by the breadth. The quantity of water is then found by multiplying the area by the velocity.

371. The increased velocity of a stream during a freshet, while the stream is confined within its banks, exhibits something of the acceleration which belongs to bodies falling freely down an inclined plane. It presents the case of a river flowing upon the top of another river, and consequently meeting with much less resistance than when it runs upon the rough unequal surface of the earth itself. The augmented force of a stream in a freshet, arises from the simultaneous increase of the quantity of water and the velocity. In consequence of the friction of the banks and beds of rivers, and the numerous obstacles they meet with in their winding course, their progress is very slow, whereas, were it not for these impediments, it would become immensely great, and its effects would be exceedingly disastrous. A very slight declivity is sufficient for giving the running motion to water. Three inches per mile, in a smooth, straight channel, gives a velocity of about three miles per hour. The Ganges, which gathers the waters of the Himalaya Mountains, the loftiest in the world, at the distance of eighteen hundred miles from its mouth, is only eight hundred feet above the level of the sea,—that is, about twice the height of St. Paul's church in London; and to fall these eight hundred feet, in its long course, the water requires more than a month. The great river Magdalena, in South America, running for a thousand miles between two ridges of the Andes, falls only five hundred feet in all that distance. The Croton aqueduct, that waters the city of New York, falls one foot per mile. The smallest inclination capable of giving motion to water is a descent of one foot in a million; or about $\frac{1}{15}$th of an inch per mile. A fall of 3 feet per mile makes a mountain torrent.

FIG. 188.



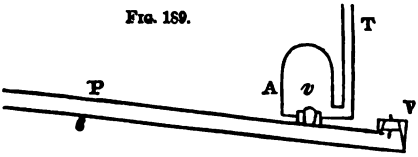
372. *Hydraulic pumps.*—The most common pumps for raising water operate on a principle of Pneumatics, and will be described in the chapter on that subject.

In the *lifting pump* (Fig. 188), the piston P, which contains a valve opening upward, works beneath the fixed valve V, by means of the frame FF, which is raised and lowered by a lever, as the piston-rod of other pumps. The piston and the fixed valve are both below the water level. When the piston descends, the water passes through its valve, and when it rises, it presses the water before it through the fixed valve, which then prevents its return. In this manner, by repeated strokes, the water can be driven to any height which the instrument can bear.

The *chain pump* consists of an endless chain with circular disks attached to it at intervals of a few inches, which raise the water before them in a tube, by means of a wheel over which the chain passes; the wheel may be turned by a crank. The disks can not fit closely in the tube, without causing too great resistance; hence, the water will soon descend to the level in the well, after the pump ceases working, and a certain velocity is requisite in order to raise it to the place of discharge.

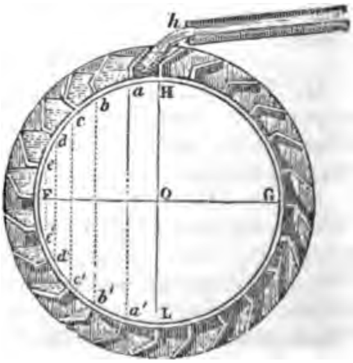
373. *The hydraulic ram.*—When a large quantity of water is moving through a pipe, and the extremity of the pipe is suddenly closed, since the water is nearly incompressible, the shock of the whole is received at the same instant, and if no escape is provided, is very likely to burst the pipe. The intensity of the shock of water when stopped is made the means of raising a portion of it above the level of the head. The instrument for effecting this, is called the *hydraulic ram.* At the lower end of a long pipe, P (Fig. 189), is a valve, V, opening downward; near it, another valve, *v*, opens into the air-vessel, A; and from this ascends the pipe, T, in which the water is to be raised. As the valve V lies open by its weight, the water runs out, till its momentum at length shuts it, and the entire column is suddenly stopped; this impulse forces the water into the air-vessel, and thence by the compressed air up the tube T. As soon as the momentum is expended, the valve V drops, and the process is repeated.

FIG. 189.



374. *Water-wheels.*—Water-wheels which revolve on a horizontal axis are of three kinds, called the *overshot*, the *under-shot*, and the *breast wheel.*

FIG. 190.

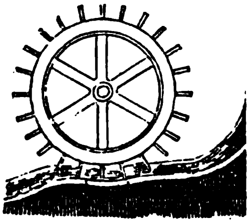


The *overshot-wheel* (Fig. 190), is constructed with buckets on the circumference, which receive the water just after passing the highest point, and empty themselves near the bottom. The weight of the full buckets on one side of the wheel, while those on the opposite side are empty, gives motion to the wheel. It is usually made as large as the fall will allow, and will carry machinery with a very small supply of water, if the fall is only considerable. The *moment* of each bucket-full constantly increases from *a*, where it is filled, to F, where its acting distance is radius, and therefore a maximum. From F downward, the moment decreases, both by loss of water and diminution of acting distance, and becomes zero at L.

There is a certain velocity with which an overshot-wheel should move in order to produce the greatest effect. If, on the one hand, the wheel is loaded so heavily that the weight of water is insufficient to move it, then of course the effect is nothing; and if, on the other hand, the velocity of the wheel were to equal that with which water would fall freely, then its pressure on the buckets would become nothing, and its moving power nothing. The best velocity that can be given to an overshot-wheel is found to be three feet per second.

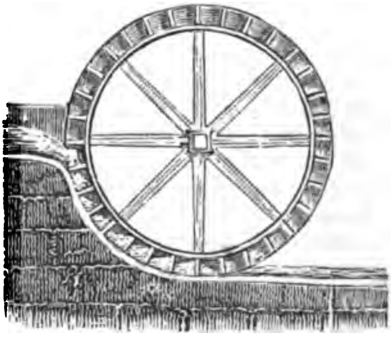
375. The *undershot-wheel* (Fig. 191), is carried, not by the weight of the water simply, as is the case in the overshot-wheel, but by the momentum of running water. Instead of close buckets for holding water, it is furnished with *float-boards*, which receive the impulse of the stream. When these are placed, as in the figure, with their planes at right angles to the rim of the wheel, the latter may turn either way; and this therefore, is the form of wheels employed in tide-mills. When the wheel is required to turn only in one direction, an advantage is gained by placing the float-boards so as to present an acute angle toward the current, by which means the water acts partly by its weight, as in the overshot-wheel. The undershot-wheel is adapted to situations where the supply of water is always abundant. It acts, moreover, with the greatest effect, when its velocity is half that of the stream.

FIG. 191.



The *breast-wheel* (Fig. 192) combines the advantages of both the others, and is therefore adapted to situations where the supply of water is generally sufficient, but not always abundant. The planes of the float-boards are at right angles to the rim of the wheel, and

FIG. 192.

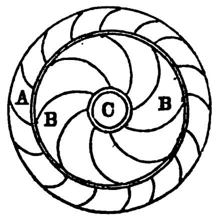


are brought so near to the mill-course that the float-boards hold water like buckets.

According to Smeaton, the effect of overshot-wheels, under the same cicumstances of quantity and fall, is, at a medium, double that of an undershot-wheel.

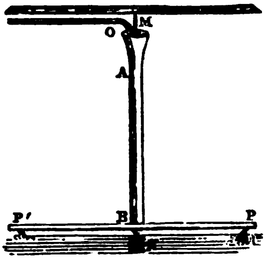
376. *The turbine.*—This is the name given to the most approved form of water-wheel, running on a vertical axis. Figure 193 shows a horizontal section through the wheel and the reservoir of water which supplies it. BB is a cylindrical reservoir, of such depth as to contain the required head of water, having openings all around it near the bottom, through which the water is discharged horizontally. In the lower part of the reservoir, are curved partitions, which give an oblique direction to the water as it issues. The wheel AA, closely surrounds these openings, presenting the concave side of its curved float-boards to the impulse of the water. These float-boards are attached to a rim closing them in on the outside, and to a covering on the upper side, while the under side is entirely open, so that the water may fall as soon as its impulse is given. The wheel is connected to its shaft by oblique spokes under the reservoir; and the shaft C, being supported at the lower end by a socket on the ground, passes up through an open tube in the center of the reservoir, to connect with the machinery above. Since the reservoir can have no support from below, it is suspended by a strong flange near the top, upon the masonry which surrounds the wheel.

FIG. 193.



377. *Barker's mill.*—This machine operates on the principle of *unbalanced hydrostatic pressure.* It consists of a vertical hollow cylinder, AB (Fig. 194), free to revolve on its axis MN, and having a horizontal tube connected with it at the bottom. Near each end of the horizontal tube, at P and P′, is an orifice, one on one side, and one on the opposite. The cylinder, being kept full of water, whirls in a direction opposite to that of the discharging streams from P and P′. This is owing to the fact that hydrostatic pressure is removed from the apertures, while on the interior of the tube, at points exactly opposite to them are pressures which are now unbalanced, but which would be counteracted by the pressures at

FIG. 194.



the apertures, if they were closed. The centrifugal force, after the machine is in rotation, has the effect to increase the pressure, and therefore the speed of rotation.

CHAPTER III.

CAPILLARITY, RESISTANCE OF FLUIDS, AND WAVES.

378. *Adhesion.*—What distinguishes a liquid from a solid, is not its want of cohesion so much as the mobility of its particles. It is proved in many ways, that the particles of a liquid strongly attract each other. It is owing to this that water so readily forms itself into drops. The same property is still more observable in mercury, which, when minutely divided, will roll over surfaces in forms most perfectly globular. When a disk of almost any substance is laid upon water, and then raised gently, it lifts a column of water after it, till at length the edge of the fluid begins to divide, and the column is detached, not in all parts at once, but by a successive rupturing of the lateral surface. It is proved that the whole attraction of the liquid would be far too great to be overcome by the force applied to pull off the disk, were it not that it is encountered by little and little, at the edges of the column.* But it is the fluid itself which has been separated in this experiment; for the upper lamina still cleaves to the disk, showing that the *adhesion* of the water to the disk is a stronger force than the *cohesion* between the particles of water. By a pair of scales we find that it requires the same force to draw off disks of a given size, whatever the materials may be, provided they are *wet* when detached. This is what might be expected, since in each case we break the attraction between two laminæ of water. But if we use disks which are not wet by the liquid, it is not generally true that those of different material will be removed by the same force; indicating that some substances adhere to a given liquid more strongly than others.

379. If a solid is wet by a certain liquid, the latter is lifted above its level when in contact with it, presenting a concave surface upward; as, for example, water by the side of the vessel containing it. But a liquid which does not wet a substance is depressed in the same circumstances, so that the edge is convex. This is the condition of mercury in contact with most solids.

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* Henry.

380. *Capillary action.*—The influence exerted by fine (*hair-like*) tubes to raise or depress liquids in them, is called *capillary action.* Whenever a liquid *wets* a solid, it will *rise* above its level in a fine tube made of that solid; when it will *not wet* it, it will be *depressed* in a capillary tube. If a tube of glass about one-tenth of an inch in diameter is dipped in water, not only are the edges raised externally and internally, but we can perceive that the whole internal column is a little elevated above the general level; and if a smaller tube is used, the column is higher. But if the same tube is plunged into mercury, the column is depressed, and the depression is greater, the more slender the tube. The principal facts relating to capillarity are the following:

1. A fine tube, when dipped into a liquid, will *raise* or *depress* it, according as it is capable of being *wet* or *not* by the liquid.
2. The substances being the same, the perpendicular *elevation* or *depression* varies *inversely as the diameter* of the tube.
3. It follows from (2) that the *product* of the *height* by the *diameter* is *constant* for given substances. The constant for glass and water is .053 sq. in. Hence, we may determine the diameter of a fine glass tube by measuring the height to which water rises in it, and dividing .053 by the height. If, for example, it rises 8 inches, the diameter is .0066 in.
4. The amount of elevation or depression does *not* depend on the *specific gravity* of the liquid. Alcohol, though lighter than water, does not rise half as high in a capillary tube.
5. Nor does the height depend on the *thickness* of the material of which the tube is made. The action seems to be due only to the particles which are contiguous to each other.
6. Fluids are raised or depressed *between plates half as far* as in tubes of the same diameter; for between plates, each vertical filament has the capillary force acting only on two sides of it, while in a tube it acts on all sides. If the plates meet in a vertical edge, the surface of the water forms a *hyperbola,* in consequence of the law, that the height varies inversely as the distance between the plates.
7. If the upper part of a tube is capillary, water is sustained to the same height, whatever the diameter of the lower part. This is found to be owing to the pressure of the air, which sustains the liquid in the large portion, after the slender portion has been closed by capillary attraction.

381. Various *phenomena* in nature and art are explained upon the principles of capillary attraction. Capillary action is not confined to tubes, but is exerted among all substances which are perforated by pores, or subdivided by fissures or interstices. On this power depend chiefly the functions of the excretory vascular system in plants and animals, and hence also the ascent of humidity through the shivered fragments of rocks, unglazed pottery, gravel, earth, and sand. Thus if the pores of the human skin (which are known to be exceedingly small), are estimated at the $\frac{1}{3000}$th part of an inch in diameter, they will support the fluids that circulate through them to the height of 120 inches, or ten feet, or higher than is required for the animal system. The ascent of the sap in trees has sometimes been ascribed to capillary attraction, their circulating vessels being a congeries of small tubes; but it is now more correctly maintained that this action is dependent, not on the mechanical structure, but upon something which is denominated the *living principle* of vegetables. Capillary action only aids this force.

Several familiar examples of capillary attraction may be added. A piece of sponge, or a lump of sugar, touching water by its lowest corner, soon becomes moistened throughout. The wick of a lamp lifts the oil to supply the flame, to the height of several inches. A capillary glass tube, bent in the form of a syphon, and having its shorter end inserted in a vessel of water, will fill itself and deliver over the water in drops. A lock of thread or of candlewick, inserted in a vessel of water in a similar manner, with one end hanging over the vessel, will exhibit the same result. An immense weight or mass may be raised through a small space, by first stretching a dry rope between it and a support, and then wetting the rope.

382. *Resistance to motion in a liquid.*—The resistance which a body encounters in moving through any fluid, arises from the inertia of the particles of the fluid, their want of perfect mobility among each other, and friction. Only the first of these admits of theoretical determination. So far as the inertia of the fluid is concerned, the *resistance* which a surface meets with in moving perpendicularly through it, *varies as the square of the velocity.* For the resistance is measured by the *momentum* imparted by the moving body to the fluid. And this momentum varies as the product of the quantity of fluid set in motion (Q), and its velocity (V); or $M \propto QV$. But it is obvious that the quantity displaced varies as the velocity of the body, or $Q \propto V$; hence $M \propto V^2$. Therefore the resistance varies as the square of the velocity.

This proposition is found to hold good in practice, where the velocity is small, as the motions of boats or vessels in water; but when the velocity becomes very great, as that of a cannon-ball, the resistance increases in a much higher ratio than as the square of the velocity. Since action and reaction are equal, it makes no difference, in the foregoing proposition, whether we consider the plane in motion and the fluid at rest, or the fluid in motion and striking against the plane at rest.

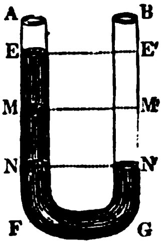
383. On account of the rapidity with which the resistance increases as the velocity is augmented, when a vessel or a steamboat is moving in water, it is only a comparatively moderate velocity that can possibly be given to it. A vessel driven by a wind which moves 60 miles an hour, is not carried forward faster than at the rate of 12 or 14 miles per hour. Steamboats are sometimes urged forward at the rate of 16 miles an hour; but to gain the additional speed over and above 12 miles, requires a great expenditure of force. If a steam-engine of 20 horse-power give a motion of 4 miles an hour, it would require one of 180 horse-power to increase the speed to 12 miles an hour. But, it must be observed, that the resistance decreases as fast when the velocity is diminished, as it increases when the velocity is augmented; and consequently, that canals may have the advantage over railways, when heavy articles are to be transported by very slow motions, although railways, encountering only the resistance of the air instead of water, have greatly the advantage when the motion is swift. A cannon-ball, on the other hand, meets with so much resistance on striking the water as to rebound.

Reckoning the resistance to increase only as the square of the velocity, it follows that twice the speed encounters four times the resistance; four times the speed, sixteen times; and ten times the speed, one hundred times the resistance. Hence a body descending freely through the air by gravity, for a great distance, does not continue to be accelerated throughout the whole distance, but is finally brought, by the resistance of the air, to a uniform motion.

384. *Waves.*—These are moving elevations of water, produced by a force which acts unequally on the surface. Water waves are of two kinds, *waves of oscillation*, and *waves of translation*. In the first kind the water has a vibratory or reciprocating motion, principally in a vertical direction, by which the columns are alternately lengthened and shortened. *Sea waves*, when formed on deep water, are the most common example of this species of wave. In the wave of translation, the particles of water are transferred forward to a new position, without any vibratory movement.

385. *Waves of oscillation.*—Sir Isaac Newton first observed the analogy between the motions of waves and the vibrations of a column of water in a recurved tube, and upon this analogy he founded his theory of waves. Let AFGB (Fig. 195) be a bent tube, of equal bore throughout, having its sides parallel to each other and perpendicular to the horizon. Suppose it to be filled with water or any fluid to the height MM′. By any pressure applied at M′, let the column be depressed to N′ and raised to E in the opposite arm. The pressure being removed, the longer column EF will preponderate and seek to regain its original level; but the ascending column will not stop at M′, but, on account of its inertia, will ascend to E′, that is, to the same height as that from which it descended on the other side. It will now descend again, and these reciprocal motions will continue until destroyed by the natural impediments to motion. On account of these, each successive vibration is shorter than the preceding, but all of them, like those of a pendulum, are performed in equal times; for the moving force is obviously proportioned to the column EM, that is, to the space through which the whole column vibrates; and when the forces are as the spaces, the times are equal.

FIG. 195.



386. Now when the surface of water is smooth and at rest, if any force (as the action of the wind or the fall of a stone) depress that surface in any particular place, the contiguous water will necessarily rise all around that place. The water which has thus been elevated, descends soon after in consequence of its gravity; and by the time it has reached the original level, it will have acquired velocity sufficient to carry it lower than that level; therefore, it now acts as another original moving force, in consequence of which, the water will be raised on both sides of it. And for the same reason, the descent of those elevated parts will produce other elevations contiguous to them, and so on. Thus the alternate rising and falling of the water in ridges, will expand all around the original place of motion; but as they recede from that place, so the ridges, as well as the adjoining hollows, grow smaller and smaller until they vanish. This diminution of size is produced by three causes, namely, by the want of perfect freedom of motion among the particles of water, by the resistance of the air, and by the remoter ridges being larger in diameter than those which are nearer.

387. *Sea waves.*—Sea waves furnish the most familiar example of what is called *transverse vibration*, so named because the *particles* of water move in lines *transverse* to the direction in which the *wave* moves; the wave advancing horizontally, while the water itself rises and falls in vertical lines. The appearance of progression in the water, of which the waves are formed, is an illusion. As portions of water rise in regular succession to the same elevation, and then again sink in the same order, the eye cannot distinguish this succession from the onward motion of one and the same mass of water. An illusion of the same kind occurs in the waves of a long rope, lying on the ground, when one end is moved up and down with rapidity.

388. From a variety of experiments and observations, it appears that the utmost force of the wind can not penetrate a great way into the water; and that even in violent storms the water of the sea is slightly agitated at the depth of twenty feet below the usual level, and probably not moved at all at the depth of thirty feet. Therefore, the actual displacing of the water by the wind can not be supposed to reach nearly so low; and hence it would seem that the greatest waves could not be so very high as they are often represented by navigators. But it must be observed that in storms waves increase to an enormous size from the *accumulation of waves upon waves;* for, as the wind is continually blowing, its action will raise a wave upon another wave, and a third wave upon a second, in the same manner as it raises a wave upon the flat surface of the water. In fact, at sea, a variety of waves of different sizes are frequently seen one upon the other, especially while the wind is actually blowing. When it blows fresh, the tops of the waves, being lighter and thinner than the other parts, are impelled forward, broken, and turned into a white foam, particles of which, called *spray*, are carried to a great distance.

While the depth of the water is sufficient to allow the oscillation to proceed undisturbed, the waves have no progressive motion, and are kept each in its place by the action of the waves that surround it. But if, by a rock rising near to the surface, or by the shelving of the shore, the oscillation is prevented, or much retarded, the waves in the deep water are not balanced by those in the shallower, and therefore acquire a progressive motion in this last direction, and form *breakers.* Hence it is that waves always break against the shore, whatever be the direction of the wind. Breakers formed over a great extent of shore, are distinguished by the name of *surf.* The surf is greatest in those parts of the earth where the wind blows always nearly in the same direction.

389. *Phases, Interference.*—The various states of the particles composing a wave are called *phases.* Corresponding parts of different waves are termed *like phases;* and those parts in which the motion is exactly reversed, are *opposite phases.* The *length* of a wave is the distance between two successive like phases; two successive phases of opposite kinds are half a wave-length apart.

The *interference* of waves denotes, in general, the resultant system, which is produced by the combination of two or more separate systems of waves. The joint effect of two systems of waves may be various, according as the different systems are more or less unlike in the length of their waves. But, if two systems are just alike, still the effect of their interference may vary, according to the coincidence or want of coincidence of the like phases. For instance, if two similar systems should exactly coincide, phase for phase, the waves would simply have double height; or, to speak in general terms, there would be double intensity in the wave-motion. But, if the phases of one system exactly coincided with the *opposite* phases of the other, then the water would be level; i. e., all wave-motion would be destroyed.

The foregoing definitions and statements are applicable, as well to waves of sound and of light, as to water-waves.

390. *The wave of translation.*—The principal characteristics of the wave of translation are, that it is solitary—i. e., it does not belong to a system, like the other kind; and that its length and velocity both depend on the depth of the water. Where the water is deeper, the wave travels faster, and its length (measured in the direction of its progress) is longer. A wave of this character is started in a canal by a moving boat; and when the boat stops, it moves on alone. A grand example of this species is found in the tide-wave of the ocean. It is called the wave of translation, because the particles of water are borne forward a certain distance, while the wave is passing, and then remain at rest.*

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* See Report on Water-waves, by J. Scott Russell, in *Rep. Brit. Assoc.*, 1844.

PART IV.—PNEUMATICS.

CHAPTER I.

MECHANICAL PROPERTIES OF THE AIR.

391. *Pneumatics.*—This is the name commonly given to that branch of Mechanics which treats of the equilibrium and motion of the gases and vapors.

Those laws, which are founded on the peculiar nature of fluids arising from the mobility of their particles, are common to Hydrostatics and Pneumatics. But under the latter head are discussed certain additional properties resulting from what is commonly termed the *elasticity* of the gaseous bodies; that is, their indefinite *dilatability* and *compressibility.*

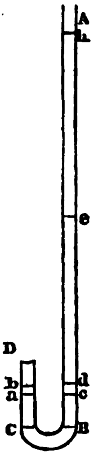
392. *Gases and vapors.*—Gaseous or aeriform bodies are those whose particles are in a condition of mutual repellency, and are held in equilibrio by this repellent force and external pressures. But most liquids and some solids, perhaps all of both classes, may, by the application of sufficient heat, be changed into the gaseous form; and by the loss of heat, they again resume their former state. And some substances, usually gaseous, may by cold and pressure be reduced to the liquid or solid form. Those which have never been thus reduced are called permanent *gases;* those which have been, are distributed into *non-permanent gases* and *vapors.*

In Pneumatics, the *air* is taken as the type of gases; and *steam*, of vapors. The expansive force of air or steam is called its *tension.*

393. *The air material.*—The air possesses the ordinary properties of matter. It obviously has *extension.* It is also *impenetrable;* for, though very compressible, it still occupies space, to the exclusion of other matter. It has *weight*, and rests with more than a ton's pressure on every square foot of the earth's surface. It shows its *inertia* in the obstruction which it presents to the motion of bodies immersed in it, and in the effects which it produces, when itself in motion. It also acts upon light, producing both reflection and refraction. By reflecting light the air becomes *visible*, when we look through a great extent of it. The sky, as it appears in the day-time, exhibits the true color of air.

394. *Mariotte's law.*—The following proposition is called, from the name of the discoverer, Mariotte's law. *At a given temperature, the volume of air is inversely as the compressing force.* It is readily illustrated by *Mariotte's tube* (Fig. 196). The end D is sealed, and A is open. Let mercury be poured in, so as just to cover the bend, and then by inclining the tube, let air in or out, till the mercury is level in both branches. The air in DC is now in its natural condition, since it just balances the atmosphere. If the tube be filled to *e*, making the air D*a* occupy half its former space, then *ec* will be found to measure 29 or 30 inches, according to the height of the barometer at the time. Thus, *two* atmospheres, one of mercury, and one of air above it, have compressed the air into *half* its volume. Again, if the tube is of sufficient length, let mercury be poured in till the inclosed air occupies D*b*, one-third of its original space; then the column *hd*, above the same level, will be found twice *ec*; that is, *three* atmospheres, two of mercury, and one of air, have reduced the same mass to *one-third* its former bulk.

Fig 196.



Since the compressing force is in each case a measure of the tension of the compressed air, it follows that *the tension of a given weight of air is as the compressing force*, and also *inversely as the volume.* And since the density of a given quantity of matter is inversely as its volume, hence *the tension of a given quantity of air is as its density.*

The law of Mariotte is supposed to apply to all the gaseous substances, so long as they retain their gaseous constitution. But when a vapor is changed, by pressure or cold, to a liquid or solid state, at that point the law ceases to be applicable.

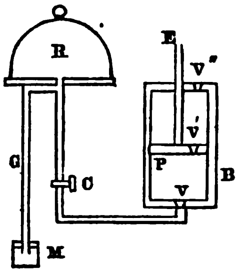
395. *Tension of air permanent.*—The elastic force of a gas remains unimpaired for any length of time. Air has been subjected to great pressures, and kept in that condition for years, and then found to have exactly the same tension, provided its temperature was the same.

396. *The diving-bell.*—The law of Mariotte is well illustrated by the diving-bell, a very useful invention, employed either for laboring or exploring under water. It consists of a large rectangular box of cast-iron, and is raised and lowered by a tackle of pulleys, having its open side downward. When it first enters the water, it is full of air of the common density·

but after descending about 34 feet, the weight of water above is equal to a second atmosphere, and the bell is now half full of water, since the air is compressed into one-half its natural volume. At the depth of 68 feet, the inclosed air occupies only *one-third* of the bell, and at 100 feet, only *one-fourth*. The instrument would therefore be nearly useless at any considerable depth, unless air were forced down by a condenser and hose. These appendages are also necessary for conveying down fresh air to take the place of that which has become vitiated by breathing, while the impure air is let off by a stop-cock.

397. *The air-pump.*—This is an instrument used for the purpose of removing the air from a vessel or receiver. It has a great variety of forms; but the following general description will apply essentially to them all. B is a cylindrical barrel, in which the piston P works air-tight, by means of the rod E. The piston-rod may be raised and depressed either by a lever, or a rack and pinion. From the bottom of the barrel, an air-pipe passes to the plate on which stands the receiver R. The entrance of the pipe is opened and closed by a valve, V; another valve, V′, is in the piston; and in some forms of pump, a third valve, V″, is placed in the top of the barrel. All these valves open upward, that is, from the receiver toward the open air; and any attempt to press the air through in the opposite direction, instantly shuts them. Whenever it is desired, the pipe may be closed by the stop-cock, C. The edge of the receiver, and the surface of the plate, are ground accurately to a plane, and thus fit each other air-tight. G is the gauge, a pipe about 33 inches long, connecting the plate with a cup, M, which contains mercury exposed to the open air.

Fig. 197.



398. *Operation.*—The operation of the air-pump is as follows. When the piston is depressed, the air in the barrel below it being prevented by the lower valve from passing into the receiver, can escape only through the piston-valve into the upper part of the barrel. When the piston is raised, the air above it cannot return, but is crowded through the third valve into the open air; while the air in the receiver and pipe, by its tension, opens the lower valve and diffuses itself equally through the receiver and barrel. Another descent and ascent of the piston only repeats the same process; and thus, by successive strokes, the air of the receiver may be nearly all removed.

399. *Rate of exhaustion.*—The quantity removed, by successive strokes, and also the quantity remaining in the receiver, diminishes in the same geometrical ratio. For, of the air occupying the barrel and receiver, a barrel-full is removed at each stroke, and a receiver-full is left. If, for example, the receiver is *three* times as large as the barrel, the air occupies *four* parts before the descent of the piston; and by the first stroke *one-fourth* is removed, and *three-fourths* are left. By the next stroke, three-fourths as much will be removed as before, and so on continually. The quantity left obviously diminishes also in the same ratio of three-fourths. In general, if B expresses the capacity of the barrel, and R that of the receiver and connecting-pipe, the ratio of each descending series is $\frac{R}{B+R}$. With a given barrel, the rate of exhaustion is obviously more rapid as the receiver is smaller. If the two were equal, ten strokes would rarefy the air more than a thousand times. In the following table, the receiver is supposed to be nine times as large as the barrel.

Number of strokes.	Part of the air expelled at each stroke.	Part remaining in the receiver.	Whole quantity expelled.
1	$\frac{1}{10}$	$\frac{9}{10}$	$\frac{1}{10}$
2	$\frac{9}{100}$	$\frac{81}{100}$	$\frac{19}{100}$
3	$\frac{81}{1000}$	$\frac{729}{1000}$	$\frac{271}{1000}$
4	$\frac{729}{10{,}000}$	$\frac{6561}{10{,}000}$	$\frac{3439}{10{,}000}$
5	$\frac{6561}{100{,}000}$	$\frac{59049}{100{,}000}$	$\frac{40951}{100{,}000}$
6	$\frac{59049}{1{,}000{,}000}$	$\frac{531441}{1{,}000{,}000}$	$\frac{468559}{1{,}000{,}000}$
7	$\frac{531441}{10{,}000{,}000}$	$\frac{4782969}{10{,}000{,}000}$	$\frac{5217031}{10{,}000.000}$

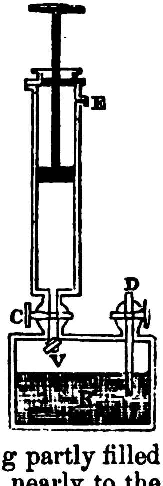
As this series never terminates, it is evident that a complete exhaustion can never be effected by the air-pump. Indeed, in practice, the vacuum is far less perfect than the theory would make it by the repetition of the blows of the piston; for when the air in the receiver becomes very much rarefied, it has not elasticity sufficient to raise the valve at the bottom of the barrel; or even if that difficulty is obviated by a different construction of the valve, still the difficulty of making the joints and valves perfectly air-tight, is such as to impair the perfection of the void. In the most improved air-pumps, the valves are made of small pieces of metal, which are opened and closed by the action of the piston itself. Also to prevent the corrosion of brass, arising from the action of the oil employed to lubricate the parts, in the place of this material, glass is sometimes used for the barrels and the plate of the pump, and the piston is made of steel.

400. *Experiments with the air-pump.*—By the air-pump a great variety of experiments may be performed, illustrative of the mechanical properties of the air. The *obstruction* of the air being removed, light and heavy bodies are seen to fall with equal rapidity; a wheel with vanes perpendicular to the plane of rotation, runs as freely as if they coincided with that plane; and water boils below blood-heat. The *weight* of a given volume of air is measured in the same manner as the weight of a liquid. The *pressure* of air in every direction is rendered apparent by many striking effects, such as lifting weights, holding together the Magdeburg hemispheres, and throwing jets of water; also, by the *difference* of pressures on the upper and lower side, bodies are shown to weigh less in air than in vacuo. And, finally, the *tension* or expansive force is exhibited by experiments equally numerous and interesting.

401. *Air-condenser.*—While the air-pump shows the tendency of air to *dilate* indefinitely, as the compressing force is removed, another useful instrument, the *pneumatic condenser*, exhibits the indefinite compressibility of air. In construction, it differs from the pump in having its valves reversed, and the receiver made very strong, so as to bear the bursting pressure to which it may be subjected.

Suppose the piston (Fig. 198) to be at the top of the barrel. On being depressed, its valve is closed by the compressed air below, while the valve at the bottom of the barrel is opened by the same force, and admits the air into the receiver. When the piston is raised, the lower valve is kept shut by the condensed air in the receiver, and that of the piston is opened by the pressure of the outer air, which thus gets admission below the piston. If the instrument is so small that the piston can be easily lifted with atmospheric pressure upon it, it is sometimes made solid, as in the figure, and the outer air enters the barrel through an aperture near the top.

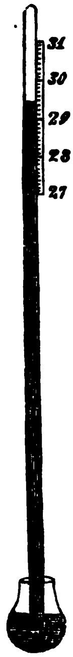
Fig. 198.



402. *Experiments with the air-condenser.*—The figure represents an arrangement for throwing jets by compressed air. The receiver, R, being partly filled with water, and the pipe, D, attached, reaching nearly to the bottom, on opening the stop-cock, a jet will be thrown to a height corresponding to the tension of the inclosed air. The condensing receiver is usually made of thick glass, and so furnished as to show by various experiments the force of compressed air; a distended rubber bag becomes flaccid, a bottle of thin glass is crushed, a water balloon sinks, &c.

Rate of condensation.—The quantity of air in the receiver of the condenser increases in an arithmetical ratio. This is obvious from the fact, that the same quantity, a barrel-full of common air, is added at every stroke. A small Mariotte's tube is often attached to the receiver, to show how many atmospheres have been admitted.

Fig. 199.



403. *Torricelli's experiment.*—Let us take a glass tube (Fig. 199), about three feet in length, closed at one end and open at the other. We fill the tube with mercury, and invert it in a vessel of the same fluid. The column immediately falls to a certain height, about twenty-nine or thirty inches, where, after vibrating a few times, it remains at rest. The space in the tube above the mercury being void of air or any other substance, is of course a vacuum, and is usually denominated the *Torricellian* vacuum, from Torricelli, an Italian philosopher, and disciple of Galileo, who by this experiment disproved the doctrine, that *nature abhors a vacuum*, and fixed the limits of the atmospheric pressure. Various precautions are necessary, in order to preserve this space free from air or any aeriform substance: when these precautions are taken, this vacuum is the most complete that we can command.

404. *Pressure of air measured.*—The column is sustained in the Torricellian tube by the pressure of air on the surface of mercury in the vessel; for the level of a fluid surface can not be preserved, unless there is an equal pressure on every part. Hence, the column of mercury on one part, and the column of air on every other equal part, must press equally. To determine, therefore, the pressure of air, we have only to weigh the column of mercury, and measure the area of the mouth of the tube. If this is carefully done, it is found that the weight of mercury is about 14.7 lbs. on a square inch. Therefore the atmosphere presses on the earth with a force of near 15 pounds to every square inch, or more than 2000 lbs. per square foot.

The specific gravity of mercury is about 13.5; and therefore the height of a column of water in a Torricellian tube should be 13.5 times greater than that of mercury, that is, about 34 feet. Experiment shows this to be true. And it was this significant fact, that *equal weights* of water and mercury are sustained in these circumstances, which led Torricelli to attribute the effect to a common force, namely, the pressure of the air.

405. *Pascal's experiment.*—As soon as Torricelli's discovery was known, Pascal of France proposed to test the correctness of his conclusion, by carrying the apparatus to the top of a mountain, in order to see if less air above the instrument sustained the mercury at a less height. This was found to be true; the column gradually fell, as greater heights were attained. The experiment of Pascal also determined the relative density of mercury and air. For the mercury falls one-tenth of an inch, in ascending 87 feet; therefore, the weight of that one-tenth was balanced by the weight of those 87 feet. And the specific gravities are inversely as the heights, i. e., as (870 × 12 =) 10440 : 1. In the same way, it is ascertained that water is 770 times as dense as air. These results can of course be confirmed by directly weighing the several fluids, which could not be done before the invention of the air-pump.

406. *The barometer.*—When the Torricellian tube and cistern are mounted in a case, and furnished with a graduated scale, the instrument is called a barometer. The scale is divided into inches and tenths, and usually extends from 26 to 32 inches, a space more than sufficient to comprehend all the natural variations in the weight of the atmosphere. By attaching a vernier to the scale, the reading may be carried to hundredths and thousandths of an inch, as is commonly done in meteorological observations. By observing the barometer from day to day, and from hour to hour, it is found that the atmospheric pressure is constantly fluctuating.

A barometer constructed with an open cistern, as already described, is adapted only for stationary use. The readings of such an instrument must obviously receive a correction for *change of level* in the cistern. But it is more convenient, as well as more accurate, to use a flexible cistern with a level-index and an adjusting screw. Before reading the instrument, the screw is turned till the mercury just touches the index, when it is known to be at the proper level. For portable use, the cistern needs to be very small, and the mercury perfectly secured from escaping, and also from dashing in the tube; for the latter purpose, the adjusting screw is turned till the mercury is forced to the top of the tube.

As the meteorological changes of the barometer are all comprehended within a range of two or three inches, much labor has been expended in devising methods for magnifying the motions of the mercurial column, so that more delicate changes in the pressure of the air might be noted. Hence the inclined tube, the wheel barometer, &c. A description of these contrivances, however, is unnecessary, as they are all found to be inferior in accuracy to the simple tube and cistern.

407. *Corrections.*—If the rigid cistern is used, a correction for *level* must be made, as already stated. As mercury is always depressed in a glass tube by capillary action, more or less according to the diameter, a correction is always necessary for *capillarity*. For each instrument, however, this may be made once for all, and applied in fixing the place of the scale. Another correction must be made at every observation, on account of the expansion and contraction of the mercury by heat and cold. This is the correction for *temperature*. It is customary to reduce the readings to the temperature of 32°. The "attached thermometer," closely connected with the barometer tube, shows the temperature of the mercury. In comparing observations made at different localities, still another correction is required, namely, for *altitude* above the sea-level.

408. *Weather marks.*—Some of the older barometers have recorded against the divisions of the scale, the states of weather, supposed to be indicated by the mercury standing at those points respectively. Such weather marks, "fair, rain, frost, wind," or whatever they are, should, however, receive no attention; since changes of weather are found to be indicated, not so much by the *height* of the column as by its *change* of height. A sudden rise from 28.5 to 29, is indicative of the same change of weather, as from 29 to 29.5. Moreover, if these marks were reliable for a given locality, they certainly could not be for another situation higher or lower, since the length of the column depends in part on the altitude of the station.

409. *Mean pressure.*—The mean pressure of the atmosphere, as indicated by the barometer, is nearly the same, at the level of the sea, in all parts of the earth, corresponding very nearly to 30 inches of mercury. This fact has been verified by numberless observations, made with the barometer in both hemispheres, from the equatorial to the polar regions. The following results for several places, in different latitudes, corrected for temperature, elevation above the level of the sea, and the influence of the earth's rotation on its axis, are nearly uniform.

	Latitude.	Bar. Pressure.
Calcutta,	22° 35′	29.776
London,	51 31	29.827
Edinburgh,	55 56	29.835
Melville Island,	74 30	29.884

But, though the mean pressure of the atmosphere is nearly the same at the level of the sea, over the whole globe, the extent of the variations to which it is liable, is exceedingly different in different parallels of latitude.. In the equatorial regions, the range of the barometer is much more limited than within the polar circles; and in the frigid zones, it is more limited than in the temperate. Within the tropics, the fluctuations of the barometer do not much exceed $\frac{1}{4}$ of an inch. At New York the variation does not much exceed $1\frac{1}{4}$ inches, while in Great Britain it is as great as 3 inches. The most extensive variations take place between the latitudes of 30° and 60°, being the zone in which the annual changes of temperature and humidity possess the widest range.

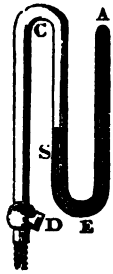
410. *Diurnal variation.*—If a long series of barometric observations be made, and the mean obtained for each hour of the day, the changes caused by weather become eliminated, and the diurnal oscillation reveals itself. It is found that the pressure reaches a maximum and a minimum twice in 24 hours. The times of greatest pressure are from 9 to 10 A. M. and P. M. In tropical climates, this variation is very regular, though small; but in the temperate zones, it is mixed up with the irregular fluctuations of weather.

411. *Gauges of the air-pump.*—The gauge of the air-pump is a barometer, and therefore properly noticed in this connection. In the form represented in Fig. 197, the mercury is at the same level in the cistern M, and tube G, before the exhaustion commences; but, as soon as a partial vacuum is made in the receiver and tube, the air resting on the mercury in M, raises the column in the tube to a height equal to the difference between the two pressures. And if a perfect vacuum could be formed by the pump, the height of the column would equal that of the barometer at the time.

Another kind of gauge is a barometer tube already filled, having its cistern open to the interior of the receiver. As soon as the pump acts, the pressure in the cistern diminishes, and the column falls in the same proportion. A complete vacuum would be indicated by a level surface of mercury in the tube and cistern.

A modified form of the last is much used, because more compact. It is called the *siphon gauge*. (Fig. 200.) Instead of a cistern, the recurved portion of the tube receives the mercury, as it sinks in the barometer; and, as before, a perfect vacuum is obtained when the fluid is at the same level in both. The tube, AE, being usually only 6 or 8

Fig. 200.



inches long, it is merely the last part of the exhaustion which is indicated; for ordinary experiments nothing more is necessary.

412. *The aneroid barometer.*—This is an instrument of modern invention. It is small and portable, somewhat resembling a chronometer. The essential part of the instrument is a flat cylindrical metallic box, whose upper surface is corrugated, and therefore elastic. The box being exhausted of air, external pressure causes the corrugated surface to sink in, to a certain extent: if the pressure increases, the surface descends a little more; if it diminishes, a little less. These minute movements are communicated by a system of levers to an index on the graduated face of the barometer. As might be expected, its range is limited, and its indications not perfectly reliable; but, for obtaining results in which accuracy is not essential, its lightness and convenient form and size recommend it, especially for portable uses.

CHAPTER II.

THE ATMOSPHERE AND ITS PHENOMENA.

413. *Weight of the atmosphere.*—From the instrumental observations described in the foregoing chapter, the weight of the entire atmosphere may be readily calculated. It is equal to that of an ocean of mercury, two and a half feet deep, covering the whole earth. Therefore, calculate first the cubical contents of such a spherical shell, and then obtain its weight from the known specific gravity of mercury.

Let $R =$ rad of the earth in feet, and $r =$ the height of mercury; then

the earth $= \frac{4}{3}\pi R^3$;

the earth + mercury $= \frac{4}{3}\pi(R + r)^3$;

the shell of mercury $= \frac{4}{3}\pi\{(R + r)^3 - R^3\} = 4\pi\left(R^2r + Rr^2 + \frac{r^3}{3}\right)$.

But, since r may be regarded as infinitesimal by the side of R, the second and third terms may be neglected, in which higher powers of r are connected with lower powers of R. Therefore the contents of the mercury in cubic feet

$$= 4\pi R^2 r = 4 \times 3.14159 \times (3956 \times 5280)^2 \times 2.5.$$

This multiplied by 62.5×13.58, the weight of a cubic foot of mercury, gives between eleven and twelve trillions of pounds, or near six thousand billions of tons. Another mode of finding the weight of the atmosphere is to multiply the surface of the earth into the number of pounds by which each unit of surface is pressed.

414. *Virtual height of the atmosphere.*—If the atmosphere were reduced to the density of that portion contiguous to the earth, it would extend to the height of about *five miles.* For, since the heights of fluid columns, when in equilibrio, are inversely as their specific gravities, $1 : 10440 :: 2.5 : 26100 = 5$ miles nearly.

415. *Decrease of density.*—But the atmosphere is very far from being throughout of uniform density. The great cause of inequality is the decreasing weight of superincumbent air at increasing altitudes. The law of diminution of density, arising from this cause, is the following:

The densities of the air decrease in a geometrical, as the altitudes increase in an arithmetical, ratio. For, let us suppose the air to be divided into horizontal strata so thin that the density of each may be considered as uniform throughout. Let a be the weight of the whole column resting on the earth; b, the weight of the column resting on the lowest stratum; c, that of the column resting on the second, &c. Then the weight of the lowest stratum is $a - b$, and the weight of the second is $b - c$, &c. Now the *densities* of these strata, and therefore their *weights* (since they are of equal thickness), are as the compressing forces; or,

$$a - b : b - c :: b : c;$$

$$\therefore\ ac - bc = b^2 - bc;\ \therefore\ ac = b^2;$$

$$\therefore\ a : b :: b : c;$$

in the same way, $b : c :: c : d$;

that is, the *weights* of the entire columns, from the successive strata to the top of the atmosphere, form a geometrical series; therefore, the *densities* of the successive strata, varying as the compressing forces, also form a geometrical series. If, therefore, at a certain distance from the earth, the air is twice as rare as at the surface of the earth, at twice that distance, it will be four times as rare, at three times that distance, eight times as rare, &c.

416. By observations on the barometer at different altitudes, aided by calculation, it is ascertained, that at the height of seven miles above the earth, the air is only one-fourth as dense as it is at the surface.* Hence, if we take an arithmetical series, increasing by seven, to denote different heights, and a geometrical series whose constant multiplier is one-fourth, to denote the corresponding densities, we may easily ascertain the density of the air at any proposed elevation.

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* Cotes, Hyd. Lect., p. 103.

Arithmetical series....	7	14	21	28	35	42	49
Geometrical series	$\frac{1}{4}$	$\frac{1}{16}$	$\frac{1}{64}$	$\frac{1}{256}$	$\frac{1}{1024}$	$\frac{1}{4096}$	$\frac{1}{16384}$

From this table it appears, that at the height of twenty-one miles, the air is sixty-four times as rare as at the surface of the earth; at the height of forty-nine miles, sixteen thousand three hundred and eighty-four times as rare; and if we pursue the calculation, we shall find that its rarity, at the moderate distance of only one hundred miles, is three hundred millions of times greater than at the earth, and of course would oppose no sensible resistance to bodies revolving in it. De Luc ascended in a balloon to such a height that his barometer fell to twelve inches. As this is about two-fifths of the mercurial column at the earth, therefore only two-fifths of the air were above him; that is, he had left *three-fifths* of the whole below. His elevation was upward of 20,000 feet.

If there were an opening into the interior of the earth, which would permit the air to descend, its density would increase in the same manner as it diminishes in the opposite direction. At the depth of about thirty-four miles, it would be as dense as water; at the depth of forty-eight miles, it would be as dense as mercury; and at the depth of about fifty miles, as dense as gold. These results are arrived at, however, on the supposition that each element of the atmosphere would retain its gaseous constitution under such enormous pressures, which is improbable.

417. *Modifications of the law.*—The foregoing law, founded on that of Mariotte, does not, however, afford *exact* data for estimating the density of the air at any given elevation, since the density is slightly affected by the following circumstances: 1. The diminishing attraction of the earth, at greater heights, causes the density to diminish faster than it would otherwise do. But this is almost insensible, even at the tops of the highest mountains, because gravity diminishes only as fast as the square of the distance from the center increases. The loss of weight near the earth's surface, as shown in Art. 15, is about a two-thousandth part of the whole for each mile. 2. A minute effect also arises from the attraction of the sun and moon, tending on the whole to diminish the density of the higher portions of the atmosphere. 3. Considerable irregularities are caused in the lower parts of the atmosphere by the changes of temperature. 4. Irregularities arise also, especially in the lower strata, from the admixture of vapors and other gaseous fluids.

Owing to these different causes of irregularity in the density of the air at different elevations, it becomes a problem of much nicety and difficulty to obtain accurate measurements of heights, by means of the barometer; but the importance of the subject has led men of science to bestow very great attention upon it. We have room only to indicate the *general principles* on which such measurements depend, leaving the details to treatises of greater extent.*

418. *The actual height of the atmosphere.*—The quantity of air on the earth is known, but not its exact height. If the law of ratios (Art. 415) were to hold good at all distances, the atmosphere would be unlimited. But on a revolving body an unlimited atmosphere is impossible. If the law of Mariotte were strictly applicable to the whole distance from the earth to the point where the centrifugal force is equal to gravity, it can be proved that the whole atmosphere would be whirled off into space. That point is at the height of 22,400 miles from the equator.† The top of the atmosphere is undoubtedly held far within that limit by the earth's attraction. For there can be no air above the point where the weight of a stratum of particles equals the repellency of the strata below. And this repellency diminishes as rapidly as the density (Art. 416), while gravity diminishes very slowly. For example, at the height of 100 miles, the repellent force would be 300 millions of times less than at the earth, while gravity is only one-twentieth less. The repulsion of the particles is diminished also by the increasing cold of the upper air; and the limit would be still lower for this reason. A calculation made many years ago, founded on the duration of twilight, led to the conclusion that the atmosphere is 45 miles high. But even if the process were perfectly satisfactory, the result could signify nothing more than that the air higher than 45 miles reflects too little light to be perceived. There is evidence that it is higher than this, though the upper surface is probably within a hundred miles; and seven-eighths of the whole occupy the first ten miles from the earth.

419. *Limit of perpetual frost.*—As we ascend from the earth, the warmth of the air constantly diminishes, till we reach a point where water would always exist in a frozen state, called the *limit of perpetual frost.* The heights of this limit for every parallel of latitude from the equator to the north pole, have been computed, partly from observation, and partly from the known mean temperature of each parallel, and the decrement of heat as we ascend in the atmosphere; and the result is expressed in the following table:

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* The necessary rules for barometrical measurements may be found in Robison's Mechanical Philosophy, vol. iii; Cavallo's El. Nat. Phil., vol. ii; Gregory's Mechanics, vol. i; Renwick's Mechanics, p. 386; and in most of the Encyclopedias, under the article *Barometer*.

† Henry on Meteorology, Patent Office Report, 1857.

Latitude.	Mean height of the limit of frost in feet.	Difference for every 5 deg. of latitude.
0°	15577	
5	15455	122
10	15067	388
15	14498	569
20	13719	779
25	13030	689
30	11592	1438
35	10664	928
40	9016	1648
45	7658	1358
50	6260	1398
55	4912	1348
60	3684	1228
65	2516	1168
70	1557	959
75	748	809
80	120	628

From this table it appears that the height of the region of perpetual frost at the equator is almost three miles; at the parallel of 35°, about two miles; and at the latitude of 54°, about one mile; while at the latitude of 80°, this region approaches very near to the earth. It is further to be remarked, that the different heights decrease very slowly as we recede from the equator, until we reach the limits of the torrid zone, when they decrease much more rapidly, the maximum being at the parallel of 40°. The average difference for every 5 degrees of latitude from 30° to 60°, is 1318; while from the equator to 30°, the average is only 664; and from 60° to 80°, it is only 891. Important meteorological phenomena depend on this fact.

420. *Cause of cold in the upper air.*—The reason for the increasing coldness of the upper air will appear from the following considerations:

1. The temperature of the space through which the earth passes, is exceedingly low. It has been estimated in different ways, and the results vary from 60° below zero to more than 200° below.
2. The intense solar rays traverse the air without heating it, and these rays of heat are absorbed by the earth.
3. When the earth has become heated, it communicates heat by contact, and by its feebler radiations to the lower strata of the air.
4. The lower strata, when heated, become specifically lighter, and are pressed up by adjacent heavier portions; but they reach the point of equilibrium within a few miles, because their temperature is rapidly lowered by the enlargement of their volume.

It is thus that the lower portions of the air are warmed indirectly by the sun's rays, while the upper parts are far less affected by the same influences.

421. *Why heat does not accumulate in the lower strata.*—It might be supposed, that, in accordance with the foregoing explanation, there would be a perpetual accumulation of heat in the earth and lower air. And it should be added, that when the earth's surface reaches a certain temperature, the intensity of its radiations becomes so great that they pass entirely through the atmosphere, and are lost in space. The amount lost in this way from year to year is precisely equal to the amount received. The difference between the intense and the feeble radiations of heat is well illustrated by the hot-bed under glass. The heat of the sun passes readily through such a substance as glass, and raises the temperature of the soil beneath. But the first feeble radiations of the ground do not penetrate the glass; and thus the temperature of the bed continually rises, until the intensity is so increased that the heat is transmitted outward as well as inward, and then a uniform temperature is preserved. So the heat of the sun comes through a window into a room and warms it; but the radiations of a stove scarcely penetrate glass at all.*

422. *Isothermal lines.*—These are imaginary lines on each hemisphere, through all those points whose mean annual temperature is the same. At the equator, the mean temperature is about 82°, and it decreases each way toward the poles, but not equally on all meridians. Hence the isothermal lines deviate widely from parallels of latitude. Their irregularities are due to the difference between land and water, in absorbing and communicating heat, to the various elevations of land, especially ranges of mountains, to ocean currents, &c. In the northern hemisphere, the isothermal lines, in passing westward round the earth, generally descend toward the equator in crossing the oceans, and ascend again in crossing the continents. For example, the isothermal of 50°, which passes through China on the parallel of 44°, ascends in crossing the eastern continent, and strikes Brussels, lat. 51°; and then on the Atlantic, descends to Boston, lat. 42°, whence it once more ascends to the N. W. coast of America. The lowest mean temperature in the northern hemisphere is not far from zero, but it is not situated at the north pole. Instead of this, there are *two* poles of greatest cold, one on the eastern continent, the other on the western, near 20° from the geographical pole. There are indications, also, of two south poles of maximum cold.

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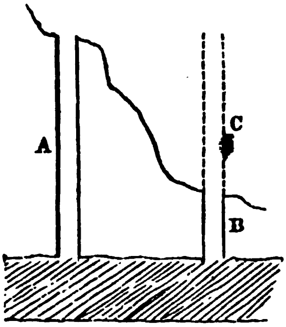
* Henry, Pat. Off. Rep., 1857.

423. *Motion of air by heat.*—Air is put in motion by the least change of temperature. Heat rarefies it, and renders it specifically lighter than the neighboring portions, by which it is forced upward. On the other hand, if air be condensed by cold, it descends, until it meets with air of the same density, where it rests. These effects naturally result from the perfect fluidity and elasticity of this substance.

424. *Shafts of a mine.*—An illustration of this principle is seen in the manner in which air circulates in the shaft or pit of a deep mine. Such a circulation is kept up briskly, even amounting sometimes to a strong wind, when two shafts or pits of unequal heights are made to communicate with each other by means of a horizontal gallery called a drift. The earth remains nearly at the same temperature, summer and winter, while the external air is hotter in summer and colder in winter than that within the mine. Now, if the air within the earth and without is of the same density, then the air of the two shafts and of the drift remains in equilibrio, the longer shaft, A, being counterbalanced by the shorter shaft, B, extended so as to embrace C, a portion of the external air, to the same height as the column A. But suppose it summer; then the air, C, being rarefied, the compound column, B + C, is lighter than A, and the circulation is down the longer, and out of the shorter shaft. And, by bringing all parts of the mine into the circulation, the whole interior will be ventilated. Again, suppose it winter; then C being colder than the air within the earth, B + C is heavier than A, and preponderates, so that the current flows in the opposite direction, down the shorter and out of the longer shaft. In spring and autumn, when the temperature of the atmosphere and the mine are nearly equal, the miners complain much of the suffocating state of the air.*

Fig. 201.



425. *Draught of chimneys.*—The motion of air in chimneys may be understood by considering the flue as one arm of a bent tube, while the external air to the same height is the other arm. Then the tendency to ascend will equal the difference in the densities of the columns of air in the opposite arms of the tube. When the air of the chimney is rarefied by heat from the fire-place, the cold air from without presses in, and thus supplies oxygen to the fire to support combustion, and carries up along with it the smoke and vapors which proceed from the fire. The smoke, it will be remarked, is carried up mechanically, by the ascending current of hot air; for smoke is itself heavier than air, and sinks when there is no current to carry it up. The *draught* of the chimney, or the strength and velocity of the ascending current, is influenced by several circumstances. 1. Within certain limits, long chimneys have a stronger draught than short ones, because they contain a longer column of rarefied air; but if very long, they cool the air too much before it has reached the top, in which case the smoke falls by its greater specific gravity. In the case of large manufactories, where a great number of fires communicate with the same flue, tall chimneys are used because they are easily kept hot throughout, and thus a very strong draught is maintained. They also serve the important purpose of conveying the noxious fumes to a great elevation in the atmosphere. Long horizontal pipes frequently have a bad draught, because they cool the smoke before it reaches the chimney. 2. A narrow throat, opening into a large pipe or funnel, makes a strong draught, because the velocity of the ascending current is thus increased, it being in different parts of the chimney inversely as the area of the section. (Art. 368.) The throat of the chimney, however, must be wide enough to admit freely all the mixed products of the ascending current, including the rarefied air, smoke, watery vapor, and so on; and, consequently, a wider throat is required for green wood than for dry, and least of all for anthracite coal, where the amount of volatile substances expelled from the fuel is comparatively small. Small funnels, being more easily rarefied than large ones, are, in general, to be preferred. When reduced, however, beyond a certain limit, they encounter too much resistance from *friction*, especially when the surface is rough. Indeed, friction impedes the circulation of air over any surface, more than it does water, since the latter fills up the inequalities and deposits a film in contact with the surface, which serves to lubricate it. Anthracite coal, on account of the small volume of gases produced in its combustion, admits of a smaller funnel than most other kinds of fuel. A large funnel is sometimes rendered unfavorable for carrying smoke, on account of a descending current of cold air from without, which meets it. This is especially the case when the top of the funnel is very large. Hence a chimney which is smoky from this cause is frequently cured by inserting in the top a smaller funnel of clay. 3. A fire-place with a low breast-work has a strong draught, because, in this case, no air can enter the chimney, except such as has felt the influence of the fire, and is thus fitted to keep the chimney warm; whereas, if the throat of the fire-place is high, much of the air that flows into it is cold, and cools the chimney, and of course diminishes the degree of rarefaction in it. Moreover, when the throat is near the fire, it becomes more heated, and thus the current of air passing through it is more rarefied, and its velocity increased. In the structure of fire-places and stoves, it is an important principle, that *as little air as possible should get into the chimney, except what passes through the fire;* for if air not heated makes its way into the current, it cools the flue, and diminishes the draught. It is another important principle, in regard to the economy of fuel, that *no more air should traverse the fire than what is necessary to support combustion.*[*] All the air that passes through the fire, over and above what undergoes decomposition, cools it, and carries a portion of the heat up the chimney. It is obvious that the air abroad must be more dense than within the flue, otherwise the current may flow downward, as is sometimes the case when a fire is kindled in a hot day.

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* Robison's Mechanical Phil., iii, 763.

426. *Ventilation of apartments.*—The air of an apartment, as it becomes vitiated by respiration, may generally be removed, and fresh air substituted, by taking advantage of the same inequality of weight in air-columns, which has been mentioned. If opportunity is given for the warm impure air to escape from the top of a room, and for external air to take its place, there will be a constant movement through the room, as in the flue of a chimney, though at a slower rate. If the external air is cold, the weight of the columns differs more, and therefore the ventilation is more easily effected, though the air may need to be warmed before being admitted to the apartment. When there is a chimney-flue in the wall of a room, with a current of hot air ascending in it, the ventilation is best accomplished by admitting the air into the flue at the upper part of the room; since it will then be removed with the velocity of the hot-air current.

427. *Wind.*—Air in motion is called wind; but the term is more particularly applied to those motions which are horizontal. Rarefaction by heat and condensation by cold are the chief causes of winds. Their distinct existence and modes of operation can frequently be discovered; and, in cases where we can discover neither, we are authorized to infer the presence of such a cause, since it is so constantly connected with the same effects in very numerous examples that daily pass before our eyes, while we are unacquainted with any other adequate causes of the same phenomena. The motion of the air, however, producing a wind, may be merely *relative*, arising from the motion of the spectator. Thus a steamboat, moving at the rate of sixteen miles an hour in a perfect calm, would appear to one on board to be facing a wind, moving at the same rate in the opposite direction; or if, in the diurnal revolution of the earth on its axis, any point of the earth's surface should move faster than the portion of the atmosphere above it, a relative wind in the opposite direction would be the result. The *direction* of the wind may be modified by various causes, the actual direction being the *resultant* of two or more currents which meet from different directions, or of several different forces.*

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[*] Count Rumford's Essays, iii, 172, &c.

428. *Land and sea breezes.*—These winds prevail in most maritime countries, but more especially in the islands of the torrid zone, blowing off from the land at night, and toward the land in the day-time. If we place a hot stone in a room (says Dr. Robison), and hold near to it a candle just extinguished, we shall see the smoke move toward the stone, and then ascend up from it. Now, suppose an island receiving the first rays of the sun in a perfectly calm morning; the ground will become warm, and will rarefy the contiguous air. If the island be mountainous, this effect will be more remarkable; because the inclined sides of the hills will receive the heat more directly. The midland air will therefore be most warmed; the heated air will rise, and that in the middle will rise fastest; and thus a current of air upward will begin, which must be supplied by air coming in on all sides, to be heated and to rise in its turn; and thus the morning sea-breeze is produced, and continues all day. This current will frequently be reversed during the night, by the air cooling and gliding down the sides of the hills, and we shall then have the land-breeze.

429. *The trade-winds.*—These are the regular currents of the torrid and warmer parts of the temperate zones, extending about to Lat. 30° on each side of the equator. On the north side they blow from the N. E., and on the south side from the S. E.; and, with some variations as to their limits, they prevail during the whole year. They are explained as follows: as the air of the torrid zone is most heated, it is crowded up by colder currents setting in from the north and south; and thus a north wind is established in the northern latitudes and a south wind in the southern. But the earth is revolving from west to east, and the air, as it moves from a higher latitude to a lower, has only so much motion eastward as the parallel from which it came. Therefore, since it really has a less motion from the west than those regions over which it arrives, it has *relatively* a motion *from the east.* This motion from the east, compounded with that from the north, produces a N. E. wind in the north latitudes; and, for a like reason, a S. E. wind in south latitudes. The narrow belt, in which these winds neutralize each other, is called the region of calms, since there is no progressive wind to aid the navigator. It is, however, a region of great rains. In certain localities within the tropics, the wind, owing to peculiar configurations of coast, and elevations of the interior, changes its direction periodically, blowing six months from one point, and six from a point nearly opposite. The *monsoons* of Southern Asia are the most remarkable example.

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* See some able remarks on this subject by Mr. W. C. Redfield, Amer. Jour., xx, 17.

The air which rises up from the torrid zone flows away to the north and south, to restore the equilibrium; and since it has at first the whole motion of the equator from the west, it will have an excess of such motion, after reaching higher parallels; hence, the upper currents, in the northern hemisphere, are from the S. W., and those in the southern hemisphere, from the N. W. In the parts of the temperate zone most remote from the equator, the movements of these returning trade-winds are felt at the surface of the earth; and hence the general prevalence of westerly winds in those latitudes.

430. *Irregular winds.*—The trade-winds, and the return currents, when the latter have descended, are in certain parts of the earth quite irregular, owing to local causes, such as mountain ranges or other inequalities of surface. And in the temperate zones generally, especially over the land, the wind blows with great irregularity, and shifts with apparent lawlessness to every point of the compass. Much of this is due to the varying distribution of the sun's heat, caused by the prevalence of clouds in one region and not in another; currents of limited extent are thus produced, which combine with the more general movement, and modify it in every possible way.

431. *Vapor.*—By the heat of the sun all the waters of the earth form above them an atmosphere of vapor, or invisible moisture, having more or less extent and tension, according to several circumstances. Even ice and snow, at the lowest temperatures, throw off some vapor.

432. *Dalton's theory.*—According to the theory of Dalton, which is generally received, any gaseous body diffuses itself by its force of tension, whether another gas occupies the same space or not; that is, the particles of one do not exert a perceptible attraction or repulsion on those of the other, but each is a vacuum to the other, except so far as it *obstructs* its movements. Therefore, at a given temperature, there can exist an atmosphere of *vapor* of the same height and tension, whether there is an atmosphere of *oxygen* and *nitrogen* or not. The vapor will not, however, form *so rapidly* among the gases, as in vacuo, because its particles meet with obstruction. Vapor, then, is not strictly *suspended* in the air, or *dissolved* by it, as formerly supposed, but exists independently. And yet it is by no means true, that there is actually the same tension of vapor as there would be if it existed alone, because considerable time is necessary for the formation of vapor, on account of mechanical obstruction presented by the air; whereas, if no air existed, the vapor would form almost instantly. It is on the same account that water will boil in a vacuum at 72°, but under the pressure of the air must be heated to 212°.

433. *Temperature and tension of vapor.*—The degree of tension of vapor forming without obstruction, depends on its temperature, but varies far more rapidly; increasing pretty nearly in a geometrical ratio, while the heat increases arithmetically. Hence, if vapor should receive its full increment of tension, while the thermometer rises 10 degrees from 80° to 90°, a vastly greater quantity would be added, than when it rises 10 degrees from 40° to 50°. On the contrary, if vapor is at its full tension in each case, much more water will be precipitated in cooling from 90° to 80°, than from 50° to 40°. On account of the obstructing effect of the air mentioned in the previous article, the *actual* differences will not always be as great as if an atmosphere of vapor existed alone.

434. *Dew-point.*—This is the temperature at which vapor, in a given case, is precipitated into water in some of its forms. If there was no air, the dew-point would always be the same as the existing temperature; since lowering the temperature in the least degree would require a diminished tension or quantity of vapor, some must therefore be condensed into water. But in the air the tension may not be at its full height, and therefore the temperature may need to be reduced several degrees before precipitation will take place. A comparison of the temperature with the dew-point is one of the methods employed for measuring the humidity of the air.

435. *Measure of vapor.*—The measure of the vapor existing at a given time, is expressed by two numbers, one indicating its *tension*,—i. e. the height of the column of mercury which it will sustain; the other, *humidity*,—i. e. its quantity per cent., as compared with the greatest possible amount at that temperature. Thus, tension = 0.6, humidity = 83, signifies that the quantity of vapor is sufficient to support sixth-tenths of an inch of mercury, and is 83 hundredths of the quantity which *could* exist at that temperature. The greatest tension possible at zero, is 0.04; at the freezing point, 0.18; at 80°, 1.0. At the lowest natural temperatures, the maximum tension is doubled every 12° or 14°; at the highest, every 21° or 22°.

436. *Instruments.*—Contrivances for measuring the moisture in the air, have usually been called *hygrometers.* Some indicate the amount of moisture by the enlargement of a substance, caused by dampness. Saussure's hygrometer is of this kind; it has a graduated arc, and an index moving over it by the expansions and contractions of a hair. The hair is lengthened by dampness, and shortened by dryness. Daniell's hygrometer operates on another principle; it has a dark bulb, which is cooled down artificially, till a film of dew forms on it; the dew-point is thus ascertained, and hence the amount of vapor in the air. The instrument most used of late years is called a *psychrometer*, which gives indications of the moisture in the air by the degree of *cold* produced in evaporation; for evaporation is more rapid, and therefore the cold occasioned by it the greater, according as the air is drier. The psychrometer consists of two thermometers, one having its bulb covered with muslin, and moistened before the observation. The wet-bulb thermometer will ordinarily indicate a lower temperature than the dry bulb; if, in a given case, they read alike, the humidity is 100. The instrument is accompanied by tables, giving tension and humidity for any observation.

437. *Precipitations of moisture.*—Whenever the air is cooled below the dew-point, a part of the vapor is deposited in the liquid or solid form. The precipitations occur under various conditions, and receive the following names: dew, frost, fog, cloud, rain, mist, hail, sleet, and snow.

438. *Dew.*—This form of deposition takes place on the surfaces of bodies, by which the air is cooled below its dew-point. It is at first in the form of minute drops, which unite and enlarge as the process goes on. Dew is formed in the evening or night, when the surfaces of bodies exposed to the sky become cold by radiation. As soon as their temperature has descended to the dew-point, the stratum of air contiguous to them deposits moisture, and continues to do so more and more as the cold increases.

Of two bodies in the same situation, that will receive most dew which radiates most rapidly. Many vegetable leaves are good radiators, and receive much dew. Polished metal is a poor radiator, and ordinarily has no dew deposited on it.

Sometimes, however, good radiators have little dew, because they are so situated as to obtain heat nearly as fast as they radiate it. Dew is rarely formed on a bed of sand, though it is a good radiator, because the upper surface gets heat by conduction from the mass below. Dew is not formed on water, because the upper stratum sinks and gives place to warmer ones.

Bodies most exposed to the open sky, other things being equal, have most dew precipitated on them. This is owing to the fact, that in such circumstances, they have no return of heat either by reflection or radiation. If a body radiates its heat to a building, a tree, or a cloud, it also gets some in return, both reflected and radiated. Hence, little dew is to be expected in a cloudy night, or on objects surrounded by high trees and buildings.

Wind is unfavorable to the formation of dew, because it mingles the strata, and prevents the same mass from resting long enough on the cold body to be cooled down to the dew-point.[*]

439. *Frost.*—When the radiating body is cooled below the freezing-point, the water deposited takes the solid form in minute crystals, and is called frost. Frost will often be found on the best radiators, or those exposed to the open sky, when only dew is found elsewhere.

440. *Fog.*—This form of precipitation consists of minute globules of water sustained in the lower strata of the air. Fog occurs most frequently over low grounds and bodies of water, where the humidity is likely to be great. If air thus humid mixes with air cooled by neighboring land, even of less humidity, there will probably be more vapor than can exist at the intermediate temperature, for the reason mentioned in Art. 433. The case may be illustrated thus. Let two masses of air of equal volumes be mixed, the temperature of one being 40°, the other 60°; and each containing vapor at the highest tension. Then the mixture will have the mean temperature of 50°, and the vapor of the mixture will also be the *arithmetical* mean between that of the two masses. But, according to the law (Art. 435), the vapor can only have a tension which is nearly a *geometrical* mean between the two, and that is necessarily lower than the *arithmetical* mean; hence the excess must be precipitated. If 8 lbs. of vapor were in one volume and 18 lbs. in the other, an equal volume of the mixture would have $\frac{1}{2}(8+18)=13$ lbs. of moisture; but at the mean temperature of 50°, only $\sqrt{8 \times 18}=12$ lbs. could exist as vapor; therefore *one pound* must be precipitated. And even if one of the masses had a humidity somewhat below 100, still some precipitation is likely to take place.

This explanation, which applies to all precipitations in the air, is usually called the Huttonian theory, from its author, Dr. Hutton of Edinburgh.

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[*] See Wells, on Dew.

441. *Cloud.*—The same as fog, except at a greater elevation. Air rising from heated places on the earth, and carrying vapor with it, is likely to meet with masses much colder than itself, and depositions of moisture are therefore likely to take place. Mountain-tops are often vailed with clouds, when all around is clear. This happens when lower and warmer strata are driven over them, and thus cooled below the dew-point. The same air, as it continues down the other side, takes up its vapor again, and is as transparent as it was before ascending. A person on the summit perceives a chilly fog driving by him, which prevents his seeing any thing else; but the fog was an invisible vapor a few minutes before reaching him, and returns to the same condition soon after leaving him. The cloud *rests* on the mountain; but all the particles, which compose it, are swiftly *crossing over*. Clouds are often above the limit of perpetual frost; they then consist of minute crystals of ice.

442. *Classification of clouds.*—The aspects of clouds are various, and depend in some measure at least on the circumstances of their formation. The usual classification is the following:

1. *Cirrus.*—This cloud is fibrous in its appearance, like *hair* or flax, sometimes straight, sometimes bent, and frequently at one end is gathered into a confused heap of fibers. The cirrus is high, and often consists of frozen particles, even in summer.
2. *Cumulus.*—This consists of compact rounded heaps, which often resemble mountain-tops covered with snow. This form of cloud is confined mostly to the summer season; it usually begins to form after the sun rises, and to disappear before it sets, and is rarely seen far from land. The cumulus is generally not so high as the cirrus.
3. *Stratus.*—*Sheets* or stripes of cloud, sometimes overspreading the whole sky, or as a fog covering the surface of the earth or water. The stratus is the most common, and usually lies lowest in the air.

4, 5, 6. *Cirro-cumulus*, *cirro-stratus*, *cumulo-stratus.*—Intermediate or combined forms.

7. *Nimbus.*—A cloud, which forms so fast as to fall in rain or snow, is called by this name.

443. *Rain.*—Whether the precipitated moisture has the form of cloud or rain, depends on the rapidity with which precipitation takes place. If currents of air are in rapid motion, if the temperatures of masses, brought into contact by this motion, are widely different, and if their humidity is at a high point, the vapor will be precipitated so rapidly, that the globules will touch each other, and unite into larger drops, which cannot be sustained. And here it should be remarked, that globules of fog and cloud are specifically as heavy as drops of rain; but they are sustained by the slightest upward movements of the air, because they have a great surface compared with their weight. A globule whose diameter is 100 times less than that of a drop of rain, meets with 100 times more obstruction in descending, since the weight is diminished a million times $(\frac{1}{100})^3$, and the surface only ten thousand times $(\frac{1}{100})^2$. So the dust of even heavy minerals is sustained in the air for some time, when the same substances, in the form of sand, or coarse gravel, fall instantly.

Mist is fine rain; the drops are barely large enough to make their way slowly to the earth. *Sleet* is frozen mist.

444. *Hail.*—When the air in which rapid precipitation occurs, is so cold as to freeze the drops, hail is produced. As hailstones are not usually in the spherical form when they reach the earth, it is supposed that they are continually receiving irregular accretions in their descent through the vapor of the air. Hail-storms are most frequent and violent in those regions where hot and cold bodies of air are most easily mixed. Such mixtures are rarely formed in the torrid zone, since there the cold air is at a great elevation; in the frigid zone, no hot air exists at any height; but in the temperate climates, the heated air of the south, and the intensely cold winds of the north, may be much more easily brought together; and accordingly, in the temperate zones it is that hail-storms chiefly occur. Even in these climates, they are most frequently found in places where such mixtures are most easily formed, as in the south of France, lying, as it does, between the Pyrenees and the Alps, which are covered with perpetual snows, while the intervening country is subject to become highly heated by the summer's sun, or is even visited, especially at a certain elevation, by occasional blasts of the hot winds that cross the Mediterranean.

Snow consists of the minute crystals of frozen cloud, united in flakes. Like all transparent substances, when in a pulverized state, it owes its whiteness to innumerable reflecting surfaces. A cloud, when the sun shines upon it, is for the same reason intensely white.

445. *Espy's theory of precipitation.*—It is believed by many who have carefully studied the phenomena of the air and vapor which envelop the earth, that precipitations, in the forms of cloud, rain, &c., are by no means wholly due to the mixing of cold and warm currents of air, but also, and perhaps principally, to changes which take place in the condition of the air itself as it ascends. This theory was propounded by the late

James P. Espy, of Washington, after much careful and ingenious investigation of meteorological phenomena.

It is known that for every 300 feet of ascent, the air is about one degree colder. If, therefore, a mass of air is transferred from the surface of the earth to a height in the atmosphere, it will be cooled to the temperature of the stratum which it reaches; not principally by giving off its caloric, but by *expanding*, and thus having its own heat reduced by being diffused through a larger space. Now, if the rising mass was saturated with moisture, this moisture would begin at once to be precipitated by the cooling which it undergoes in consequence of expansion. If, instead of being saturated, its dew-point is a certain number of degrees below its temperature, it must ascend far enough to be cooled to the dew-point, before precipitation of its moisture will take place. Suppose, for instance, the temperature at the earth is 70°, and the dew-point is 65°; then after the warm air has risen 1500 feet (5 × 300 ft.), it will become 5° cooler, and contain all the moisture which is possible at that temperature. At that point precipitation begins, and forms the base of a cloud. The clouds, called *cumulus*, which are seen forming during many summer forenoons, are the precipitations of columns rising from warm spots of earth so high that they are cooled below their dew-point. But the movement and the precipitation do not stop here; for, as moisture is precipitated, its latent heat is given off in large quantities, which elevates the temperature of the mass, and causes it to rise still higher, and precipitate still more of its moisture. As it becomes rarer, it spreads laterally, and causes the cumulus often to assume the overhanging form which distinguishes that species of cloud.*

446. *Northeast storms.*—Mr. Redfield has investigated, with great success, the phenomena of violent storms, especially of *Atlantic hurricanes*, and has shown that they are generally, if not always, great whirlwinds. They usually take their rise in the equatorial region eastward of the West India Islands; they spin like a top (or more like those little whirls which we sometimes observe to take up leaves and other light substances), advancing slowly to the northwest, until they approach the coast of the United States near the latitude of 30°, and then veer to the northeast, running nearly parallel to the American coast, and finally spend themselves in the northern Atlantic. It is a remarkable fact, that their rotary motion is always in one direction, namely, from right to left, or against the sun. This motion is also far more violent, especially in the central parts of the storm, than the progressive motion. The rotary motion may amount to 200 miles or more per hour, while the forward motion of the storm is not more than 20 or 30 miles. The able and ingenious papers of Mr. Redfield on this subject, may be found in the different volumes of the American Journal of Science.

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* Espy, Philosophy of Storms. Henry, Patent Office Report, 1858.

Mr. Espy does not admit the rotary motion of storms supposed by Mr. Redfield, but maintains that *the winds blow from all directions toward the center of the storm;* that the air ascends in the center in a continual column; that being cooled as it rises, its vapor condenses, forming cloud; that the latent heat given out on condensation expands the ascending column, and causes it to rise still higher, condensing more vapor; and finally, that when the original body of air is hot and largely charged with vapor, and rises rapidly to the higher and colder regions of the atmosphere, the effects are proportionally violent, giving rise to tornadoes, thunder-storms, and water-spouts.*

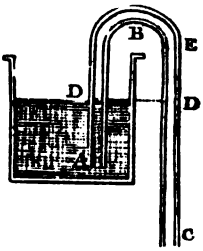
CHAPTER III.

MECHANICAL AGENCIES OF AIR AND STEAM.

447. In consequence of our power of forming a vacuum, either by the exhaustion of air, or the condensation of steam, and of employing the expansive energies of these substances, they become important prime movers in various kinds of machinery.

448. *The siphon.*—If a tube having two branches be filled with water, and the mouth of one branch be immersed in water, the fluid will be discharged through the tube, so long as the outer end is lower than the fluid level. Such a tube is called a siphon. It may be filled either by suction, or by pouring water into it, keeping the orifices closed till the end is immersed; or, when the siphon is large, each orifice is shut, and water is poured in through an opening in the top of the bend. The principle of its operation is as follows: the atmospheric pressure at D, equals that at C; the former, diminished by the weight of a column of the height BD, exerts pressure outward at B; the latter, diminished by a column of the height BC, opposes it at the same point. It is

FIG. 202.



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* Espy, on the Philosophy of Storms.

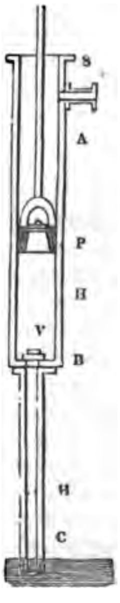
obvious that the first pressure is greater than the second, by the weight of CD. Hence, the water will run with a velocity due to the difference of the heights of the columns.

The limiting height above the water level is 34 feet, because the atmosphere can not sustain a column of water higher than that. For the same reason, a siphon for mercury cannot operate higher than 30 inches.

449. *Intermitting springs.*—Springs which flow freely for a time, and then cease for a certain interval, when they flow again, are found in some cases to act on the principle of the siphon. Suppose a reservoir or hollow in the interior of a hill, having a siphon-shaped outlet. It is obvious, upon hydrostatic principles, that no water will be discharged until the fluid has reached a level in the reservoir as high as the top of the bend in the outlet. Then it will begin to run out, and will continue to run, until the water has descended to the level of the outlet; after which, no more water will be discharged until enough has collected to reach the higher level, as before.

450. *Pneumatic pumps.*—Pumps for raising water, which are in most common use, operate on the principle of forming a vacuum in a pipe, into which the water is then raised by atmospheric pressure. The exhaustion of the pipe is effected in several ways; but the most simple is *the piston*, which is therefore generally employed.

FIG. 203.



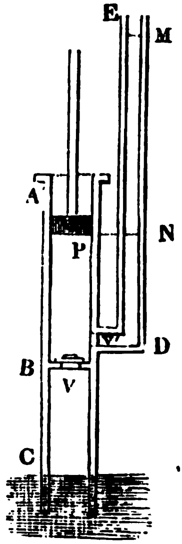
451. *The suction-pump.*—This pump is constructed as follows: SB (Fig. 203) is an exhausting tube, in which a piston, P, rises and falls; in the piston is a valve, and at the bottom of the tube another, V, each opening upward. The lower valve closes the end of the suction-pipe, BC, and must be less than 34 feet high perpendicularly from the water. The discharge-pipe, S, may be at any convenient height above the piston. Suppose now that the piston is in contact with the lower valve. When it is lifted, the air below is rarefied, and a column, CH, is raised, by the weight of the outer air, to such a height that its weight, added to the diminished tension of the air above it, equals the atmospheric pressure. When the piston descends, the air below it is prevented from returning by the lower valve, and escapes through the piston. The piston being raised again, the water rises still higher, till at length it passes through the valve, and the piston dips into it; after which it is lifted directly, without the intervention of the air.

452. *Calculation of the force.*—Let the whole atmospheric pressure be represented by 34 (its equivalent in feet of water), and the height of water, CH, by h. Since the tension of air in the tube, added to h, equals 34, therefore the tension $= 34 - h$; and this force is exerted upward on the lower side of the piston; while the downward pressure on the top of the piston $= 34$. The difference of the two $= h$, which is, therefore, the height of water, whose downward pressure is to be overcome. We arrive at the same result, if the water is above the piston at A, when $h = \text{AC}$. For, in this case, the pressure upward on $\text{P} = 34 - \text{PC}$; while the downward pressure $= 34 + \text{AP}$; and the difference between them is $\text{PC} + \text{AP} = h$. Therefore, in every case, the force required to lift the piston and column of water, is that which would be required to lift the same weight in any other way. The atmosphere has no other agency than to furnish a convenient mode of applying the force.

If $d =$ the diameter of the piston, in decimals of a foot, then $\frac{1}{4}\pi d^2 =$ its area; $\frac{1}{4}\pi d^2 h =$ the cubic *feet* of water; and $\frac{1}{4}\pi d^2 h \times 62.5 =$ the *pounds* of water.

453. *The forcing-pump.*—The piston of the forcing pump is solid, and the upper valve V′ (Fig. 204) opens into a side pipe, DM. In the ascent of the piston, the water is raised as in the suction-pump; but in its descent a force must be applied to press the water which is above the lower valve into the side pipe.

Fig. 204.

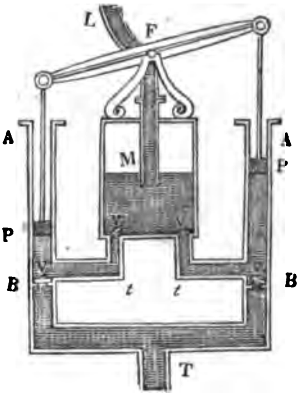


454. *Calculation of the force.*—Suppose the water to reach M, and let the height of M above C be called h. As in the suction-pump, the force necessary to *elevate* the piston is equal to the weight of PC. When it is *depressed*, PV balances ND, and force must be applied sufficient to lift NM. Therefore, the whole force expended in raising and depressing the piston, is that requisite to raise the entire column, $\text{PC} + \text{MN} = h$. As the weight of the piston opposes the force in raising, and aids it in descending, its effect on the whole is neutralized. The number of pounds of water raised, as before $= \frac{1}{4}\pi d^2 h \times 62.5$.

455. *Pump regulator.*—In both kinds of piston-pump, the force is applied by a series of impulses, and the water therefore does not flow continuously from the discharge-pipe. A continuous flow is easily obtained, however, by raising the water first into an air-vessel, where it condenses the air, whose expansive force continues to discharge the water between the strokes of the pump.

456. *The fire-engine.*—This important machine generally consists of one or more forcing-pumps, with a regulating air-vessel; though the arrangement of parts is exceedingly varied. Fig. 205 will illustrate the principles of its construction. As the piston, P, ascends, the water from T is raised through the valve, V, by atmospheric pressure. As P descends, the water in AB is driven through V′ into the air-vessel, M, whence by the condensed air it is forced out without interruption through the hose pipe, L. The piston on the other side operates in the same way by alternate movements. The force of several men may be applied at once by hand bars, called *brakes*, suitably attached to the lever, F.

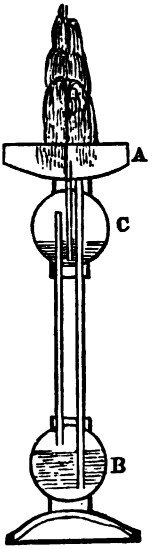
FIG. 205.



457. *Hero's fountain.*—The condensation in the air-vessel, from which water is discharged, may be produced by the weight of a column of water. An illustration is seen in Hero's fountain, Fig. 206. A vertical column of water from the vessel, A, presses into the air-vessel, B, and condenses the air, according to the height of AB. From the top of this vessel, an air tube conveys the force of the compressed air to a second air-vessel, C, which is nearly full of water, and has a jet-pipe rising from it. Since the tension of air in C is equal to that in B, a jet will be raised, which, if unobstructed, would be equal in height to the compressing column, AB.

This plan has been employed to raise water from a mine in Hungary, and hence called "the Hungarian machine."

FIG. 206.



458. *Steam as a power.*—We have already seen that water evaporates at all the natural temperatures, forming vapor of a feeble tension. If the temperature is artificially raised, the vapor receives the name of *steam*, and its tension increases at a rapid rate, being doubled, on the average, for every increment of 27°; so that at 212° the tension equals atmospheric pressure, or 30 inches of mercury. By boiling the water in strong vessels, its tension may be increased beyond known limits. Thus we have in the *tension*, or *expansive force* of steam, a source of great power. Again, if steam is suddenly cooled, so as to return to water, nearly the whole space occupied by it becomes a vacuum. This gives opportunity to employ atmospheric pressure; and thus steam becomes, *indirectly*, a source of limited power.

459. *Volume of steam.*—At 212°, one cubic *inch* of water forms 1700 cubic inches, or nearly one cubic *foot* of steam. And the tension of such steam is equal to atmospheric pressure, or about 15 lbs. per inch. But if the steam is formed in a close vessel, its force will increase and its volume diminish, according to the temperature of boiling, as in the following table:

Temperature.	Atmospheres of tension.	No. of inches of steam for 1 of water.
212°	1	1700
250	2	850
273	3	567
291	4	425
306	5	340

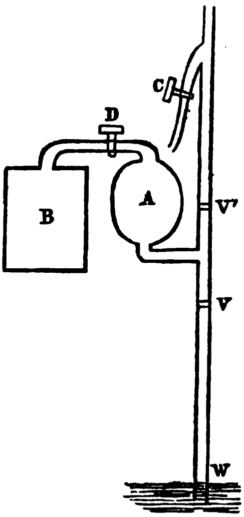
What temperature would be requisite to convert water into steam without enlarging its volume at all, is not known, but its force would be 1700 times that of steam at 212°; that is, it would sustain a column of mercury = 1700 × 30 inches = 4250 feet, = more than three-fourths of a mile high.

460. *Latent heat.*—The quantity of heat required to raise water from the freezing to the boiling point, is 180° Fah. To convert the same mass of water into steam, in the open air, requires 1000° more. All this 1000° of caloric becomes latent; the steam is no hotter than the water. If the steam is formed under greater pressures, and therefore at higher temperatures, it contains less latent, and more sensible heat.

On the other hand, when common steam is condensed to water, the 1000° of heat, before latent, are given off to surrounding bodies, and raise their temperature.

461. *Savary's steam-engine.*—About the year 1700, Savary, of England, obtained a patent for a machine to raise water by means of steam. Fig. 207 will illustrate its principle. A metallic vessel, A, is filled with steam from the boiler, B; and then, after the stop-cock, D, is shut, cold water from the jet-pipe, C, flows over the vessel, and condenses the steam within; upon which the atmosphere at W forces up a column of water through the valve V, into the vessel A, and fills it. The jet-pipe is then closed, and the steam-pipe opened. The elastic force of the steam on the surface of water in A, drives it through the valve V′, to a height depending on the tension of the steam. The stop-cocks, C and D, being alternately opened and shut by an attendant, the process is continually repeated. Savary's engines were able to raise water near 100 feet; and, by placing one above another at intervals of 100 feet, he employed them to some extent in draining mines.

FIG. 207.



462. *Newcomen's atmospheric engine.*—About ten years later, Newcomen proposed to work the mining pumps by steam; and for more than half a century the atmospheric engine, which he invented, was used for the purpose. The steam, having only atmospheric tension, was brought from the boiler into the cylinder beneath the piston. A jet of cold water was then thrown in, which condensed the steam and formed a vacuum, while the atmosphere above caused the piston to descend to the bottom. The steam was again let in beneath, which equalized the pressure of the air above, and the piston was drawn up by the weight of the pump-rod on the opposite end of the working-beam. The jet of cold water was again introduced, and the same movements were repeated.

463. *Imperfection in these engines.*—In Savary's engine, there was great waste arising from letting steam directly upon cold water. It would be condensed, and therefore powerless, till it had heated the surface of the water to the boiling point; and after it began to press the water down, the cold walls of the vessel would continue to condense some part of the steam, and enfeeble its action. The same difficulty in a less degree existed in Newcomen's engine. Cold water was injected into the same space occupied by steam, in order to condense it; and the condensation would not be complete till the cylinder was reduced to about 72°, because water boils in a vacuum at that point. And the next time that steam was introduced it would be condensed by the cold cylinder, and therefore inoperative, till the cylinder was again heated to 212°. Or, if it was not cooled so low, then the vacuum was imperfect, and the operation of the engine proportionally sluggish.

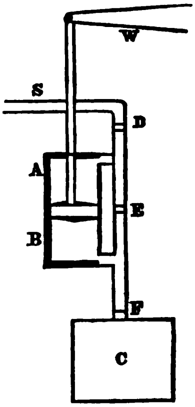
464. *Single-acting engine of Watt.*—The principal improvements in the steam-engine were made by James Watt, philosophical-instrument maker in Glasgow, who, in 1763, devised means for overcoming the great difficulty under which the atmospheric engine had always labored; that is, he succeeded in forming a perfect vacuum beneath the piston, without cooling the cylinder.

1. He employed the *condenser*, a separate vessel, into which were admitted both the steam from the cylinder, and the cold water for condensing that steam.
2. He covered the top of the cylinder, causing the piston-rod to move through a stuffing-box, and thus depressed the piston by the *force of steam*, instead of the weight of the air.
3. He *encased* the cylinder, either with a non-conducting material as wood, or a second metallic cylinder, with steam between the two.

All these contrivances were directed to the accomplishment of one object, the preservation of a uniform high temperature in the steam-cylinder.

465. *Operation.*—Fig. 208 exhibits the relations of the cylinder and condenser. The steam-pipe, S, which comes from the boiler (not in the figure), has in it three stop-cocks, D, E, F, which are opened and shut by the machine at the proper moments. Between D and E the pipe communicates with the upper part, A, of the cylinder; between E and F is a similar communication with the lower part, B; and the extremity of the pipe at F, opens into the condenser, C. The remote end of the working-beam, W, is loaded, so that when the piston is free from steam-pressure it will be drawn up to the top. Suppose the piston at the top, and steam beneath it; D and F are opened, and E is shut; as C is very cold, the steam from B flashes into it, being condensed as fast as it enters, and the space B in the cylinder, in an instant becomes a vacuum. In the mean time the steam from the boiler presses into A, and pushes down the piston with nothing to oppose its motion. D and F are then shut, and E is opened; there is no communication either with the boiler or the condenser; but, as there is free circulation be-

FIG. 208.



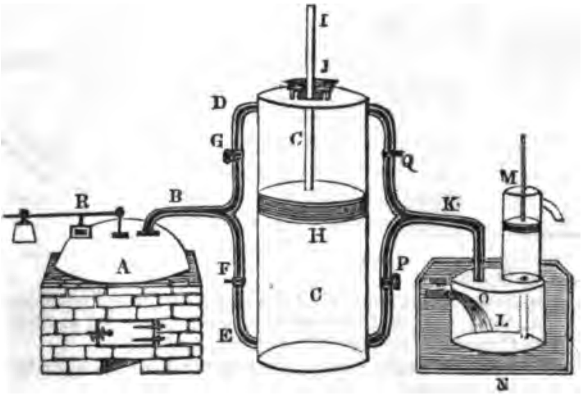
tween A and B, the piston is subject to equal pressures above and below, and is therefore drawn up by the beam.

As Watt substituted steam-pressure for weight of the air, the engine was literally changed from an *atmospheric* to a *steam*-engine.

466. *Defect of the single-acting engine.*—Though Watt had effectually saved from waste the force of the steam, and given energy to the steam-engine, yet it was only *single-acting;* that is, the steam urged the piston only one way, and it returned merely by the *cessation* of the force. Such an intermittent power answered indeed for pumping water, to which labor alone steam was as yet applied; but another step was wanting, to render the engine useful for general purposes. Watt almost immediately made the improvement, by which the steam-power is employed to urge the piston alternately in both directions.

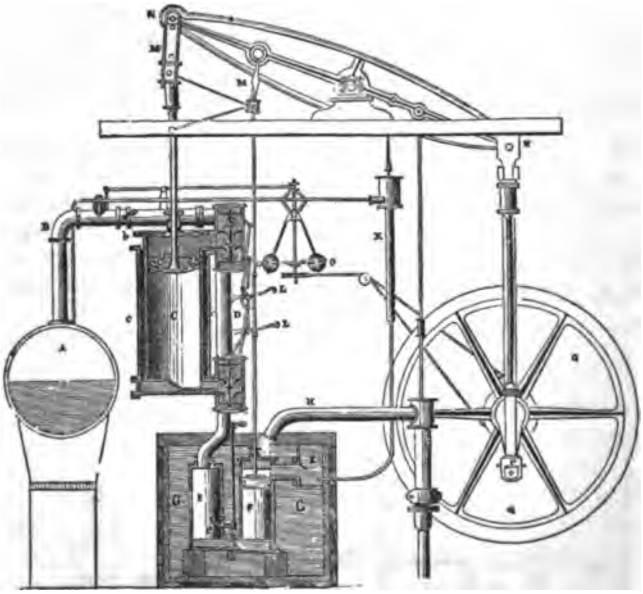
467. *Double-acting engine of Watt.*—In this most improved form of the steam-engine, both the steam-pipe and the condenser have alternate communication with the top and bottom of the cylinder. Thus, not only is the piston depressed by steam, while a vacuum is below, but also raised by steam, while a vacuum is above. Fig. 209 presents the essential parts, as follows: A, the boiler; B, the steam-pipe, dividing into the branches, D, E; K, the eduction-pipe, with similar branches, and opening at O, into the condenser, L; N, the cold-water cistern, in which the condenser is immersed; M, the air-pump, for clearing the condenser of air, and water of condensation; H, the piston; I, the piston-rod, working through the stuffing-box, J; F, G, P, Q, the stop-cocks for forming the four communications; R, the safety-valve, loaded to the required

FIG. 209.



tension of steam. Suppose all parts of the cylinder and pipes to be filled with steam. Then, if G and P are opened, the piston descends; as soon as it has reached the bottom, G and P are closed, F and Q are opened, and the piston ascends. In the mean time, the pump, M, is in constant action, to remove the accumulating water from the condenser. The latent heat, given off in condensation, rapidly heats the water of the cistern, N; and it must therefore be continually furnished with new supplies, by a cold-water pump, not represented in the figure. In Fig. 210 may be traced more fully the details of the engine.

FIG. 210.



A. The **BOILER.**
B. The **STEAM-PIPE,** conveying the steam to the cylinder, having a steam-cock, *b*, to admit or exclude the steam at pleasure.
C. The **CYLINDER,** surrounded with the *jacket*, *c c*.
D. The **EDUCTION-PIPE,** communicating between the cylinder and the condenser.
E. The **CONDENSER,** with a valve, *e*, called the *Injection-cock*, admitting a jet of cold water, which meets the steam the instant the latter enters the condenser.
F. The **AIR-PUMP.**
G. G. **COLD-WATER CISTERN,** for the condenser, filled by
H. The **COLD-WATER PUMP.**
I. The **HOT-WELL,** containing water from the condenser.
K. The **HOT-WATER PUMP,** which returns the water of condensation to the boiler.
L. L. **LEVERS,** which open and shut the valves in the channel be-

tween the induction-pipe, cylinder, eduction-pipe, and condenser; which levers are raised or depressed by projections attached to the piston-rod of the air-pump.

M. M. Apparatus for Parallel Motion, a beautiful contrivance, by which the piston-rod is maintained constantly in a perpendicular position, while the end of the working-beam which carries it, moves in the arc of a circle. It is more common, however, to keep the rod in a straight line by means of guides.

N. N. The Working-beam.

O. O. The Governor (Art. 297).

P. The Crank (Art. 300).

Q. Q. The Fly-wheel (Art. 294).

468. *Steam-valves.*—What are commonly called valves in steam machinery are not strictly such, since they are not opened by the pressure of a fluid in one direction, and closed by a pressure in the opposite direction. On the contrary, they are opened and shut by the action of an eccentric cam on the principal axis. The *puppet valve* is the frustum of a cone, fitting into a conical socket, and opens the pipe by being raised. The *sliding valve* does not rise, but slides over the aperture. The *rotary valve* is a cylinder lying across the pipe, and having an aperture through it, so that by a quarter revolution it opens or shuts the pipe. The *throttle valve* is a partition in the pipe, turning on an axis, so as to lie cross-wise or lengthwise.

The valves represented in Fig. 210 are puppet valves. But these have mostly given way to the sliding valve, made in the form of the letter D, and hence called the D-valve, which is so contrived as to regulate all the four communications in the steam and eduction pipes.

469. *Their adjustment.*—Watt discovered that a further economy, both in the use of steam, and in the wear of machinery, arose from the proper adjustment of the valves. If the steam enters the cylinder during the whole length of a stroke of the piston, its motion is necessarily *accelerated*; and is therefore swiftest at the instant before being stopped; thus the machinery receives a violent shock. If the valve is adjusted to *cut off* the steam when the piston has made one-third of its stroke, the diminishing tension exerts about force enough, during the remaining two-thirds, to keep up a uniform motion. Two important advantages are thus gained; the wear of the machinery is greatly diminished, and two-thirds of the fuel are saved. The *cut-off* should be regulated in each engine, according to friction, and other obstructions.

470. *High-pressure engine.*—The engine of Watt, already described, is properly called a *low-pressure* engine, because the steam, having a vacuum on the opposite side of the piston, works at the least possible tension. For many purposes, especially those of locomotion, it is advantageous to dispense with the large weight and bulk of machinery necessary for condensation, and do the work with steam of a higher tension. In Fig. 209, if K, L, M, N, and other parts connected with them, but not represented, are removed, then the valves P and Q will alternately open the top and bottom of the cylinder to the air. Therefore the steam in that part of the cylinder which is open to the air, will have a tension of 15 lbs. per inch; and, consequently, the steam on the opposite side of the piston must have a tension 15 lbs. per inch greater than before, in order to do the same work.

471. *Applications of steam-power.*—For more than half a century, the only use of the steam-engine was to work the water-pumps of the English mines. But the genius of Watt has rendered it available for nearly every purpose which requires the use of machinery. Every description of machine, for the heaviest and the lightest operation, may have a steam-engine for its prime mover. Near the beginning of the present century, it began to be used for locomotion on land and water; and at the present day, both traveling and the transportation of merchandise are principally accomplished by means of steam.

472. *Estimation of steam-power.*—It is customary to express the power of a steam-engine by comparing it with the number of horses whose strength it equals. In making this comparison, Watt took as a measure of *one horse-power*, the ability to raise 2,000,000 lbs. through the height of *one foot* in an hour; or 2,000,000 *foot-pounds* per hour. It is obviously immaterial what the respective factors for feet, and for pounds, are, if the product only equals 2,000,000. For example, 2000 lbs. through 1000 ft., or 5000 lbs. 400 ft. per hour, &c., is equal to one horse-power. It is found, that the available force of *one cubic foot* of water, when changed to steam, is about equal to 2,000,000 foot-pounds; that is, to one horse-power. Hence, an engine of fifty horse-power is one which can change fifty cubic feet of water into steam in one hour.

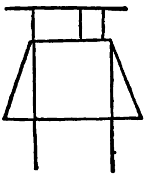
THE MOTION OF CURRENTS IN FLUIDS.

473. There are some phenomena relating to currents moving through a fluid, either of the same or a different kind, which belong alike to hydraulics and pneumatics; a brief account of these is presented here.

474. *Motion communicated to the surrounding fluid.*—If a stream is driven through a medium, it carries along the adjoining particles by *friction* or *adhesion*. The experiment of Venturi illustrates this kind of action, as it takes place between the particles of water. A reservoir filled with water, has in it an inclined plane of gentle ascent, whose summit just reaches the edge of the reservoir. A stream of water is driven up this plane with force sufficient to carry it over the top; but in doing so, it takes out continually some part of the water of the reservoir, and will in time empty it to the level of the lowest part of the stream. A stream of air through air produces the same effect, as may be shown by the flame of a lamp near the stream always bending toward it. In like manner, water through air carries air with it; when a stream of water is poured into a vessel of water, air is carried down in bubbles; and cataracts carry down much air, which as it rises forms a mass of foam on the surface. The strong wind from behind a high waterfall is owing to the condensation of air brought down by the back side of the sheet.

475. *Ventilators.*—If the stream passes across the end of an open tube, the air within the tube will be taken along with the stream, and thus a partial vacuum formed, and a current established. It is thus that the wind across the top of a chimney increases the draught within. To render this effect more uniformly successful, by preventing the wind from striking the interior edge of the flue, appendages, called *ventilators*, are attached to the chimney top. A simple one, which is generally effectual, consists of a conical frustum surrounding the flue, as in Fig. 211, so that the wind, on striking the oblique surface, is thrown over the top in a curve, which is convex upward. The same mechanical contrivance is much used for the ventilation of public halls, and the holds of ships. A horizontal cover may be supported by rods, at the height of a few inches, to prevent the rain from entering.

FIG. 211.



476. *A stream meeting a surface.*—Though the moving fluid may be elastic, yet, when it meets a surface, it tends to *follow* it, rather than to rebound from it. This effect is partly due to adhesion, and partly to the resistance of the medium in which the stream moves. It will not only follow a plane or concave surface, but even one which is convex, provided the velocity of the current is not too great, or the curvature too rapid. A stream of air, blown from a pipe upon a plane surface, will extinguish the flame of a lamp held in the direction of the surface beyond its edge, while, if the lamp be held elsewhere near the stream, the flame will point toward the stream, according to Art. 474. Hence, snow is blown away from the windward side of a tight fence, and from around trees.

477. *Diminution of pressure on the surface.*—When a stream is thus moving along a surface, the fluid pressure on that surface is slightly diminished. This is proved by many experiments. If a curved vane be suspended on a pivot, and a stream of air be directed tangentially along the surface, it will move toward the stream, and, may be made to revolve rapidly, by repeating the blast at each half revolution. What is frequently called the *pneumatic paradox*, is a phenomenon of the same kind. A stream of air is blown through the center of a disk, against another light disk, which, instead of being blown off, is forcibly held near to it by the means. The pressure is diminished by all the radial streams along the surface contiguous to the other disk, and the full pressure on the outside preponderates. Another form of the experiment is to blow a stream of air through the bottom of a hemispherical cup, in which a light sphere is lying loosely. The sphere can not be blown out, but, on the contrary, is held in, as may be seen by inverting the cup, while the blast continues. It appears to be for a reason of the same sort, that a ball or a ring is sustained by a jet of water. It lies not on the *top*, but on the *side* of the jet, which diminishes the pressure on that side of the ball, so that the air on the outside keeps it in contact. The tangential force of the jet causes the body to revolve with rapidity. A ball can be sustained a few inches by a stream of air.

478. *Formation of vortices.*—As a current reaches the termination of the surface along which it was flowing, a *vortex* or whirl is likely to occur in the surrounding medium behind the edge of the surface. Vortices are formed on water, whose flow is obstructed by rocks; and often when the obstructing body is at a distance below the surface, the whirl which is established there, is communicated to the top, so that the vortex is seen, while its cause is out of sight. There is a depression at the center, caused by the centrifugal force; and if the rotation is rapid, a spiral tube is formed, in which the air descends to great depths. These are called *whirlpools.* In a similar manner whirls are produced in the air, when it pours off from a surface. The eddying leaves on the leeward side of a building in a windy day, often indicate such a movement, though it may have no permanency. The vortex is perhaps repeatedly broken up and reproduced.

479. But vortices are also formed by counteracting currents in an open medium. When an aperture is made in the middle of the bottom of a vessel, as the water runs toward it, the filaments encounter each other, and usually, though not invariably. they establish a rotary motion, and form a whirlpool. Vortices are a frequent phenomenon of the atmosphere, sometimes only a few feet in diameter, in other instances, some rods or even miles in width. The smaller ones, occurring over land are called *whirlwinds*; over water, *water-spouts*. The mode of their production is not well understood, but they probably originate in currents which do not exactly oppose each other, but act as a *couple* of forces, tending to produce rotation. (Art. 57.)

The burning of a forest sometimes occasions whirlwinds, which are borne away by the wind, and maintain their rotation for miles. As the pressure in the center is diminished by the centrifugal force, substances heavier than air, as leaves and spray, are likely to be driven up in the axis, and floating substances, as cloud, will for the same reason descend. The rising spray and the descending cloud, frequently mark the progress of a vortex in the air, as it moves over a lake or the ocean. Such a phenomenon is called a water-spout. Some philosophers attribute these appearances to *electrical attraction* between a cloud and the earth.

PART V.—ACOUSTICS.

480. *Acoustics* is the science which treats of the nature and laws of sound.

It has for its object, to explain the nature of sound, and the mode of its production—its propagation through air and other media—its reflection from surfaces—the character and relations of musical sounds—and the philosophical principles of music.

◆

CHAPTER I.

NATURE OF SOUND, AND ITS PROPAGATION IN AIR.

481. *Sound.—Vibrations.*—The word *sound* has two significations, either the *impression* produced on the organ of hearing, or that *progressive vibratory movement* in a medium by which the impression is occasioned.

By careful observations, it is ascertained that *vibrations* in the medium surrounding the ear are essential to hearing; and these vibrations are always traceable to the body in which the sound originates. A body becomes a source of sound, by producing an impulse or a series of impulses on the surrounding medium, and thus throwing the medium itself into motion. A single sudden impulse causes a *noise*, with very little continuance; an irregular and rapid succession of impulses, a *crash*, or *roar*, or *continued noise* of some kind; but if the impulses are rapid and perfectly equidistant, the effect is a *musical sound.* In most cases of the last kind, the impulses are vibrations of the body itself; and whatever affects these vibrations, is found to affect the sound emanating from it; and if they are destroyed, the sound ceases.

If we rub a moistened finger along the edge of a tumbler nearly full of water, or draw a bow across the strings of a viol, we can procure sounds which remain undiminished in intensity as long as the operation by which they are excited is continued. In both cases the vibrations are visible; those of the tumbler are plainly seen as crispations on the water to which they are communicated; the string appears as a broad shadowy surface. If a wire or light piece of metal rests against a bell, or glass receiver, when ringing, it will be made to rattle. If sand be strewed on a horizontal plate while a bow is drawn across its edge, the sand will be agitated, and dance over the surface, till it finds certain places where vibrations do not exist. Near an organ-pipe the tremor of the air is perceptible, and pipes of the largest size jar the seats and walls of an edifice. Every species of sound may be traced to impulses or vibrations in the sounding body.

482. *Sonorous bodies.*—Two qualities in a body are necessary, in order that it may be sonorous. It must have a form favorable for vibratory movements, and sufficient strength of elasticity.

The favorable *forms* are in general rods and plates, rather than very compact masses, like spheres and cubes; because the particles of the former are more free to receive lateral movements than those of the latter, which are constrained on every side. But even a thin lamina may have a form which allows too little freedom of motion, such as a spherical shell, in which the parts mutually support each other. If the shell be divided, the hemispheres are bell-shaped and very sonorous.*

The *elasticity* of some materials is too imperfect for continued vibration; thus lead, in whatever form, has no sonorous quality. In other cases, where the elasticity is nearly *perfect*, yet it is a *feeble* force, and hence the vibrations are slow and inaudible. (Art. 96.) Thus india-rubber is almost perfectly elastic, but its force is feeble, and occasions but little sound. So, fibers of wool and cotton, and textures made of them, though highly elastic, have only small elastic force.

483. *Air as a medium of sound.*—There must not only be a vibrating body, as a *source* of sound, but a medium for its *communication* to the organ of hearing. The ordinary medium is air. Let a bell mounted with a hammer and mainspring, so as to continue ringing for several minutes, be placed on a thick cushion under the receiver of an air-pump. The cushion, made of several thicknesses of woolen cloth, is necessary to prevent communication through the metallic parts of the instrument. As the process of exhaustion goes on, the sound of the bell grows fainter, and at length ceases entirely. From this experiment we learn, that sound can not be propagated through a vacant space, even though it be only an inch or two in extent; and also that air conveys sound more feebly as it is more rare. The latter is proved by the faintness of sounds on the tops of high mountains. Saussure relates, that on Mont Blanc the firing of a pistol resembles the report of a cracker. Travelers among the Alps often observe that at great elevations a gun can be heard only a small distance. The fact that meteoric bodies are sometimes heard when passing over at the height of 40 or 50 miles, does not conflict with the above statements; for the velocity of meteors is hundreds of times greater than any other velocities which occur within the earth's atmosphere. On the other hand, when air has more than the natural density, it conveys sound with more intensity, and therefore to a greater distance. In a diving-bell sunk to a considerable depth, a whisper is painfully loud.

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* Millington's Natural Philosophy, p. 123.

484. *Velocity of sound in air.*—Sound occupies an appreciable time in passing through air. This is a fact of common observation. The flash of a distant gun is seen before the report is heard. Thunder usually follows lightning after an interval of many seconds; but if the electric discharge is quite near, the lightning and thunder are almost simultaneous. If a person is hammering at a distance, the perceptions of the blows received by the eye and the ear do not generally agree with each other: or if in any case they do agree, it will be observed that the first stroke seen is inaudible, and the last one heard is invisible; for it requires just the time between two strokes for the sound of each to reach us. Many careful experiments were made in the 18th century to determine the velocity of sound; but as the temperature was not recorded, they have but little value. During the present century, the velocity has been determined by several series of observations in different countries, and all reduced for temperature to the freezing-point. The agreement between them is very close, and the mean of all is 1090 feet per second at 32° F.* To find the velocity for a higher temperature, 0.96 feet should be added for every degree.† Therefore, in most New England climates, the velocity of sound varies more than 100 feet during the year on account of temperature. Probably the celebrated experiments of Derham, in London, 1708, who made the velocity 1142 feet, were performed in the heat of summer.

485. Wind of course affects the velocity of sound by the addition or subtraction of its own velocity, estimated in the same direction, because it transfers the medium itself in which the sound is conveyed. This modification, however, is only slight, for sound moves ten times faster than wind in the most violent hurricane. Besides wind and temperature, other circumstances seem to affect the velocity too little to be perceived.

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* Peirce on Sound, p. 7.

† Ibid., p. 32.

Neither pressure, moisture, nor any change of weather, alters the *velocity* of sound, though they may affect its *intensity*, and therefore the distance at which it can be heard. Falling snow and rain *obstruct* sound, but do not *retard* it. All *kinds* of sound—the firing of a gun—the blow of a hammer—the notes of a musical instrument, or of the voice, however high or low, loud or soft, are conveyed at the same rate. That sounds of different pitch are conveyed with the same velocity was conclusively proved by Biot, in Paris, who caused several airs to be played on a flute at one end of a pipe more than 3000 feet long, and heard the same at the other end distinctly, and without the slightest displacement in the order of notes, or intervals of silence between them.

486. *The calculated velocity.*—For several years there was a large unexplained difference between the calculated velocity of sound and the actual velocity as determined by experiment. While the latter is, as already stated, 1090 feet per second at the freezing-point, calculation gave 916 feet. The difference was at length explained by La Place, who ascertained that it arises from the heat developed in the air by the compression which it undergoes. The calculation regarded the elasticity as varying with the density alone, according to Mariotte's law, assuming that the temperature remained unchanged. But it is a well-known fact, that when air is compressed, a part of its latent heat becomes sensible, and raises its temperature. If the condensation is gradual, the heat is radiated or conducted off, especially if in contact with other bodies; but the heat developed in the propagation of sound has little opportunity to escape, and, though without continuance, augments the elasticity of the air, so as to add 174 feet to the velocity of sound in it.

487. *Diffusion of sound.*—Sound produced in the open air tends to spread equally in all directions, and will do so whenever the original impulses are alike on every side. But this is rarely the case. In firing a gun, the first impulse is given in one direction, and the sound will have more intensity, and be heard further in that direction than in others. It is ascertained by experiment, that a person speaking in the open air can be equally well heard at the distance of 100 feet directly before him, 75 feet on the right and left, and 30 feet behind him; and therefore an audience, in order to hear to the best advantage, should be arranged within limits having these proportions. But, as will be seen hereafter, this rule is not applicable to the interior of a building.*

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* Henry, Proc. Amer. Assoc., Albany, 1856.

Sound is also heard in certain directions with more intensity, and therefore to a greater distance, if an obstacle prevents its diffusion in other directions. On one side of an extended wall, sound is heard further than if it spread on both sides; still further, in an angle between two walls; and to the greatest distance of all, when confined on four sides, and limited to one direction, as in a long tube. The reason in these several cases is obvious; for a given force can produce a given amount of motion, and if the motion is prevented from spreading to particles in some directions, it will reach more distant ones in those directions in which it does spread. Speaking-tubes confine the movement to a slender column of air, and therefore convey sound to great distances, and are on this account very useful in transmitting messages and orders between remote parts of manufacturing edifices and public houses.

488. *Nature of acoustic waves.*—The vibrations of air, in the transmission of sound, are of the kind called *longitudinal;* that is, the particles vibrate longitudinally with regard to the movement of the sound; whereas, in water-waves, the particle-motion is transverse to the wave-motion. (Art. 387.) If, for example, sound is passing from A to B (Fig. 212), the particles between A and *a* are all crowding forward, and produce condensation at *a*; those at *a*, in their turn, crowd upon those forward of them; and thus the condensation is transferred onward toward B. But there is also a rarefaction resulting from the condensation; for the increased density at *a*, causes the particles, that have crowded toward that point, to rebound, while those in advance of them move forward; thus, the particles at any place separate at one moment, as much as they crowded upon each other the moment before. In Fig. 213, the whole line of particles is supposed to be already in a state of vibration; condensed at *a* and *c*; rarefied at A, *b*, and B. From A to *a*, the particles are moving forward; from *a* to *b*, they are rebounding; from *b* to *c*, advancing, &c. The point of condensation, *c*, has traveled from A to its present position, and each of the other phases is following it; and yet no one particle has

FIG. 212.

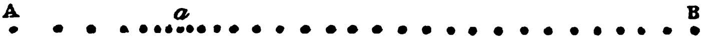
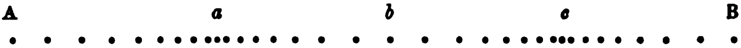


FIG. 213.



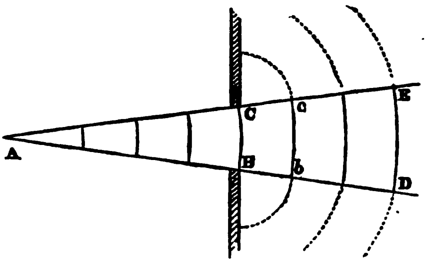
any other motion than a small vibration back and forth in the line, near its original place of rest.

As, in water-waves, we distinguish carefully between the motion of the *wave*, and the motion of the *water* which forms the wave, so here, the wave-motion is totally different from the motion of the air itself. The wave, i. e. the state of condensation and subsequent rarefaction, travels swiftly forward; but the particles of air, which suffer these condensations and rarefactions, simply tremble in the line of that motion.

489. Since the motion is propagated in all directions alike, the entire system of waves around the point where sound originates, consists of spherical strata of air alternately condensed and rarefied. As the quantity set in motion in these successive layers increases with the square of the distance, the amount of motion communicated to each particle must diminish in the same ratio. Hence, the intensity of sound varies inversely as the square of the distance.

490. If air-waves are allowed to pass through an opening in an obstructing wall, they are not entirely confined within the radii of the wave-system produced through the opening; but spread with diminished intensity in lateral directions. Points on the edges of the opening, as B and C (Fig. 214), may be considered as sources of sound; and if they be made centers of concentric spheres, whose radii are equal to the length of the wave, B*b*, or C*c*, and its multiples, then these spherical surfaces will represent the lateral systems of waves which are diffused on every side of the direct beam, BD, CE. But the sound is in general more feeble as the distance from BD, or CE, is greater, and in certain points is destroyed by interference. This spreading of sound in lateral directions may be called the *inflection* of sound.

FIG. 214.



CHAPTER II.

PROPAGATION OF SOUND IN OTHER MEDIA THAN AIR.

491. It has been ascertained, by numerous experiments, that elastic substances, whether gaseous, liquid, or solid, will convey sound, and some of them with much greater velocity than air.

492. *Gaseous bodies, as media of sound.*—Let a spherical receiver, having a bell suspended in it, be exhausted of air, till the bell ceases to be heard; then fill it with any gas or vapor instead of air, and the bell will be heard again. By means of an organ-pipe blown by different gases, it can be learned with what velocity sound would move in each kind of gas experimented upon, because the pitch of a given pipe depends upon the velocity of the waves, as will be seen hereafter. In hydrogen, sound is exceedingly feeble, but moves nearly three times as fast as in air. Momentary development of heat by compression produces, in all gaseous bodies, the effect of increasing the velocity of sound.

493. *Liquids as media.*—Many experimenters have determined the circumstances of the propagation of sound in water. Franklin found that a person with his head under water, could hear the sound of two stones struck together, at a distance of more than half a mile. In 1826, Colladon made many careful experiments in the water of Lake Geneva. The results of these and other trials, are principally the following:

1. Sounds produced in the air are very faintly heard by a person in water, though quite near; and sounds originating under water are feebly communicated to the air above, and in positions somewhat oblique, are not heard at all.

2. Sounds are conveyed by water with a velocity of 4700 feet per second, at the temperature of 47° F., which is more than four times as great as in air. The calculated and the observed velocity of sound in water agree so nearly with each other, that there appears to be no appreciable effect arising from heat developed by compression.

3. Sounds conveyed in water to a distance, lose their sonorous quality. For example, the ringing of a bell gives a succession of short sharp strokes, like the striking together of two knife blades. The musical quality of the sound is noticeable only within 600 or 700 feet. In air, it is well known that the contrary takes place; the blow of the bell-tongue is heard near

by, but the continued musical note is all that affects the ear at a distance.

4. Acoustic *shadows* are formed; that is, sound passes the edges of solid bodies nearly in straight lines, and does not turn around them except in a very slight degree. In this respect, sound in water resembles light much more than it does sound in air.

To enable the experimenter to hear distant sounds without placing himself under water, Colladon pressed down a cylindrical tin tube, closed at the bottom, thus allowing the acoustic pulses in the water to strike perpendicularly on the sides of the tube. In this way, the faintest sounds were brought out into the air. It appears to be true of sound as of light, that it can not pass from a denser to a rarer medium at large angles of incidence, but suffers nearly a total reflection.

494. *Solids as media.*—Solid bodies of high elastic energy are the most perfect media of sound which are known. An iron rod—as, for instance, a lightning-rod—will convey a minute sound from one extremity to the other, with much more distinctness than the air. If the ears are stopped, and one end of a long wire is held between the teeth, a slight scratch or tap on the remote end will sound very loud. The sound in this case travels through the wire and the bones of the head to the organ of hearing. The *stethoscope*, an instrument used by physicians for determining whether the lungs or heart have a diseased or healthy action, illustrates the conduction of sound by solids. The instrument is a tubular rod of wood, one end of which is pressed upon the chest of the patient, while the ear is applied to the other. The movements of the vital organs are thus distinctly heard, and the character of those movements readily distinguished. The sound of earthquakes and volcanic eruptions is transmitted to great distances through the solid earth.

495. In one important particular, solids differ from fluids—namely, in the fixed relations of the particles among themselves. These relations are usually different in different directions; hence, sound is likely to be transmitted more perfectly in some directions through a given solid, than in others. The scratch of a pin at one end of a stick of timber seems loud to a person whose ear is at the other end. The sound is heard more perfectly in the direction of the grain than across it. In crystallized substances, it is unquestionably true, that the vibrations of sound move with different speed and with different intensity, in the line of the axis, and in a line perpendicular to it.

496. The velocity of sound in cast-iron is about 11,000 feet per second,—ten times greater than in air. This was determined by Biot, in his experiments on some aqueduct pipes in Paris, already alluded to. A blow upon one end was brought to an observer at the other through two channels, and seemed to be two blows. One sound traveled in the air within the tube, the other in the iron itself of which the pipe was made. The observed interval of time between the two sounds, and the known velocity of sound in air, enabled Biot to calculate with ease the rate of sound in iron. The pitch of sound produced by rods and tubes of different materials, when vibrating longitudinally, enables us to determine with tolerable accuracy the velocity of propagation in those substances respectively.

497. *Mixed media.*—In all the foregoing statements, it has been supposed that the medium was homogeneous; in other words, that the material, its density, and its structure, continue the same, or nearly the same, the whole distance from the source of sound to the ear. If abrupt changes occur, even a few times, the sound is exceedingly obstructed in its progress. When the receiver is set over the bell on the pump plate, the sound in the room is very much weakened, though the glass may not be one-eighth of an inch in thickness, and is an excellent conductor of sound. The vibrations of the internal air are very imperfectly communicated to the glass, and those received by the glass pass into the air again with a diminished intensity. If a glass rod extended the whole distance from the bell to the ear, the sound would arrive in less time, and with more loudness, than if air occupied the whole extent. For a like reason, walls, buildings, or other intervening bodies, though good conductors of sound themselves, obstruct the progress of sound in the air. This explains the facts mentioned in Art. 493, that sound in air is heard faintly in water, and *vice versa.* When the texture of a substance is loose, having many alternations of material, it thereby becomes unfit for transmitting sound. It is for this reason that the bell-stand, in the experiment just referred to, is set on a cushion made of several thicknesses of loose flannel, that it may prevent the vibrations from reaching the metallic parts of the pump. The waves of sound, in attempting to make their way through such a substance, continually meet with new surfaces, and are reflected in all possible directions, by which means they are broken up into a multitude of crossing and interfering waves, and are mutually destroyed.* A tumbler, nearly filled with water, will ring clearly; but, if filled with an effervescing liquid, it will lose all its sonorous quality, for the same cause as before. The alternate surfaces of the liquid and gas, in the foam, confuse the waves, and deaden the sound.

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* Herschel, on Sound, Ency. Metr.

CHAPTER III.

REFLECTION OF SOUND.

498. *Echoes.*—Sound is *reflected* by hard bodies, producing the well known phenomenon called an *echo*. If a straight line be drawn from the sounding body to the reflecting surface, representing the course of the sound before reflection, and another straight line be drawn from the reflecting surface, in the direction of the sound after reflection, these two lines will make equal angles with that surface; that is, when sound is reflected, *the angle of reflection is equal to the angle of incidence.*

The surfaces of various bodies, solid and fluid, have been found capable of reflecting sounds, viz., the sides of hills, houses, rocks, banks of earth, the large trunks of trees, the surface of water, especially at the bottom of a well, and sometimes even the clouds. It is, therefore, evident that in an extensive plain, or at sea, where there is no elevated body capable of reflecting sounds, no echo can be heard. It is hence easy to see why the poets, who convert echo into an animated being, place her habitation near mountains, rocks, and woods.

499. The most frequent instances of the reflection of sound, are from surfaces which may be considered as plane. In these, the sound issuing from any point seems, after reflection, to proceed from a point equally distant, and similarly situated, on the other side of the reflecting surface; the phenomena differing a little according to the position of the speaker, with respect to the body which occasions the reflection. If a person's voice strike any surface perpendicularly, it will be reflected back in the same line; and the time occupied between the utterance of the sound and its arrival again at the speaker, will be equal to the time in which the sound travels through twice the distance between the speaker and the reflecting surface, and hence the interval may be made a measure of the distance. If the sound of the voice returns to the speaker in two seconds, when the temperature is 74°, the distance of the reflecting surface is 1130 feet, and in that proportion for other intervals. Thus, the breadth of a river may be ascertained when there is an echoing rock on the further shore. A perpendicular mountain's side, or lofty cliffs, such as frequently skirt the sea-coast, sometimes return an echo of the discharge of artillery, or of a clap of thunder, to the distance of many miles. The number of syllables that can be pronounced in the interval will be repeated distinctly; but a greater number would be blended with the commencement of the echo.

500. When a single obstacle reflects the sound, the echo is *simple;* when there are several obstacles disposed at suitable distances, the echo is *complex.* Echoes of the latter kind have been observed which repeated the original sound forty times. Two parallel walls which mutually reverberate the sound, may produce a double or complex echo, with regard to an auditor placed in the intermediate space. The sound of artillery and of thunder, is frequently prolonged by reverberations in an uneven country.

The rolling of thunder has been attributed to echoes among the clouds; and that such is the case, has been ascertained by direct observation on the sound of a cannon. Under a perfectly clear sky, the explosion of guns is heard single and sharp; while when the sky is overcast, or when a large cloud comes overhead, the reports are accompanied by a continued roll, like thunder, and occasionally a double report arises from a single shot.*

The continued sound of distant thunder, which is sometimes prolonged for many seconds, is not always owing to reverberation, but frequently arises simply from the different distances of the same flash. Although the progress of a flash of lightning through the air were absolutely instantaneous, still, if its path were in a line that would carry it further from the ear in one place than in another, there would be a corresponding difference in the times at which the sound generated in different portions of the path would reach the ear. Herschel observes, that if (as is almost always the case) the flash be crinkled, and composed of broken rectilinear and curvilinear portions, some concave, some convex to the ear,—and especially if the principal trunk separates into many branches, each breaking its own way through the air, and each becoming a separate source of thunder,—all the varieties of that awful sound are easily accounted for.†

501. *Concentrated echoes.*—The divergence of sound from a plane surface continues the same as before, that is, in spherical waves, whose center is at the same distance behind the plane as the real source is in front. But concave surfaces in general produce a concentrating effect. A sound originating in the center of a hollow sphere, will be reflected back to the center from every point of the surface. If it emanates from one focus of an ellipsoid, it will, after reflection, all be collected at the other focus. So, if two concave paraboloids stand facing each other, with their axes coincident, and a whisper is made at the focus of one, it will be plainly heard at the focus of the other, though inaudible at all points between. In the last case, the sound is twice reflected, and passes from one reflector to the other in parallel lines. All these effects are readily proved from the principle that the angles of incidence and reflection are equal.

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* Herschel, on Sound. † Ibid.

The *speaking trumpet* has been supposed, by most writers on sound, to owe its peculiar properties to its multiplying sound by numerous reflections. Hence is suggested the form of a paraboloid, the place of the mouth being at its focus. The vibrations emanating from the mouth would then be reflected into straight lines parallel with the axis of the trumpet, and would thus go forward in a collected body to a distant point. And, since such a form is also favorable for collecting distinct sounds into one point, the same figure is proposed as the most suitable for the *ear trumpet.* But the sound of these instruments may be regarded as merely the longitudinal vibration of a body of air, to which momentum is given in the direction of the axis, not by reflection from the sides, but by the direct impulse of the mouth (Art. 487).

502. Concave surfaces cause all the curious effects of what are called *whispering galleries*, such as the dome of St. Paul's, in London. In many of these instances, however, there seems to be a continued series of reflections from point to point along the smooth concave wall, which all meet simultaneously (if the curves are of equal length) at the opposite point of the dome; for the whisperer places his mouth, and the hearer his ear, close to the wall, and not in a focus of the curve. The *Ear of Dionysius* was probably a curved wall of this kind in the dungeons of Syracuse. It is said that the words, and even the whispers, of the prisoners were gathered and conveyed along a hidden tube to the apartment of the tyrant. The sail of a ship when spread, and made concave by the breeze, has been known to concentrate and render audible to the sailors the sound of a bell 100 miles distant. A concave shell held to the ear concentrates such floating sounds as may exist in the air, and is suggestive of the murmur of the ocean.

503. *Reflection of sound in rooms.*—If a rectangular room has smooth, hard walls, and is unfurnished, its reverberations will be loud and long-continued. Stamp on the floor, or make any other sudden noise, and its echoes passing back and forth will form a prolonged musical note, whose pitch will be lower as the apartment is larger. This is called the *resonance* of the room. Now, let furniture be placed around the walls, and the reverberations will be weakened and less prolonged. Especially will this be the case if the articles be of the softer kinds, and have irregular surfaces. Carpets, curtains, stuffed seats, tapestry, and articles of dress have great influence in destroying the resonance of a room. The appearance of an apartment is not more changed than is its resonance, by furnishing it with carpet and curtains. The blind, on entering a strange room, can, by the sound of the first step, judge with tolerable accuracy of its size and the general character of its furniture.

The reason why substances of loose texture do not reflect sound well, is essentially the same as what has been stated (Art. 497), for their not transmitting well; they are not homogeneous,—the waves are reflected in all directions by successive surfaces, interfere with each other, and are destroyed.

504. *Halls for public speaking.*—In large rooms, such as churches and lecturing halls, all echoes which can accompany the voice of the speaker syllable by syllable, are useful for increasing the volume of sound; but all which reach the hearers sensibly later, only produce confusion. It is found by experiment, that if a sound and its echo reach the ear within *one-sixteenth* of a second of each other, they seem to be one. Hence, this fraction of time is called the *limit of perceptibility.* Within that time an echo can travel about 70 feet more than the original sound, and yet appear to coincide with it. If an echoing wall, therefore, is within 35 feet of the speaker, each syllable and its echo will reach every hearer within the limit of perceptibility.* Walls intended to aid by their echoes should be smooth, but not too solid; plaster on lath is better than plaster on brick or stone; the first echo is louder, and the reverberations less. Drapery behind the speaker deprives him of the aid of just so much echoing surface. A lecturing hall is improved by causing the wall behind the speaker to change its direction, on the right and left of the platform, at a very obtuse angle, so as to exclude the rectangular corners from the room. The voice is in this way more reinforced by reflection, and there is less resonance arising from the parallelism of opposite walls. Paneling, and any other recesses for ornamental purposes, may exist in the reflecting walls without injury, provided they are plane, and parallel to the general surface. The ceiling should not be so high that the reflection from it would be delayed beyond the limit of perceptibility. Concave surfaces, such as domes, vaults, and broad niches, should be carefully avoided, as their effect generally is to concentrate all the sounds they reflect. An equal diffusion of sound throughout the apartment, not concentration of it to particular points, is the object to be sought in the arrangement of its parts.

505. As to distant parts of a hall for public speaking, the more completely all echoes from them can be destroyed, the more favorable is it for distinct hearing. It is indeed true, that if a hearer is within 35 feet of a wall, however remote from the speaker, he will hear a syllable, and its echo from that wall, as one sound; but to all the audience at greater distances from the same wall, the echoes will be perceptibly retarded, and fall upon subsequent syllables, thus destroying distinctness. The distant walls should, by some means, be broken up into small portions, presenting surfaces in different directions. A gallery may aid in effecting this; and the seats of the gallery and of the lower floor may rise rapidly one behind another, so that the audience receive directly much of the sound which would otherwise go to the remote wall, and be reflected. Especially should no large and distant surfaces be *parallel* to nearer ones, since it is between parallel walls that prolonged reverberation occurs.*

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* Henry.

CHAPTER IV.

MUSICAL SOUNDS, AND MODES OF PRODUCING THEM.

506. *Musical sounds.*—When the impulses of a sounding body upon the air are equidistant, and of sufficient frequency, they produce what is termed a musical sound. In most cases these impulses are the isochronous vibrations of the body itself, but not necessarily so; it is found by experiment that blows or puffs, of any species whatever, if they are more than about 15 or 20 per second, and possess the property of *isochronism*, cause a musical tone. For example, the snapping of a stick on the teeth of a metallic wheel would seem as unlikely as any thing to produce a musical sound; but when the wheel is in rapid motion, the succession causes a pure musical note. Equidistant *echoes* often produce a musical sound, as when a person stamps on the floor of a rectangular room, finished, but unfurnished (Art. 503). So, on a walk by the side of a long baluster fence, a sudden sharp sound, like the blow of a hammer on a stone, brings back a tone more or less prolonged, resembling the chirp of a bird. It is occasioned by successive equidistant echoes from the balusters of the fence. A flight of steps will sometimes produce the same effect, the tone being on a lower key than that from the fence, as it should be.

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* See a valuable paper on "Acoustics applied to Public Buildings," by Prof. J. Henry. Proceedings of the Amer. Assoc., Albany, 1856. The lecture-room of the Smithsonian Institution, constructed under the direction of Prof. Henry, in accordance with the principles of that paper, is pronounced by those who have attended and delivered lectures in it, to be the best room of the same capacity in the country for easy speaking and distinct hearing.

507. What is called the *pitch* of a musical sound, or its degree of acuteness, is owing entirely to its rate of vibration. Other qualities of sounds are due to other, and often unknown circumstances; but rapidity of vibration is the only condition on which the pitch depends. In comparing one musical sound with another, if the number of vibrations per second is greater, the sound is more acute, and is said to be of a *higher pitch;* if the vibrations are fewer per second, the sound is graver, or of a *lower pitch.*

Musical sounds have occupied the attention of philosophers more than any other class of sounds. The superior precision with which the ear can estimate any variation in pitch, renders these sounds more easily compared; and the vibrations of the sonorous bodies which produce them, are, on account of their superior simplicity of form, more easily investigated.*

508. *The monochord.*—If a string of uniform size and texture is stretched on a box of thin wood, by means of a pulley and weight, the instrument is called a *monochord,* and is useful for studying the laws of vibrations in musical sounds. The sound emitted by the vibrations of the whole length of the string is called its *fundamental* sound.

If the string be drawn aside from its straight position, and then released, one component of the force of tension urges every particle back towards its place of rest; but the string passes beyond that place, on account of the momentum acquired, and deviates as far on the other side; from which position it returns, for the same reason as before, and continues thus to vibrate till obstructions destroy its motion. By the use of a bow, the vibrations may be continued as long as the experimenter chooses.

The pitch of the fundamental sound of musical strings, is found by experience to depend on three circumstances; the *length* of the string,—its *weight* or quantity of matter,—and its *tension.* The tone becomes more acute as we increase the tension, or diminish either the length or the weight. The operation of these several circumstances may be seen in a common violin. The pitch of any one of the strings is raised or lowered by turning the screw so as to increase or lessen its tension; or, the tension remaining the same, higher or lower notes are produced by the same string, by applying the fingers in such a manner as to shorten or lengthen the string which is vibrating; or, both the tension and the length of the string remaining the same, the pitch is altered by making the string larger or smaller, and thus increasing or diminishing its weight.

The time of a *single vibration* (in distinction from a *double* vibration), is the time occupied by a string in passing from the extreme point on one side to the extreme point on the other. Independently of calculation, it is easy to see, that with a given weight per inch, and a given tension, the string will vibrate *slower*, if *longer*, since there is more matter to be moved, and only the same force to move it; and for a similar reason, the length and tension being given, it will also vibrate *slower*, if *heavier*. On the other hand, if length and weight are given, it will vibrate *faster*, if the *tension* is *greater;* because a greater force will move a given quantity at a swifter rate.

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* Ed. Encycl., art. *Acoustics.*

509. *Time of a vibration.*—The mathematical formula, which has been obtained with much labor, is the following, in which T = the time of a single vibration; L = the length of the string in inches; w = the weight of one inch of the string; t = the tension in lbs.; and g = the force of gravity = 386 inches:

$$T = L \left(\frac{w}{gt}\right)^{\frac{1}{2}}.$$

As the *distance* of the string from its quiescent position (called the *amplitude* of its vibration), does not form an element of the algebraic expression, which is thus found for the time of a vibration, it follows that the time is independent of the amplitude. Hence, as in the pendulum, the vibrations of a string, fixed at both ends, are performed in equal times, whether the amplitude of the vibrations be greater or smaller. It is on this account that the pitch of a string does not alter, when left to vibrate till it stops. The excursions from side to side grow less, and therefore the sound more feeble, till it ceases; but the *rate* of vibration, and therefore the pitch, remains unaltered to the last. This property of *isochronism*, independent of extent of excursion, is common to sounding bodies generally, and is owing to what may be called *the law of elasticity*, *that the restoring force*, *acting on any particle*, *varies directly as its distance from the place of rest.* For example, each particle of the string, if removed twice as far from its place of rest, is urged back by an accelerating force twice as great, and therefore returns in the same time.

510. The number of vibrations performed by a string in a second of time, being inversely as the time of one vibration, it is expressed by the reciprocal of the formula denoting the time; so that if N represents the number of vibrations, we shall have the following expression: $N = \frac{\sqrt{gt}}{L\sqrt{w}}$.

The frequency of vibration which this equation gives, is found to agree very exactly with the result of experiments.

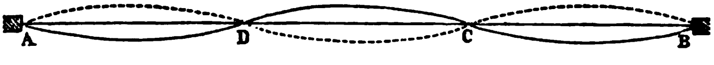
To express the *relation* between the vibrations of different strings, we merely reject the constant, g; and then we have

$$N \propto \frac{\sqrt{t}}{L\sqrt{w}}$$

According, then, as we diminish the length of a string, and the weight of an inch of it, or increase its tension, we increase its frequency of vibration; but equal changes in these circumstances do not produce equal effects. Thus, if in different strings, their tension and the weight of an inch remain the same, *their frequency of vibration will be inversely as their lengths;* for then $N \propto \frac{1}{L}$. If we make the length one-third, we triple the number of vibrations, and so for any other ratio. If the length and tension remain the same, $N \propto \frac{1}{\sqrt{w}}$, or, *the number of vibrations is inversely as the square roots of the weights;* consequently a string four times as heavy as another will vibrate half as fast. The bass strings in most instruments have fine wire wound round them to increase their quantity of matter, for wound wire has more flexibility and fullness of tone than solid wire of the same weight. If the length and the weight of equal portions be the same, then $N \propto \sqrt{t}$, or *the frequency of vibration is as the square root of the tension.* Therefore, we must give the string of a violin four times the tension in order to make it vibrate twice as fast.

511. *Vibrations of a string in parts.*—The monochord may be made to vibrate in parts, the points of division remaining at rest; and this mode of vibration may even coexist with the one already described. Of course the sound produced by the parts will be on a higher pitch, since they are shorter, while the tension and the weight per inch remain unaltered. It is a noticeable fact, that the parts are always such as will exactly measure the whole without a remainder. Hence the vibrating parts are either halves, thirds, fourths, or other aliquot portions. The sounds produced by any of these modes of vibration are called *harmonics*, for a reason which will appear hereafter.

Fig. 215.



Suppose a string (Fig. 215) to be stretched between A and B, and that it is thrown into vibration in three parts. Then while

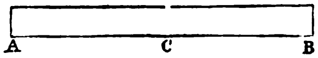
AD makes its excursion on one side, DC will move in the opposite direction, and CB the same as AD; and when one is reversed, the others are also, as shown by the dotted line. In this way D and C are kept at rest, being urged toward one side by one portion of string, and toward the opposite by the next portion. But the string may at the same time vibrate as a whole; in which case D and C will have motion to each side of their former places of rest, while relatively to them the three portions will continue their movements as before. The points C and D are called *nodes;* the parts AD, DC, and CB, are called *ventral segments.* By a little change in the quickness of the stroke, the bow may be made to bring from the monochord a great number of harmonic notes, each being due to the vibrations of certain aliquot parts of the string. By confining a particular point, however, at the distance of $\frac{1}{2}$, $\frac{1}{3}$, or other simple fraction of the whole from the end, the particular harmonic belonging to that mode of division may be sounded clear, and unmingled with the others.

512. *Vibrations of a column of air.*—When a musical sound is produced by a pipe of any kind, it is the column of inclosed air which must be regarded as the sounding body. A condensed wave is caused, by some mode of excitation, to travel back and forth in the pipe, followed by a rarefied portion; and these waves affect the surrounding air much in the same way as the alternate excursions of a string. That it is the air, and not the pipe itself, which is the source of sound, is proved by using pipes of various materials—the most elastic, and the most inelastic—as glass, wood, paper, and lead; if they are of the same form and size, the tone in each case has the same pitch.

In order to examine the manner in which the air-columns in pipes perform their vibrations, it is convenient to consider them in three classes: 1st. Pipes which are closed at both ends; 2d. Those which are closed at one end and open at the other; 3d. Those which are open at both ends.

513. *Both ends of the pipe closed.*—Suppose the ends of the pipe, ACB (Fig. 216), to be closed, and an impulse in some way to be communicated at the center, C; then the motion of the column will consist of a constant and regular fluctuation of the whole mass to and fro within the pipe, the air being always condensed in one half, while it is rarefied in the other. While the condensed pulse moves from B to C, the point of rarefaction runs from A to C, where they pass each other; hence, at the middle of the pipe there is no change of density, since every degree of condensation is at that point

FIG. 216.



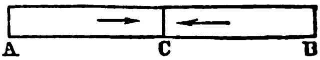
met by an equal degree of rarefaction of the other half of the general wave. At the extremities, A and B, there is alternately a *maximum* of condensation and of rarefaction, each being reflected and returning, to meet again at C. At all points between the center and the ends, there is alternate condensation and rarefaction, but in a less degree according to the distance from the ends.

On the other hand, the *excursions* of the particles are greatest at C, and nothing at A and B, where all motion is prevented by the fixed stoppers by which the pipe is closed. Between the ends and the center, the amplitude of vibration is greater, as the distance from the center is less.

The pitch of such a pipe will be lower, as the pipe is longer, because the waves have a greater distance to travel between the successive reflections, and hence there will be a smaller number per second. So also, lowering the temperature lowers the pitch, since the wave then travels more slowly, and suffers fewer reflections in a second.

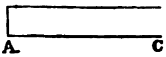
514. *Vibrations of a column of air in parts.*—The same is true of a column of air as of a string, that it may vibrate in parts; and also, that different modes of vibration may coexist in the same column. For example, if two condensed pulses should start simultaneously from A and B (Fig. 217), they would reach the middle point, C, at the same instant, and would produce condensation there, without any movement of C either to the right or left; and then would be reflected from C toward the ends. Thus, the center, in this case, is exactly like the ends, having alternate condensations and rarefactions, but no amplitude of vibration. The pulses will run through each half *twice*, while a wave would go over the whole length *once;* and therefore the pitch will be raised accordingly. AC and BC are called *ventral segments*, and C is called a *node*, or *nodal section.* Even while there is a wave running the whole length, AB, there may coexist with it another system of subordinate waves in each half, just as we have seen to be true of a string. And an air-column may not only vibrate in *two* equal parts, but in *three*, *four*, or any greater number. In very long pipes this is more likely to occur.

FIG. 217.



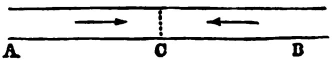
515. *One end of the pipe closed, the other open.*—If, while the column AB is vibrating as a whole, an aperture is made at the center, or even if the pipe is divided there, so that the aperture extends entirely round it, this will not interrupt the oscillation already described, because there is neither rarefaction nor condensation at the point C, and hence no tendency there to lateral motion. The means employed for exciting vibrations may therefore be applied at the open section. Let the pipe AB (Fig. 216), remaining stopped at A and B, be divided at the middle, C; and let the half pipe BC be removed, while the exciting cause remains at C (Fig. 218), then the vibrations in AC will still continue, and the pitch be unaltered. For now the condensed pulse, on reaching C, will be returned to A by the vibrating disk or spring which excites it, and will make a second reflection at A at the same instant as it would have done at B in the whole pipe AB; thus the same movements are performed now in *one* half which were before performed alternately in the *two*. Hence it is, that a pipe with only one end closed, and a pipe of twice its length, with both ends closed, give the same pitch.

FIG. 218.



516. *Both ends of the pipe open.*—When both ends of a pipe are open, it may still produce a musical tone, by having a node in the center of it, thus forming two pipes like the one last described. When the vibration is established in such a pipe, the pulses from the ends move simultaneously toward C (Fig. 219), and again from it after reflection. Thus C is a fixed point, where the greatest condensation and rarefaction occur alternately, like A in Fig. 218. It therefore has the same pitch as AC alone, stopped at C and open at A. If a solid partition be inserted at C, it causes no change of pitch.

FIG. 219.



517. In comparing with each other the three kinds of pipe which have been described, it is observable, that the *first* kind (stopped at both ends), and the *third* kind (open at both ends), is each a double pipe of the *second* kind (open at one end, and stopped at the other). For, if two pipes of the second kind be placed with their open ends together, as we have seen, they form one of the first kind, and there is no change of pitch. Again, if the two be placed with the closed ends in contact, they form a pipe of the third class; since the partition may remain or be removed, without affecting the mode of vibration. Hence, a pipe open at both ends, and one of the same length closed at both ends, each yields the same fundamental note as a pipe of half their length, open only at one end.

The second and third species of pipes may have several nodes, and vibrate in a series of parts, just as has been stated in regard to the first kind (Art. 514).

518. *Modes of exciting vibrations in pipes.*—There are two methods of making the air-column in a pipe to vibrate: one by a stream of air blown across an orifice in the pipe, the other by a metallic plate called a reed. A familiar example of the first is the *flute.* A stream of air from the lips is directed across the *embouchure,* so as just to strike the opposite edge; this causes a wave to move through the tube. The stream of air, like a spring, vibrates so as to keep time with the movement of the wave to and fro, while at each pulse it renews that movement, and makes the sound continuous. For higher notes, the stream must be blown more swiftly, that by its greater elastic force, it may be able to conform to the more rapid vibration of the column. A large proportion of the pipes of an organ are made to produce musical tones essentially in the same way as the flute.

The *clarinet* is an example of vibrations in an air-column by a reed. In that instrument the reed is often made of wood; when the air is blown past its edge into the tube, the reed is thrown into vibration, and by it the column of air. The strength of elasticity in the reed should be such that its vibrations will keep time with the excursions of the wave in the column. What are called the reed pipes of the organ are constructed on the same principle.

519. *Vibrations of rods and laminæ.*—A plate of metal called a reed is much used for musical purposes in connection with a column of air, as already stated. Except in such connection, the sounds of wires and laminæ are generally too feeble to be employed in music. But their vibrations have been much studied, on account of the interesting phenomena attending them.

520. *Wires.*—If one end of a steel wire is fastened in a vise and vibrated, while a thin blade of sunlight falls across it, its path may be traced. It will not ordinarily vibrate in a plane, but in some irregular figure. A frequent mode is, two series of ellipses, whose axes are inclined to each other, the vibrations passing alternately from one system to the other several times before running down. If the structure of the wire were the same in every part across its section, and if the fastening pressed equally on every point around it, the orbit of each particle would be a series of ellipses, whose major axes are on the same line. If, moreover, there was no obstruction to the motion, and the law of elasticity could obtain perfectly, it would vibrate in the same elliptic orbit forever, the force toward the center being directly as the distance. It is easy to cause the wire, in the experiment just described, to vibrate also in parts; in which case each atom, while describing the elliptic orbit, will perform several smaller circuits, which appear as waves on the circumference of the larger figure.

521. *Chladni's plates.*—If a square plate of glass or elastic metal, of uniform thickness and density, be fastened by its center in a horizontal position, and a bow be drawn on its edge, it will emit a pure musical tone; and by varying the action of the bow, and touching different points of the edge with the finger, a variety of sounds may be obtained from it. The plate necessarily vibrates in parts; and the lowest pitch is produced when there are two nodal lines parallel to the sides, and crossing at the center, thus dividing the plate into four square ventral segments. The position of the nodal lines, and the forms of the segments, are beautifully exhibited by sprinkling writing-sand on the plate. The particles will dance about rapidly till they find the lines of rest, where they will presently be collected. For every new tone the sand will show a new arrangement of nodal lines; and as two or more modes of vibration may coexist in plates, as well as in strings and columns of air, the resultant nodes will also be rendered visible. Again, by fastening the plate at a different point, still other arrangements will take place, each distinguishable by the position of its nodal lines and the pitch of its musical note. The form of the plate itself may also be varied, and each form will be characterized by its own peculiar systems. Chladni, who first performed these interesting experiments, delineated and published the forms of *ninety* different systems of vibration in the square plate alone.

If a fine light powder, as lycopodium (the pollen of a species of fern), be scattered on the plate, it is affected in a very different manner from heavy sand. It will gather into conical heaps on those portions of the segments which have the greatest amplitude of vibration; the particles which compose the heaps performing a continual circulation, down the sides of the cones, along the plate to the center, and up the axis. If the vibration is violent, the heaps will be thrown up from the plate in little clouds over the portions of greatest motion. The cause of this singular effect was ascertained by Faraday, who found that in an exhausted receiver the phenomenon ceased. It is due to a circulation of the air, which lies in contact with a vibrating plate. The air next to those parts which have the greatest amplitude, is at each vibration thrown upward more powerfully than elsewhere, and surrounding particles press into its place, and thus a circulation is established; and a fine light powder is more controlled by these atmospheric movements than by the direct action of the plate.

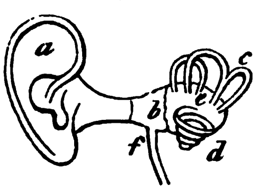
522. *Bells.*—If a thin plate of metal takes the form of a cylinder or bell, its fundamental note is produced when each ring of the material changes from a circle to an ellipse, and then into a second ellipse, whose axis is at right angles to the former.

It thus has four ventral segments and four nodal lines, the latter lying in the plane of the axis of the bell or cylinder. If the rings which compose the bell were all detached from one another, they would have different rates of vibration according to their diameter, and hence would produce tones of various pitch; but, being bound together by cohesion, they are compelled to keep the same time, and hence give but one fundamental tone. But a bell, especially if quite thin, may be made to emit a series of harmonic sounds, by dividing up into a greater number of segments. It is obvious that the number of nodes must always be *even*, because two successive segments must move in opposite directions in one and the same instant; otherwise the point between them could not be kept at rest, and therefore would not be a node. Besides the principal tone of a church-bell, one or two subordinate sounds on a different pitch may usually be detected. A glass bell, suitably mounted for the lecture-room, will yield *ten* or *twelve* harmonics, by means of a bow drawn on its edge.

523. *The voice.*—The vocal organ is complex, consisting of a cavity called the *larynx*, and a pair of membranous folds like valves, called the *glottis*, which admit the air to the larynx from the windpipe below. In the act of breathing, the folds of the glottis lie relaxed and separate from each other, and the air passes freely between them, without producing vibration. But in the effort to form a vocal sound, these folds approach each other, forming between them a narrow space, like a button-hole, and become tense, so that the current of air throws them into vibration. These vibrations are enforced by the consequent vibrations in the air of the larynx above; and thus a fullness of sound is produced, as in many musical instruments, in which a reed, and the air of a cavity, perform synchronous vibrations, and emit a much louder sound than either could do alone. The natural kèy of a person's voice depends mainly on the length of the edges of these folds or valves. The yielding nature of all the parts, and the ability, by muscular action, to change the form and size of the cavity and the tension of the valves, give great variety to the pitch, and the power of adjusting it with precision to every shade of sound within certain limits. No instrument of human contrivance can be brought into comparison with the organ of voice. After the voice is formed by its appropriate organ, it undergoes various modifications, by means of the palate, the tongue, the teeth, the lips, and the nose, before it is uttered in the form of articulate speech.

524. *The ear.*—The parts of the *ear*, and the progress of sound to the sentient nerve, may be simply described as follows:

FIG. 220.



1. There is externally a wide-mouthed tube or ear-trumpet, *a* (Fig. 220), for catching and concentrating the pulses of sound. In many animals it is movable, so that they can direct it to the place from which the sound comes.

2. The sound concentrated at the bottom of the ear-tube falls upon a membrane, which completely separates the outer ear from the cavity, *b*, beyond it. The membrane and cavity constitute what is called the *tympanum*, or drum of the ear. That its motion may be free, the air contained within the drum has free communication with the external air by the open passage *f*, called the *Eustachian tube*, leading to the back part of the mouth. A degree of deafness ensues when this tube is obstructed by wax.

3. The vibrations of the tympanum are conveyed further inwards by a chain of four bones (not here represented on account of their minuteness), reaching from the center of the tympanum to the oval door or window of the *labyrinth*, *e*.

4. The labyrinth, or complex inner compartment of the ear, over which the nerve of hearing is spread as a lining, is full of water; and therefore, when the vibrations of the tympanum acting through the chain of bones (3) are communicated to this fluid, they are instantly felt over the whole cavity. The labyrinth consists of the *vestibule*, *e*, the three *semi-circular canals*, *c*, imbedded in the hard bone, and of a winding cavity, *d*, called the *cochlea*, like that of a snail shell, in which fibers, stretched across like harp-strings, constitute the *lyra*. The exact uses of these various parts are not yet perfectly known. The membrane of the tympanum may be pierced, and the chain of bones may be broken, without loss of hearing; but if the fluid of the cochlea flows out, the hearing is destroyed.

CHAPTER V.

MUSICAL SCALES.—THE RELATIONS OF MUSICAL SOUNDS.

525. *Numerical relations of the notes.*—To obtain the series of notes which compose the common scale of music, it is convenient to use the monochord. Calling the sound, which is given by the whole length of the string, the *fundamental*, or *key note*, of the scale, we measure off the following fractions of the whole for the successive notes; namely, $\frac{8}{9}$, $\frac{4}{5}$, $\frac{3}{4}$, $\frac{2}{3}$, $\frac{3}{5}$, $\frac{8}{15}$, $\frac{1}{2}$. If the whole, and these fractions, are made to vibrate in order, the ear will recognize the sounds as forming the series called the *gamut*, or *diatonic scale.* And the interval between the fundamental and each of the others, is named according to its distance inclusively. Thus, the interval from the whole (= 1) to $\frac{8}{9}$, is called the *second;* from 1 to $\frac{4}{5}$, the *third*, &c.; therefore, from 1 to $\frac{1}{2}$, the *eighth*, or *octave.* Now, as the number of vibrations varies inversely as the length of the string, the numbers corresponding to the notes respectively, are expressed by the same fractions inverted, 1, $\frac{9}{8}$, $\frac{5}{4}$, $\frac{4}{3}$, $\frac{3}{2}$, $\frac{5}{3}$, $\frac{15}{8}$, 2. Reducing these to a common denominator, and using the numerators (since they have the same ratios), we have the following series, 24, 27, 30, 32, 36, 40, 45, 48, to express in the simplest manner the relative numbers of vibrations in the notes of the scale, however produced. The sounds represented by these numbers are not arbitrarily chosen to form the scale, but they are demanded by the ear, and constitute the basis of the music of all ages and nations.

526. In examining the ratio of each two successive numbers in the foregoing series, we find these ratios have three different magnitudes, as follows:

24 : 27, 32 : 36, and 40 : 45, is each as 8 : 9;
27 : 30, and 36 : 40, ... as 9 : 10;
30 : 32, and 45 : 48, ... as 15 : 16.

Of these seven intervals, the first five, though not all equal, are called *whole tones;* and the last two, though larger than half of either whole tone, are nevertheless called *semitones.* In the diatonic scale, the semitones are always the intervals between the 3d and 4th, and the 7th and 8th.

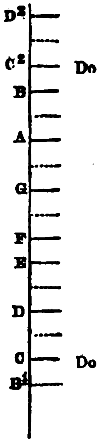
527. *Repetition of the scale.*—The eighth note of the scale so much resembles the first in sound, that it is regarded as a repetition of it, and called by the same name. Beginning, therefore, with the half string, where the former series closed, let us consider the sound of that as the fundamental, and take $\frac{8}{9}$ of it for the second, $\frac{4}{5}$ of it for the third, &c.; we then close a second series of notes on the quarter-string, whose sound is also considered a repetition of the former fundamental. Each fraction of the string used in the second scale is obviously half of the corresponding fraction of the whole string, and therefore its note an octave above the note of that. This process may be repeated indefinitely, giving the *second octave*, *third octave*, &c. Ten or eleven octaves comprehend all sounds appreciable by the human ear; the vibrations of the extreme notes of this entire range have the ratio of $1 : 2^{10}$, or $1 : 2^{11}$; that is, 1 : 1024, or 1 : 2048. Hence, if 16 vibrations per second produce the lowest appreciable note, the highest varies from 16,000 to [illegible],000. It was ascertained by Dr. Wollaston, that the highest limit is different for different ears; so that when one person complains of the piercing shrillness of a sound, another maintains that there is no sound at all. The lowest limit is indefinite for a different reason; the sounds are heard by all, but some will recognize them as low musical tones, while others only perceive a rattling or fluttering noise. Few musical instruments comprehend more than six octaves, and the human voice has only from one to three, the male voice being in pitch an octave lower than the female.

528. *Modes of naming the notes.*—There is one system of names for the notes of the scale, which is fixed, and another which is movable. The first is by the seven letters, A, B, C, D, E, F, G. The notes of the second octave are expressed by the same letters, in some way distinguished from the former. The best method is to write by the side of the letter the numeral expressing that index of 2, which corresponds to the octave: as A_2, A_3, &c, in the octaves above; $A_{\bar{2}}$, $A_{\bar{3}}$, in those below.

The second mode of designation is by the syllables, *do*, *re*, *mi*, *fa*, *sol*, *la*, *si*. These express merely the *relations* of notes to each other, *do* always being the fundamental, *re* its second, *mi* its third, &c. In the natural scale, *do* is on the letter C, *re* on D, &c.; but by the aid of interpolated notes, the scale of syllables may be transferred, so as to begin successively with every letter of the fixed scale.

529. *The chromatic scale.*—Let the notes of the diatonic scale be represented (Fig. 221) by the horizontal lines, C, D, &c.; the distance from C to D being a tone, from D to E a tone, E to F a semitone, &c. It will be observed that the fundamental, C, is so situated that there are *two* whole tones above it, before a semitone occurs, and then *three* whole tones before the next semitone. C is therefore the letter to be called by the syllable *do*, in order to bring the first semitone between the 3d and 4th, and the other semitone between the 7th and 8th, as the figure represents them. Now, that we may be able to transfer the scale of relations to every part of the fixed scale (which is necessary, in order to vary the character of music, without throwing it beyond the reach of the voice), the whole tones are bisected, and two semitone intervals occupy the place of each. The dotted lines in the figure show the places of the interpolated notes, which, with the original notes of the diatonic scale, divide the whole into a series of semitones. This is called the *chromatic* scale. The interpolated note between C and D is written C♯ (C sharp),

Fig. 221.



or D♭ (D flat), and so of the others. As the whole tones lie in groups of *twos* and *threes*, so the new notes inserted are grouped in the same way. This explains the arrangement of the *black keys* by twos and threes alternately in the key-board of the organ and piano-forte. The white keys compose the diatonic scale, the white and black keys together, the chromatic scale. It is obvious that on the chromatic scale any one of the twelve notes which compose it may become *do*, or the fundamental note, since the required series, 2 tones, 1 semitone, 3 tones, 1 semitone, can be arranged to succeed each other, at whatever note we begin the reckoning.

530. *Chords and discords.*—When two or more sounds, meeting the ear at once, form a combination which is agreeable, it is called a *chord;* if disagreeable, a discord. The disagreeable quality of a discord, if attended to, will be perceived to consist in a certain roughness, or harshness, however smooth and pure the simple sounds which are combined. On examining the combinations, it will be found, that if the vibrations of two sounds are in some very simple relations, as 1 : 2, 1 : 3, 2 : 3, 3 : 4, &c., they produce a chord; and the lower the terms of the ratio, the more perfect the chord. On the other hand, if the numbers, expressing the relations of the sounds, are large, as 8 : 9, or 15 : 16, a discord is produced. It appears that concordant sounds have *frequent* coincidences of vibrations. If, in two sounds, there is coincidence at every vibration of each, then the pitch is the same, and the combination is called *unison.* If every vibration of one coincides with every alternate vibration of the other, the ratio is 1 : 2, and the chord is the *octave*, the most perfect possible. The *fifth* is the next most perfect chord, where every second vibration of the lower meets every third of the higher, 2 : 3. The *fourth*, 3 : 4, the *major third*, 4 : 5, the *minor third*, 5 : 6, and the *sixth*, 3 : 5, are reckoned among chords; while the *second*, 8 : 9, and the *seventh*, 8 : 15, are harsh discords. What is called the *common chord*, consists of the 1st, 3d, and 5th, combined, and is far more used in music than any other. *Harmony* consists of a succession of chords, or rather, of such a succession of combined sounds, as is pleasing to the ear; for discords are employed in musical composition, their use being limited by special rules. Many combinations, which would be too disagreeable for the ear to dwell upon, or to finish a musical period, are yet quite necessary to produce the best effect; and without the relief which they give, perfect harmony, if long continued, would satiate.

531. *Temperament.*—The subject of musical temperament is complicated, and generally regarded as difficult; but the principles of it are simple, and are properly noticed in the philosophy of musical sounds. The necessity of temperament arises entirely from the fact that the whole tones are *unequal*, and the semitones not truly *half-tones*. Taking the numbers in Art. 525, as representing the notes in the key of C, we have $\begin{cases} C, D, E, F, G, A, B, C_2, D_2, E_2, \\ 24, 27, 30, 32, 36, 40, 45, 48, 54, 60, \end{cases}$ &c. Now, suppose we wish to make D, instead of C, our key-note; then it is obvious that E will not be exactly correct for the second on the new scale. For the fundamental to its second is as 8 : 9; and 8 : 9 :: 27 : 30.375, instead of 30. Therefore, if D is the key-note, we must have a new E, slightly above the E of the original scale. So we find that A, represented by 40, will not serve to be the 5th in the new scale; since 2 : 3 :: 27 : 40.5, which is a little higher than A (=40). After adding these and other new notes, to render the intervals all exactly right for the new key of D, if we proceed in the same manner, and make E (=30) our key-note, and obtain its second, third, &c., exactly, we shall find some of them differing a little, both from those of the key of C, and also of the key of D. Using in this way all the twelve notes of the chromatic scale in succession for the fundamental, it appears that several different E's, F's, G's, &c., are required, in order to make each scale perfect. In instruments, whose sounds can not be modified by the performer, like the organ and piano-forte, as it is considered impossible to insert all the pipes or strings necessary to render every scale perfect, such an *adjustment* is made as to distribute these errors equally among all the scales. For example, E is not made a perfect *third* for the key of C, lest it should be too *imperfect* for a *second* in the key of D, and for its appropriate place in other scales. This equalizing of errors among the several scales is called *temperament*. The errors, when thus distributed, are too small to be observed by most persons; whereas, if an instrument was tuned perfectly for any one scale, all others would be intolerable.

532. Temperament has no application except to instruments of fixed keys, as the organ and piano-forte; for, where the performer can control and modify the notes as he is playing, he can make every key perfect, and then there are no errors to be distributed. The flute-player can roll the flute slightly, and thus humor the sound, so as to cause the same fingering to give a precisely correct *second* for one scale, a correct *third* for another, and so on. The player on the violin does the same, by touching the string in points slightly different. The organs of the voice, especially, can be adjusted to make the intervals perfect on *every* scale. In these cases there is no *tempering*, or dividing of errors among different scales, but a *perfect adjustment*, by which all error is avoided.

533. *Harmonics.*—The fact has been mentioned, that a string, or a column of air, may vibrate in parts, even while vibrating as a whole. It only remains to show the musical relations of the sounds thus produced. When a string vibrates in parts, it divides into halves, thirds, fourths, or other *aliquot* parts. Now, a half-string produces an *octave* above the whole, making the most perfect chord with it. The third of a string being two-thirds of the half-string, produces the *fifth* above the octave, a very perfect chord. The quarter-string gives the *second octave;* the fifth part of it, being $\frac{4}{5}$ of the quarter, gives the *major third* above the second octave; and the sixth part, being $\frac{2}{3}$ of the quarter, gives the *fifth* above the second octave. Thus, all the simpler divisions, which are the ones most likely to occur, are such as produce the best chords; and it is for this reason that the sounds are called *harmonics.* The same is true of air-columns and bells. The Æolian harp furnishes a beautiful example of the harmonics of a string. Two or more fine smooth cords are fastened upon a box, and tuned, at suitable intervals, like the strings of a violin; and the box is placed in a narrow opening, where a current of air passes. Each string at different times, according to the intensity of the breeze, will emit a pure musical note; and, with every change, will divide itself in a new mode, and give another pitch, while it will frequently happen that the vibrations of different divisions will coexist, and their harmonic sounds mingle with each other.

534. *Communication of vibrations.*—The acoustic vibrations of one body are readily communicated to others, which are near or in contact. We have already noticed that the vibrations of a reed will excite those of a column of air in a pipe. If two strings, which are adapted to vibrate alike, are fastened on the same box, and one of them is made to sound, the other will sound also more or less loudly, according to the intimacy of their connection. The vibrations are communicated partly through the air, and partly through the materials of the box. So, if a loud sound is uttered near a piano-forte, several strings will be thrown into vibration, whose notes are heard after the voice ceases. The noticeable fact in all such experiments is, that the vibrations thus communicated from one body to another, cause sounds which *harmonize* with each other, and with the original sound. For the rate of vibration will either be identical, or have those simple relations which are expressed by the smallest numbers. Let a person hold a pneumatic receiver, or a large tumbler before him, and utter at the mouth of it several sounds of different pitch; and he will probably find some one pitch, which will be distinctly reinforced by the vessel. That particular note, which the receiver by its size and form is adapted to produce, will not be called forth by a sound, that would be discordant with it. The melodeon, seraphine, and instruments of like character, owe their full and brilliant notes to reeds, each of which has its cavity of air adapted to vibrate in unison with it. It sometimes happens, that the second body, vibrating as a whole, would not harmonize with the first, and yet will give the same note by some mode of division. Thus it is, that all the various sounds of the monochord, and of the strings of the viol, are reinforced by the case of thin wood, upon which they are stretched. The plates of wood divide by nodal lines into some new arrangement of ventral segments for every new sound emitted by the string. In like manner, the pitch of the tuning-fork, and all the rapid notes of a music-box, are rendered loud and full by the table, in contact with which they are brought. The extended material of the table is capable of division into a great variety of forms, and will always give a sound in unison with the instrument which touches it.

535. If two sounding bodies are nearly, but not precisely on the same key, they will sometimes, when brought into close contact, be made to harmonize perfectly. The vibrations of the more powerful will be communicated to the other, and control its movements so that the discordance, which they produce when a few inches apart, will cease, and concord will ensue. Two diapason pipes of an organ, tuned a quarter-tone or even a semitone from unison, so as to jar disagreeably upon the ear, when one inch or more asunder, will be in perfect unison, if they are in contact through their whole length. Even the slow oscillations of two watches will influence each other; if one gains on the other only a few beats in an hour, then, if they are placed side by side on the same board, they will beat precisely together, and of course keep time alike.

536. *Crispations of fluids.*—Among the numerous acoustic experiments illustrating the communication of vibrations, none are more beautiful than those in which the vibrations of glass rods are conveyed to the surface of a fluid. Let a very shallow pan of glass or metal be attached to the middle of a thin bar of wood, three or four feet long, and resting near its ends on two fixed bridges; let water be placed in the pan, and a long glass rod standing in it, or on the wood, be vibrated longitudinally, by drawing the moistened fingers down upon it; the liquid immediately shows that the vibrations are communicated to it. The surface is covered with a regular arrangement of heaps, called *crispations*, which vary in size with the pitch of sound, which is produced by the same vibration. If the pitch is higher, they are smaller, and may be readily varied from three or four inches in diameter, to the fineness of the teeth of a file. Crispations of the same character are also formed in clusters on the water in a large tumbler or glass receiver, when the finger is drawn along its edge; every ventral segment of the glass produces a group of hillocks by the side of it on the surface of the water.

537. *Interference of waves of sound.*—Whenever two sounds are moving through the air, every particle will, at a given instant, have a motion which is the resultant of the two motions which it would have had, if the sounds were separate. These motions may conspire, or they may oppose each other. The word *interference* strictly refers to the latter case, though it is sometimes used in a general sense, for the *resultant effect*, whatever it may be. The *beats*, which are frequently heard in listening to two sounds, indicate the points of maximum condensation produced by the union of the condensed parts of both systems of waves. And the sounds are considered discordant, when these beats are just so frequent as to produce a disagreeable fluttering or rattling. If too near, or too far apart for this, they are regarded practically as concordant. And when the beats are too close to be perceived separately, yet the peculiar adjustment of condensations of one system with those of the other, according as *one* wave measures *two*, or *two* waves measure *three*, or *four* measure *five*, &c., is at once distinguished by the ear, and recognized as the chord of the *octave*, the *fifth*, the *third*, &c. When a sound and its octave are advancing together, there are instants in which any given particle of air is impressed with two opposite motions, and other alternate moments when both motions are in the same direction. For the waves of the highest sound are half as long as those of the lowest; hence, while every *second* condensation of the former coincides with *every* condensation of the latter, the alternate ones of the former must be at the points of greatest rarefaction of the latter; and this can not occur without opposite movements of the particles. If two simultaneous sounds have the same pitch, i. e., the same length of wave, they ordinarily *run together*, so that like phases in the two systems are coincident, and the compound sound (called *unison*), simply has twice the loudness of one of them alone. But, by a delicate mode of experiment, these two systems of waves, having equal lengths, and equal intensities, may be made to suffer an exact interference, and destroy each other. In this curious case, two sounds produce *silence*, on the same principle that two systems of water-waves may produce *level water*.

538. *Absolute number and length of waves for each note.*—Though the vibrations of any musical note are too rapid to be counted, yet the number may be ascertained in several ways. One of the readiest methods is by means of a little instrument called the *siren*, invented by De La Tour. The pulses are produced by streams of air driven through holes in a revolving wheel. The revolutions of the wheel are recorded by machinery, and the number of vibrations in each revolution is known from the number of holes through which the air rushes. When such a velocity of revolution is given as to produce the required pitch, then the revolutions of the index per minute may be counted, and the number of *vibrations* in the same time will be known, and therefore the number per second. In this and other ways it is ascertained, that the numbers corresponding to the letters of the scale are the following:

C,	D,	E,	F,	G,	A,	B,	C_2,	D_2,
128,	144,	160,	170,	192,	214,	240,	256,	288.

The highest note of the above series, D_2, 288, is the lowest on the common or D-flute. There is not, however, a perfect agreement of pitch in different countries, and among different classes of musicians. Accordingly, C, which is given above as corresponding to 128 vibrations per second, has several values, varying from 127 to 131.

To find the *length* of acoustic waves for any given pitch, we have only to divide the velocity of sound in one second by the number of vibrations which reach the ear in the same length of time. For example, at the temperature of 60°, sound travels 1118 feet per second; therefore the length of waves of low D on the flute $= 1118 \div 288 =$ nearly *four* feet. The waves of the lowest musical note are about 70 feet long; and of the highest, less than half an inch.

PART VI.—ELECTRICITY.

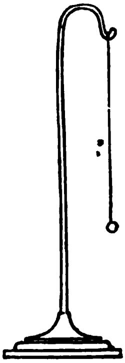
CHAPTER I.

GENERAL PRINCIPLES OF THE SCIENCE.

539. *Definitions and remarks.*—The name *Electricity* is derived from ἤλεκτρον, *amber*, the substance in which electrical properties were first discovered. The most general effect by which the presence of electricity is manifested, is *attraction.* Thus, when a glass tube is rubbed with a dry silk or woolen cloth, it acquires the property of attracting light bodies, as cotton, feathers, &c. When by any process a body is made to give signs of electricity, it is said to be *excited.* When a body receives the electric fluid from an excited body, it is said to be *electrified.* Since there is found to be a great difference in bodies in regard to the power of transmitting electricity, all bodies are divided into two classes, conductors and non-conductors. *Conductors* are bodies through which the electric fluid passes readily; *non-conductors* are bodies through which the electric fluid either does not pass at all, or but very slowly. The latter bodies were formerly denominated *electrics*, because it is by the friction of bodies of this class that electricity is usually excited. An electrified body is said to be *insulated,* when its connection with other bodies is formed by means of non-conductors, so that its electricity is prevented from escaping. Instruments employed to detect the presence of electricity are denominated *electroscopes;* such as are employed to estimate its comparative quantity, are called *electrometers.*

540. *Electroscopes.*—The *pendulum electroscope* is represented in Fig. 222, consisting of a glass standard, supported by a base, and bent into a hook at the top, from which is suspended a fine silk thread sustaining a pith ball. Fig. 223 represents the *gold-leaf electroscope.* Within a glass receiver, having a metallic base and cover, are suspended two strips of gold leaf connected with the metal at the top. When an electrified body is

FIG. 222.



brought near, the leaves separate, or, if separated, collapse or separate more, for reasons to be explained hereafter. *Coulomb's electroscope* (Fig. 224) is still more delicate. From the cover of a glass cylinder is suspended, by a long and exceedingly fine wire, a slender needle of *gum lac*, H, on each end of which is a minute pith ball. On the same level as the needle, a rod, A, with balls on the ends, is inserted through the side of the cylinder. If the rod is electrified in the slightest degree, the attracting or repelling influence overcomes the feeble force of torsion in the wire, and causes the needle to revolve.

FIG. 223.

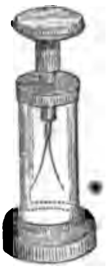
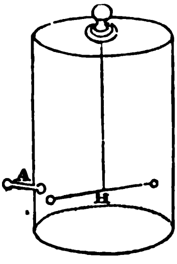


FIG. 224.



The following modifications of the two first are convenient for some purposes; a wire with a ball on the top, having a thread and pith ball hanging by the side of it; and two threads with pith balls suspended together below a conductor.

541. *Modes of exciting electricity.*—Electricity may be excited or developed in several ways. It is often manifested during the *changes of state* in bodies, such as liquefaction and congelation, evaporation and condensation. Some bodies even are excited by mere *pressure;* others by the *contact* or *separation of different surfaces.* Most *chemical combinations* and *decompositions* are also attended by the evolution of electricity, which manifests its presence to delicate electroscopes. But the method most employed is *friction;* by which it may be developed in a greater or less degree from all substances; and from some, it may in this way be easily and abundantly obtained.

If we rub a piece of amber, sealing-wax, or any other resinous substance, on dry woolen cloth, or fur, or silk, and bring it toward an electroscope, it will give signs of electricity. A glass tube may be excited in a similar manner. Moreover, if we bring the excited tube near the face, it imparts a sensation resembling that produced by a cobweb. If the tube is strongly excited, it will afford a spark to the knuckle, accompanied by a snapping noise. A sheet of white paper, first dried by the fire, and then laid on a table and rubbed with india-rubber, will become so highly excited as to adhere to the wall of the room, or any other surface to which it is applied. Indeed, friction is so constantly attended by electricity, that in favorable weather the fluid is abundantly indicated on brushing our clothes, which thus are made to attract the light downy particles that are floating in the air. If we hold an *iron* or *brass* rod in the hand, and rub it with silk, we shall not succeed in discovering any sign of electricity; but this, as we shall presently see, is because the fluid is conducted away as fast as developed.

That form of electricity which is discussed in this treatise, being usually excited by friction, is sometimes called *frictional electricity*, to distinguish it from the *galvanic electricity*, which is developed by chemical action. The former is also called *statical electricity*—the latter, *dynamical electricity*.

542. *The two electrical states.*—When friction has taken place between two bodies, we find them in electrical conditions, which are in some remarkable particulars *unlike each other.* These are distinguished by the names *vitreous* and *resinous*, according to one theory, and *positive* and *negative*, according to another. The first is called the theory of *Du Fay*, having been suggested by Du Fay of France, though afterward matured by Symmer of England. It ascribes all electrical phenomena to the agency of *two* fluids, specifically different from each other, and pervading all bodies. In unelectrified bodies these two fluids exist in combination, and exactly neutralize each other. By the separation of the two fluids it is that bodies are electrified; and it is by the reunion of the two fluids that the electricity is discharged, or bodies cease to be excited. The second hypothesis was proposed by *Franklin*, and is called by his name. It ascribes all electrical phenomena to the agency of *one* fluid, which, as in the other case, is supposed to pervade all bodies, being naturally in a state of equilibrium. A body is electrified when it has either more or less of the fluid than its natural share; in the former case it is *positively*, in the latter case *negatively*, electrified. The expressions, *positive electricity* and *negative electricity*, if employed at all, must have reference to Du Fay's theory, because they imply *two* fluids; but to prevent confusion in explaining phenomena, it is better to avoid them altogether.

543. These two conditions are always developed *simultaneously;* one of the bodies rubbed being electrified with the vitreous, the other with the resinous electricity. For example, glass rubbed with silk, has itself the *vitreous* electricity, or is *positively* electrified, while the silk has the *resinous* electricity, or is *negatively* electrified.

The foregoing law holds true universally; but the kind of electricity which each substance acquires, depends upon the substance against which it is rubbed. If we rub dry woolen cloth against *smooth* glass, it acquires the resinous, and the glass the vitreous electricity; but if we rub the same cloth against *rough* glass, it becomes positively, while the glass becomes negatively electrified. The following table contains a number of substances, arranged in such a way that when they are rubbed against each other, any substance in the list before another becomes positively, and any substance below it negatively electrified:

1. Fur of a cat,
2. Smooth glass,
3. Woolen cloth,
4. Feathers,
5. Wood,
6. Paper,
7. Silk,
8. Lac,
9. Rough glass,
10. Sulphur.

The fur of a cat, when rubbed against any of the bodies in the table, always affords the vitreous, and the sulphur always the resinous electricity. Feathers become negative when rubbed against the fur of a cat, smooth glass, or woolen cloth; but positive when rubbed against wood, paper, silk, lac, rough glass, or sulphur.

It is sometimes found, however, that the previous electrical condition of one of the bodies will invert the order stated in the table. For example, if silk, which has been rubbed upon *smooth* glass, and has therefore taken the negative state, should then be rubbed on *rough* glass, it would probably retain that state, and the rough glass become positively electrified.

544. *Mutual action.*—Bodies electrified *in different ways attract*, and *in the same way repel* each other. Thus if an insulated pith ball, or a lock of cotton, be electrified by touching it with an excited glass tube, it will immediately recede from the tube, and from all other bodies which afford the vitreous electricity, while it will be attracted by excited sealing-wax, and by all other bodies which afford the resinous electricity. If a lock of fine long hair be held at one end, and brushed with a dry brush, the separate hairs will become electrified, and will repel each other. In like manner, two insulated pith balls, or any other light bodies, will repel each other when they are electrified the same way, and attract each other when they are electrified different ways.

Hence it is easy to determine whether the electricity afforded by a given body is vitreous or resinous; for, having electrified the electroscope by excited glass, then all those bodies, which, when excited, *attract* the ball, afford the resinous, while all those which *repel* the ball, afford the vitreous electricity.

545. *Conduction.*—Electricity passes through some bodies with the greatest facility; through others with difficulty, or scarcely at all; and others have a conducting power intermediate between the two. As the conducting quality exists in different substances in all conceivable degrees, it is impossible to draw a dividing line between them, so as to arrange all conductors on one side, and all non-conductors on the other. The following brief table contains the most prominent of the two classes; the first column in the order of *conducting* power, the second in the order of *insulating* power:

Conductors.	Insulators.
The metals,	Lac, amber, the resins,
Charcoal,	Sulphur,
Plumbago,	Wax,
Water, damp snow,	Glass, precious stones,
Living vegetables,	Silk, wool, hair, feathers,
Living animals,	Cotton, paper,
Smoke, steam,	Air, the gases,
Moist earth, stones.	Baked wood.

As air is rarefied, its insulating power is diminished, and the more complete the vacuum, the more freely does electricity pass through it. Hence, it is believed that the fluid moves without any obstruction at all through a perfect vacuum.

546. *The Leichtenberg figures.*—When a spark of electricity is laid upon a non-conductor, it will, by its own self-repellency, extend itself a little distance along the surface. The *Leichtenberg figures* furnish a visible illustration of this fact, and also show that the two fluids diffuse themselves in very different forms. Lay down sparks of vitreous electricity from the knob of the Leyden jar upon a plate of resin, and near them, some sparks of resinous electricity. Then blow upon the plate the mingled powders of sulphur and red-lead. The sulphur, by the agitation of passing through the air, will be electrified resinously, and attracted therefore by the vitreous sparks; the red-lead, vitreously electrified, will be attracted by the resinous. Thus the spots on which the electricities are placed will appear in their exact forms by means of the colored powders attached to them. The vitreous resemble stars, or rather a group of crystals shooting out from a nucleus; the resinous spots are circles with smooth edges; and the size of the electrified spots in each case depends on the quantity of fluid in the spark.

547. *Modes of insulating.*—Solid insulating supports are usually made of glass; and to improve their insulating power, they are sometimes covered with shell-lac varnish. Insulating threads for pith balls, or cords for suspending heavier bodies, are made of silk. The best insulator for suspending any very small weight, is a single fiber of silk, a hair, or a fine thread of lac. The air must be dry, or no care whatever relating to apparatus can insure success in electrical experiments; and therefore, in a room occupied by an audience, especially if the weather is damp, it is necessary to dry the air artificially by fires. If the air were a good conductor, it is probable that no facts in this science would ever have been discovered.

548. The *sphere of communication* is the space within which a spark may pass from an electrified body, in any direction from it. It is sometimes called the striking distance. The *sphere of influence* is the space within which the power of attraction of an electrified body extends in every way, beyond the sphere of communication. A glass tube strongly excited will exert an influence upon the gold-leaf electroscope at the distance of ten or even twenty feet, although a spark could not pass from the tube to the cap of the electroscope at a greater distance than a few inches.

The electricity which a body manifests by being brought near to an excited body, without receiving a spark from it, is said to be acquired by *induction.* When an insulated conductor, unelectrified, is brought into the neighborhood of an insulated charged conductor, its electricity undergoes a new arrangement. The end of it next to the excited conductor, assumes a state of electricity opposite to that of the excited conductor; while the further extremity assumes the same state. Suppose the excited conductor is electrified positively. The end of the insulated conductor next to it becomes negative, and the remoter end positive; and intermediate between these two points, there occurs a place not electrified in either way: this is called the *neutral point.* The principle of induction will be more fully discussed under the subject of the *Leyden jar.*

549. *The two theories.*—In view of the foregoing first principles, the following statements may be presented, as embodying the substance of Franklin's theory.

1. One electric fluid pervades all matter, each portion of matter having what is called its natural share.
2. A body is said to be positively electrified, when it has more than its natural share; negatively, when it has less.
3. The electric fluid is self-repellent.
4. All matter, when destitute of electricity, is also self-repellent (though this repellent force is very feeble in comparison with the ordinary force of cohesion).
5. Electricity, and matter destitute of electricity, attract each other.

The following are the principles of Du Fay's theory.

1. Two electric fluids, naturally mingled and neutralized, pervade all matter, called the vitreous and the resinous electricities.
2. A body is said to be electrified vitreously (or positively),

when it has the vitreous electricity in excess; resinously (or negatively), when it has the resinous fluid in excess.

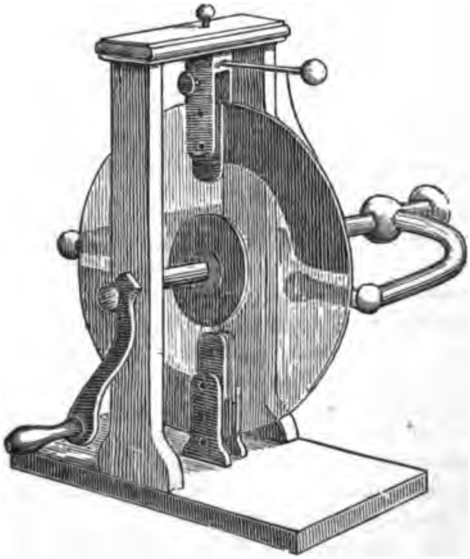
3. Electricities of the same kind repel each other.
4. Electricities of different kinds attract each other.

CHAPTER II.

ELECTRICAL MACHINES.—LAW OF FORCE.—LAW OF DISTRIBUTION.

550. *The plate machine.*—In order that glass may be conveniently subjected to friction for the development of electricity, it is made in the form of a circular *plate*, and mounted on an axis, which is supported by a wooden frame, and revolved by a crank, while *rubbers* press against its surface. Fig. 225 represents one of the many forms which have been adopted.

FIG. 225.



The pressure of the rubbers is equalized by their being placed at top and bottom, and on both sides of the glass. The vitreous electricity on the glass is taken off upon the *prime conductor*, which is made of brass and supported by glass pillars. The part of the prime conductor which is nearest the plate, terminates in a few sharp points, which give ready transmission to the fluid. But all other parts are carefully rounded in cylindrical and spherical forms, without edges or points, as these tend to dissipate the electricity. The glass, as it revolves from the rubbers to the points of the prime conductor, is protected by a silk cover, to prevent the electricity from escaping into the air. The rubbers are made of soft leather, attached to a piece of wood or metal, and are occasionally coated with an *amalgam* of zinc, tin, and mercury, or with the bi-sulphuret of tin, which is one of the best exciters on glass. The diameter of the plate varies from 1¼ to 3 feet; but in some of the largest it is 6 feet, and two plates are sometimes mounted on one axis.

551. *The cylinder machine.*—In many electrical machines of the smaller sizes, a glass cylinder is employed, having a length considerably exceeding its diameter. In the cylinder machine, the rubber is applied to one side, and the prime conductor receives the fluid from the opposite. The rubber is sometimes mounted on a glass pillar, so that it can be insulated, whenever it is desired.

552. *The hydro-electric machine.*—It was discovered in 1840, that a steam-boiler electrically insulated, gave out sparks, and that the steam issuing from it was also electrified. Hence resulted the construction of *the hydro-electric machine.* It consists of a boiler mounted on glass pillars, and furnished with a row of jet-pipes, and a metallic plate, against which the steam strikes. The prime conductor, to which the steam-plate is attached, is electrified positively, and the boiler itself negatively. Professor Faraday ascertained that the electricity in this case is developed, not by evaporation or condensation, but by the friction of watery particles in the jet-pipes. That the machine may act with energy, it was found necessary to make the interior of the jet-pipes angular, and quite irregular.

553. *The quadrant electrometer.*—In order to measure the intensity of electricity in the prime conductor, there is set upon it, whenever desired, a *quadrant electrometer;* this consists of a pillar about six inches high, having a graduated semicircle attached to one side, and a delicate rod and ball suspended from the center of the semicircle. As the conductor becomes electrified, the rod is repelled from the pillar, and the arc passed over indicates rudely the degree of electrical intensity.

554. *Experiments with the machine.*—When an electrical machine is skillfully fitted up, and works well, on turning it, circles of light surround the plate, or cylinder, and brushes or pencils of light emanate copiously from the cushion and other parts of the machine. The circles of light consist of electric sparks, which discharge themselves between the excited surface and the rubber, their passage being so rapid as to appear like a continued line, like that of a small stick ignited at the end and whirled in the air. The brushes of light arise from the facility with which the fluid escapes from points or thin edges. When electricity is escaping plentifully from the machine, a peculiar odor is often noticed, which was formerly supposed to be the odor of electricity; but it is now considered to arise from a gas called *ozone*, which is liberated not only by electrical excitation, but also by chemical means.

The experiments on electrical attraction and repulsion described in Chapter I., may now be repeated in a more striking manner, and various others added, which can be shown only when the electricity is developed in considerable quantities.

555. We enumerate a few of the effects of electricity, as they are exhibited by the machine, confining ourselves for the present to those experiments which relate to attraction and repulsion, and the passage of the spark, reserving an account of others of a different character for future sections.

1. When the machine is turned, a downy feather, or a lock of cotton held in the hand by a conducting thread, will be strongly attracted toward the excited surface.

2. A skein of thread, or lock of fine hair, looped, and suspended by the loop from the prime conductor, will exhibit strong repulsions between the threads or hairs. If the thread or hair is very dry, it will not be readily electrified; but if brought in from a cold room at the moment when wanted, a little moisture will be precipitated upon it. For the same reason, most electrical articles *can not* be used when first brought from a cold to a warm room.

3. The quadrant electrometer (Art. 553), being attached to the prime conductor, the conducting powers of different substances may be readily tried. Thus, an iron rod held in the hand, and applied to the prime conductor, will cause the index of the electrometer to fall instantly; and the same effect will follow the application of any metallic rod. A wooden rod of the same dimensions, will cause the index to descend more slowly; and a glass rod will hardly move it at all. These experiments show that iron is a perfect, wood an imperfect conductor, and glass a non-conductor. In the same manner the conducting powers of a stick of sealing-wax, a roll of silk, or cloth, and of various other bodies, may be illustrated.

4. If a pith ball, or feather, or any other light body, held by a silk thread, be presented to the prime conductor, it will first be attracted and then repelled, and it can not again be brought into contact with the electrical conductor, until its electricity is

discharged by communicating with the finger, or some unelectrified conductor.

5. By placing light bodies between an electrified conductor and an uninsulated body, they may be made to move with great rapidity backward and forward, from one surface to the other, being alternately attracted and repelled by the electrified surface. By this means are performed electrical dances, the ringing of bells, and a variety of interesting and amusing experiments.

6. If the rubber be *insulated* while the machine is turned, the rubber and the glass cylinder, or plate, will be found to be in different electrical states; an insulated body attracted by the one will be repelled by the other.

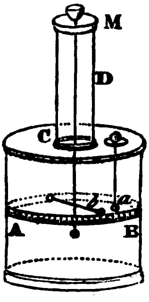
Bodies are electrified positively by connecting them with the glass, by means of the prime conductor, and negatively by connecting them with the rubber, the latter being insulated, and the prime conductor uninsulated.

7. If the parts of an electrified body are movable, they tend to separate, being endued with the power of mutual repulsion. Thus, a lock of cotton is separated into its minutest fibers. Melted sealing-wax, when attached by a wire to the prime conductor, is divided into filaments so small as to resemble red wool. Water, dropping from a capillary siphon-tube, on being electrified, is made to run out in a great number of exceedingly fine streams. Water, spouting from an air fountain, is divided into a number of rays, presenting the appearance of a brush.

8. A portion of electrified air, in consequence of the mutual repulsion between its particles, expands, and when at liberty to escape, becomes rarefied. Thus, a current of air may be set in motion from an electrified point, or small ball, or be made to issue from the neck of a bottle.

556. *Coulomb's torsion balance.*—In the latter part of the last century, Coulomb, of France, investigated the laws of *torsion* in fine wires, and applied those laws to the measurement of the forces of electricity and magnetism. When a wire is twisted from its natural position, the elastic force with which it returns, i. e. its torsion, varies inversely as the length, and directly as the fourth power of the thickness. Therefore, this force is more feeble as the wire is longer, and of less diameter; and may be used to measure repelling or attracting forces, which are inconceivably minute. The construction of the instrument is as follows. In order to guard the suspended needle of lac from the agitations of the air, it is protected by a glass cylinder, AB (Fig. 226), having a movable lid, C, from the

FIG. 226.



center of which rises a smaller glass cylinder, D, which covers the suspending thread; this latter cylinder is surmounted by a graduated circle, M, upon which moves a pointer or index, connected at the center with the suspending thread, which is twisted when the index is turned. The lid, C, is perforated with a hole to allow access to the pith ball of the needle. In the figure, this opening is represented as closed by the handle of a movable rod of glass or lac, which insulates the ball *a*, by which electricity is conveyed to the ball *b* of the needle. On a level with the needle is a circular band, graduated into degrees and minutes.

557. *Electrical force and distance.*—Adjustment is now made, so that while the wire is in its natural condition, the needle touches *a*, and is at zero, and the index at top also at zero on the circle M. Let a minute charge of electricity be communicated to *a*, and it will repel *b*, and cause it, after a few oscillations, to settle at a certain distance; suppose, for instance, at 36°. The index at M is now turned in the opposite direction, until the needle is brought within 18° of the ball *a*. In order to bring it thus near, the index has to be turned 126°, which added to the 18°, makes the whole torsion 144°, or *four times* as great as before. Therefore, at *one-half* the distance there is *four times* the repulsion. In like manner, it is found that at *one-third* the distance there is *nine times* the repulsion. In general, *electrical repulsion varies inversely as the square of the distance.*

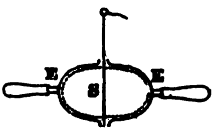
In a manner somewhat similar to the foregoing, it was conclusively proved by Coulomb, that electrical *attraction* obeys the same law of distance, though there is more practical difficulty in performing the experiments. But if the electrified body *a* is placed outside of the circle described by *b*, so that the latter is allowed to vibrate both to the right and left, the square of the *number* of vibrations in a given time becomes a measure of the attractive force, as in the case of the pendulum (Art. 161).

558. *Waste of electricity from an insulated body.*—In making accurate investigations like the foregoing, in which considerable time is necessarily occupied, a difficulty arises from the loss of the electrical charge. The *first*, and most obvious source of waste is the moisture in the air, which conducts away the fluid; but this may be nearly avoided by setting into the cylinder a cup of dry lime, or other powerful absorbent of moisture. A *second* is the imperfect insulation afforded by even the most perfect non-conductors. A *third* is the mobility of the air, whose particles, when they have touched the electrified body, and become charged, are repelled, taking away with them the charge they have received. The loss in these ways is very slight, when the charge is small, and allowance can be made for it with a good degree of accuracy. But when bodies are highly charged, they lose their electricity at a rapid rate.

559. *An electrical charge lies at the surface.*—This is proved in many ways. A *hollow* ball, no matter how thin, will receive as large a quantity of electricity as a *solid* one. Hence it is, that the prime conductor of the electrical machine, and metallic articles of electrical apparatus generally, are made of sheet brass, for the sake of lightness.

Fig. 227 represents a metallic ellipsoid, S, suspended by a silk thread, and covered with two thin metallic caps, EE, furnished with insulating handles. The caps being removed, let the ellipsoid be electrified, and the caps again applied; then the whole charge will be found to have passed to their outside surface, and will be taken off with them, if they are removed a second time.

FIG. 227.



If cavities are sunk into a solid conductor, no sensible quantity of electricity is found at the bottom of such cavities. In experiments of this kind, Coulomb found his torsion balance (Fig. 226) of great service. A *proof plane*, as he termed it, that is, a minute piece of gilt paper cemented upon the end of a slender rod of lac, was first touched to that part of an electrified body which was to be examined, and then applied to the ball of the instrument. The distance to which the needle was repelled, indicated the intensity of electricity at the point in question. The charge taken from the bottom of an abrupt cavity was never sufficient to move the needle.

Another proof that the charge occupies only the outside surface, is that the intensity diminishes as the *surface* is enlarged, while the *mass* of the conductor remains the same. A metallic ribbon rolled upon an insulated cylinder may be unrolled, and thus the surface enlarged to any extent. An electroscope standing on the instrument will fall as the ribbon is unrolled, and rise when it is again rolled up.

560. *Distribution of a charge on the surface.*—Developed electricity resides at the surface of a body, as we have seen, but is not uniformly diffused over it, except in the case of the sphere. In general, the more *prominent* the part, and the more *rapid its curvature*, the more intensely is the fluid accumulated there.

In a long slender rod, Coulomb found that nearly the whole charge was collected at the extremities. On the surface of an ellipsoid it was found to be arranged according to a very simple law; namely, *the quantity of the charge at each point varies as the diameter through that point.* But the tendency to escape increases at a more rapid rate, and varies as the *square* of the diameter. Hence it is that electricity is so rapidly dissipated from *points*, which may be regarded as the extremities of ellipsoids indefinitely elongated. If the surface of a body is partly convex and partly concave, the distribution is still more unequal; nearly all the charge collects on the convex parts; and if the concavities are deep or abrupt, like those mentioned in Art. 559, no sign of electricity is discovered in them.

561. *The charge held on the surface by atmospheric pressure.*—The mutual repellency, which drives the particles asunder till they reach the surface of the conductor, tends to make them escape in all directions from that surface; and it is the air alone which prevents. For if one extremity of a charged and insulated conductor extends into the receiver of an air-pump, the charge is dissipated by degrees, as the receiver is exhausted; and when the exhaustion is as complete as possible, the most abundant supply from the machine fails to charge the conductor. As the atmospheric pressure is limited to about 15 lbs. per square inch, so the amount of charge is limited which can be retained on a conductor of given form. Hence the reason for the well-known fact, that the prime conductor receives all the charge which it is capable of retaining in one or two turns of the machine. All that is gained over and above this, by continuing to turn, flies off through the air.

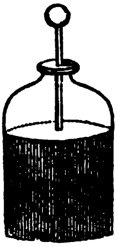
As no charge could be retained if the air were a conductor (Art. 547), so neither could there be if the air did not exist at all.

CHAPTER III.

ACCUMULATION OF ELECTRICITY BY INDUCTION.

562. *The Leyden jar.*—This article of electrical apparatus consists of a glass jar (Fig. 228), coated on both sides with tinfoil, except a breadth of two or three inches near the top, which is sometimes varnished for more perfect insulation. Through the cork passes a brass rod, which is in metallic contact with the inner coating, and terminates in a ball at the top.

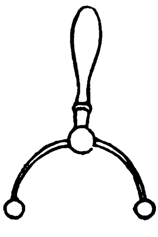
FIG. 228.



On presenting the knob of the jar near to the prime conductor of an electrical machine, while the latter is in operation, a series of sparks passes between the conductor and the jar, which will gradually grow more and more feeble, until they cease altogether. The jar is then said to be *charged.* If now we take the *dis-*

charging rod (which is a crooked wire, armed at each end with a knob, and insulated by a glass handle, as in Fig. 229), and apply one of the knobs to the outer coating of the jar, and bring the other to the knob of the jar, a flash of intense brightness, accompanied by a loud report, immediately ensues. If, after the lapse of a few minutes, the rod be applied a second time, a feeble spark passes, being the *residuary charge*, after which all signs of electricity disappear, and the jar is said to be *discharged.*

Fig. 229.



If, instead of the discharging rod, we apply one hand to the outside of the charged jar, and bring a knuckle of the other hand to the knob, a sudden *shock* is felt, convulsing the arms, and when sufficiently powerful, passing through the breast.

563. The Leyden jar derives its name from the place of its discovery. In the year 1746, while some philosophers of Leyden were performing electrical experiments, one of them happened to hold, in his hand, a tumbler partly filled with water, to a wire connected with the prime conductor of an electrical machine. When the water was supposed to be sufficiently electrified, he attempted, with the other hand, to detach the wire from the machine; but as soon as he touched it, he received the electric shock. It was by imitating this arrangement that the Leyden jar was constructed; for here was a glass cylinder, having good conductors on both sides, viz. the hand on the outside, and water on the inside, which were prevented from communicating with each other by the non-conducting powers of the glass. A metallic coating, as tin-foil or sheet-lead, was substituted for the two conductors, and a jar for the tumbler, and thus the electrical jar was constructed.

564. *Phenomena of the Leyden jar.*—The following are the principal facts concerned in the explanation of the theory of the Leyden jar:

1. The jar is usually charged by bringing the knob near to the prime conductor, while the outer coating is in electrical communication with the earth.

A jar, if overcharged, will discharge itself spontaneously. If the uncoated part is clean and dry, the discharge will be instantaneous, loud, and brilliant; if the glass is soiled or damp, the discharge will be gradual, with a hissing noise, or even without any sound at all.

2. If the outer coating is insulated by setting the jar on an insulating stand, only two or three sparks will pass to the knob, and the jar fails of being charged.

3. The two coatings of the jar are always in opposite electrical states. In the usual way of charging, the *inner* coating has the *vitreous* electricity, and the *outer* the *resinous*; a jar is said in that case to be *vitreously* or *positively* charged.

4. A jar is charged *resinously* or *negatively* by holding the knob in the hand, and presenting the outside to the prime conductor; or, if the rubber can be insulated, the knob of the jar may be brought near the rubber, while the outer coating communicates with the earth.

5. By insulating a series of jars, and connecting the outside of the first with the knob of the second, the outside of the second with the knob of the third, and so on, they are all charged vitreously; but the charge is more and more feeble from the first to the last.

6. If one jar is charged *vitreously*, and another *resinously*, and a communication is made between the two knobs, no discharge will take place, unless the other two coatings are also in communication with each other.

If two jars are prepared as above, and the outer coatings are in communication, a pith ball, suspended by a long thread between the knobs, will vibrate back and forth, being alternately attracted and repelled by each, and carrying on a slow discharge. The *electrical spider*, *electrical bells*, *see-saw*, and other articles of like kind, may be made to continue their oscillations for hours without renewing the charge.

7. If one of two jars be charged, and the other not, and if the inner coatings be brought into communication, and also the outer coatings, the charge of the first jar will be instantly diffused over the two, with the report of a discharge. In this way a charge may be halved, or divided in any other ratio, according to the relative surfaces of the jars.

8. The coatings of a charged jar may be removed, and almost the whole charge will remain on the glass; another pair of coatings may then be applied, and the discharge made by means of them.

9. In pure dry air, a jar will retain its charge for days or even weeks.

10. A pane of glass, coated on both sides with a margin of two inches all around, may be used as a Leyden jar; and, if cemented upright in a base, is for some purposes more convenient than a jar. It is called the *Franklin plate*.

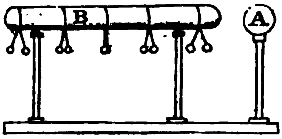
11. Let a small pendulum electroscope, hanging by the side of a brass rod, be connected with the inner coating of a jar, and another with the outer; after the jar has been charged and insulated for a few minutes, the electroscopes are raised to the same height. Touch one of the coatings, and the electroscope connected with it falls, and the other rises higher; then touch the other, and the electroscope on that coating falls, and the

first rises; and this alternation may continue for hundreds of times, a small spark passing to the finger at each contact, and the jar being slowly discharged.

The principle which furnishes an explanation of the foregoing phenomena, is called *induction*, a term which was defined in Art. 548, but which will be more fully presented in the following articles.

565. *Elementary experiment.*—Where an electrified body is placed near one which is unelectrified, but not within the sphere of communication, the natural electricities of the latter are decomposed, one being attracted toward the former, the other repelled from it. (Art. 549.) Thus the ends become electrified by the *influence* of the first body, without *receiving* any electricity from it. This is proved by the following experiment. Let A (Fig. 230) be charged with vitreous electricity, and let the insulated conductor, B, be furnished with several electroscopes, as represented. Those nearest the ends will diverge most, and the others less according as they are nearer the center, where there is no sign of electricity. By taking off small quantities with the proof-plane, and testing them, it is found that resinous electricity occupies the end nearest to A, and vitreous the remote end. Remove the bodies to a distance from each other, and B returns to its unelectrified condition; bring them near again, and B is electrified as before. As this electrical state is *induced* upon the conductor by the electrified body in its vicinity, without any *communication* of electricity, it is said to be electrified by *induction*. If A is first charged with the resinous electricity, the two electricities of B will be arranged in reversed order; the vitreous will be attracted to the nearest end, the resinous repelled to the farthest.

FIG. 230.



566. *Series of actions and reactions.*—If A is itself an insulated conductor, the foregoing is not the entire effect; for a reflex influence is exerted by the electricity in the nearer end of the conductor. Let A have a vitreous charge, as at first. After the resinous fluid is attracted to the nearer end of B, it in turn attracts the vitreous charge of A, and accumulates it on the nearest side, leaving the remote side less strongly charged than before. This is shown by electroscopes attached to the opposite sides of A. The charge of A, being now nearer, will exert more power on B, separating more of its original electricities, and thus making the nearest end more strongly resinous, and the remote end more strongly vitreous than before; and this new arrangement of fluids in B causes a second reaction upon A, of the same kind as the first. Thus an indefinite diminishing series of adjustments takes place in a single moment of time.

567. *Division of the conductor.*—Suppose, that before the experiment begins, B is in two parts, with ends in contact; the entire series of mutual actions takes place as already described. Now, while A remains in the vicinity, let the parts of B be separated; then the resinous fluid is secured in the nearest half, and the vitreous in the other. And if A is *now* removed, the electricities of B can not return to their natural state; but the resinous charge is diffused over the half which was nearest, and the vitreous over the more distant half. Thus each kind of electricity can be completely separated from the other by means of induction.

568. *Effect of lengthening the conductor.*—If the conductor, B, is *lengthened*, the accumulation on the adjacent parts of the two bodies is somewhat increased. The vitreous electricity, which, at the remote end of the shorter conductor, operated in some degree by its repulsion to prevent accumulation on the nearest side of A, is now driven to a greater distance; and therefore a larger charge will come from the remote to the nearer side of A, which in turn attracts more resinous to the nearer end of B, and thus a new series of actions and reactions takes place in addition to the former. To obtain the greatest effect from this cause, the conductor, B, is connected with the earth, that is, it is *uninsulated;* then the vitreous part of its decomposed electricities is driven to the earth, and the resinous part is attracted to the nearer end; so that, when the series of adjustments is completed, the remote end of the conductor is in the neutral state. This experiment is performed by touching the finger to the conductor, after it has become electrified by induction. The electroscope nearest to A instantly rises a little higher, and the distant ones collapse.

If the original charge in A was *resinous* instead of *vitreous*, the foregoing experiments are in all particulars the same, except that the order of the two fluids is reversed.

569. *Disguised electricity.*—The electricity which occupies the surface of the prime conductor, or any other body electrified in the ordinary way, and which is kept from diffusing itself in every direction only by the pressure of the air (Art. 561), is called *free* electricity; for it will instantly spread over the surface of other conductors, when they are presented, and therefore will be lost in the earth, the moment a communication is made. But the electricity which is accumulated by the inductive influence, is not free to diffuse itself; the same attractive force which has condensed it, still holds it as near as possible to the original charge; and if we touch the electrified body with the hand, the electricity does not pass off; it is therefore called *disguised* electricity. In this respect the two fluids on the contiguous sides of A and B are alike; either may be touched, or in any way connected with the earth, but, unless communication is made between them, they hold each other in place by their mutual attraction, and show none of the phenomena of free electricity.

570. *A series of conductors.*—If another insulated conductor, C, is placed near to the remote end of B, and A is charged with the vitreous fluid, then that extremity of B nearest to C is inductively charged with vitreous, as already stated. Hence, the electricities of C are separated, the resinous approaching B, and the vitreous withdrawing from it; there is therefore the same arrangement of fluids in both bodies, but a less intensity in C than in B. For, on account of distance, the vitreous is not so intensely accumulated at the remote end of B, as in the original body A, and therefore a less force operates on C than on B. The same effects are produced in a less and less degree in an indefinite series of bodies; and the shorter they are, the more nearly equal will be the successive accumulations.

571. *An electrified body attracts an unelectrified body.*—This fact, which is the first to be noticed in observing electrical phenomena (Art. 539), is explained by induction. If B is light, and delicately suspended, a consequence of the arrangement of fluids already described is, that B will move toward A. For, according to the law of distance (Art. 557), the resinous, in the nearer part, is attracted more strongly than the vitreous, in the remote part, is repelled; hence the body yields to the greater force, and moves toward A. That the attracted fluid does not leave the body, B, behind, and go to A, is owing to the fact, noticed in Art. 561, that the body and the electricity are confined to each other by atmospheric pressure.

572. *Induction according to Franklin's theory.*—The mode of explaining induction presented in the foregoing articles, is in accordance with the theory of Du Fay, which supposes the existence of two fluids (Art. 549). The one-fluid theory of Franklin would account for the phenomena in the following manner: the body, A, is positively electrified; that is, it is charged with more electricity than naturally belongs to it. This excess on A repels some of the natural share of the fluid from the nearer end of B to the farther end; therefore the nearer end is negatively, and the farther end positively electrified. A reaction is now exerted by the *material* of B on the electricity with which A is charged, and attracts a portion of it to the side nearest to B (Art. 549). This new supply repels a second portion to the farther end of B, leaving the nearer end more destitute than before, so that it attracts another part of A's charge toward itself, and so on.

If A had been at first negatively electrified, then the matter of A would attract some of the fluid, naturally belonging to B, from the distant to the nearer part, by which means the nearer end becomes positively electrified, and the remote one negatively electrified. The excess now existing in the nearer end repels more of the native electricity of A to the farther side, leaving the nearer side more destitute than at first, and consequently a second attraction is exerted on the fluid in B; and thus the process goes on.

573. *Mode of greatly increasing the inductive action.*—In the experiments as now described, the inductive influence is feeble, and the accumulation of electricities very small; for the bodies present toward each other only a limited extent of area, and they are necessarily as much as four or five inches distant, in order to prevent the fluid from passing across. By giving the bodies such a form, that a large extent of surface may be equidistant, and then interposing a solid non-conductor, as glass, between them, so that the distance may be reduced to one-eighth of an inch or less, it is easy to increase the attracting and repelling forces many thousands of times. Let A be connected with one coating of a Franklin plate (Art. 564, No. 10), and B with the other, and we have the desired arrangement. If now A forms a part of the prime conductor of an electrical machine, so that the vitreous electricity may be supplied as fast as it is accumulated by induction on the surface near to B, we are prepared to notice the remarkable phenomena of the Leyden jar. If the amount of surface and the thickness of glass are the same, the particular form of the instrument is immaterial; but, for most purposes, a vessel or jar is more convenient than a pane of glass of equal surface, and is generally employed for electrical experiments.

574. *Theory of the Leyden jar.*—This instrument accumulates and condenses great quantities of electricity on its surfaces, upon the principle of mutual attraction between unlike fluids, one of which fluids is furnished by the machine, the other obtained from the earth by induction. That the process of charging may be distinctly traced, let us first suppose the outer coating insulated. A spark of the vitreous passes from the prime conductor to the inner coating; this tends to repel the vitreous from the outer coating; but as the latter can not escape, it remains to prevent, by its counter-repulsion, any addition to the charge of the inside; and thus the process stops. But now connect the outer coating with the earth, and immediately some of its vitreous electricity, repelled by the charge on the inside, passes off, while its resinous is attracted close upon the glass, and gives room for the accession of more from the earth. The slight condensation of resinous upon the outside, by its attraction, condenses the vitreous of the inner coating, and allows a second spark to pass in from the prime conductor. This produces the same effect as the first, and a second addition of resinous is made to the outer coating, the latter being obtained from the earth as before. These actions and reactions go on in a diminishing series, till there is a great accumulation of the two fluids, held by mutual attraction as near each other as possible, on opposite sides of the glass. The jar in this condition is said to be charged.

The foregoing explanation is according to Du Fay's theory. To apply Franklin's, we proceed thus: A spark of the fluid from the prime conductor positively electrifies the inner coating. This addition repels some of the natural supply from the outer coating to the earth, and renders that coating in a slight degree negative. The *material* of the outer coating, thus partly deprived of fluid, attracts the fluid of the inner coating (Art. 549), and condenses it somewhat on the interior of the glass, giving room for another spark of electricity to enter from the prime conductor. This second quantity acts like the first, driving some more from the outside to the earth, and leaving the outer coating more void of electricity than before; and hence again, the *matter* of the outer coating is in a condition to attract electricity, and a second condensation occurs on the inside, and so on.

575. *Explanation of phenomena.*—The presentation of the theory of the Leyden jar accounts for several of the statements in Art. 564. A few remarks may be needed in relation to others. It is clear that the two electricities of the inner coatings (6), though they attract each other, will not rush together so long as each is held by the force of the nearer unlike fluid on the opposite side of the glass. *Neither* pair will mingle, till opportunity is given for *both* pairs to do so. The two electricities in (7) are held disguised by their mutual attractions; but will instantly diffuse themselves, according to their self-repellent nature, if allowed to continue within each other's influence. In (8), the two fluids remain on the opposite sides of the glass when the coatings are taken off, for the same reason that they were first condensed there, namely, that they powerfully attract each other. And, for a like reason, a charge is long retained in a jar (9), provided moisture does not conduct the fluids round the edge of the glass. The threads (11) are raised by the small quantities of electricity which are *free;* i. e. not disguised on the glass. When the vitreous coating is touched, the free electricity on that surface is taken off, and therefore the thread on that side falls; and more resinous is set free on the opposite side, because the force which disguised it is a little diminished; hence, the thread on the resinous side rises higher. When the resinous coating is touched, the same effect occurs in reversed order.

The *residuary charge* is probably to be explained as follows: The charge, at first limited to the coatings, gradually diffuses itself on the uncoated glass for a little distance, according to the length of time the jar remains charged. At the first discharge, only the electricity which is in contact with the coatings is taken off, and that which lies on the glass slowly diffuses itself back again, and is spread over the whole coated surface; so that after the lapse of a minute or two, a sensible discharge occurs on applying the rod a second time.

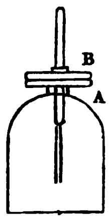
576. *The battery.*—Leyden jars are made of various sizes, from a half-pint to one or two gallons. But when a great amount of surface is needed, it is more convenient, and, in case of fracture by violent discharge, more economical, to connect several jars, so that they may be used as one. Four, nine, twelve, or even a greater number of jars, are set in a box, whose interior is lined with tinfoil, so as to connect all the outer coatings together. Their inner coatings are also connected, by wires joining all the knobs to one central knob or ring, on which the quadrant electrometer stands. Care is necessary in discharging batteries, that the circuit is not too short and too perfect, since the violence of discharge is liable to perforate the jars. A chain, three or four feet long in the circuit, will generally prevent the accident.

577. *Different routes of discharge.*—If two or more circuits are opened at once between the two coatings of a charged jar or battery, the discharge will take one or another, or divide between them, according to circumstances. If the circuits are alike except in length, the discharge will follow the *shorter*. If they differ only in conducting quality, the electricities will take the *best conductor*. If the circuits are interrupted, and in all respects alike, except that the conductors of one are pointed at the interruptions, and of the others not pointed, the discharge will follow the line which has *pointed conductors*. If the circuits are *very attenuated* (as very fine wire, or threads of gold leaf), the charge is liable to divide among them.

578. *The gold-leaf electroscope.*—The principle of induction explains the construction of some other instruments besides the Leyden jar; as the gold-leaf electroscope, the electrical condensers, and the electrophorus. The first has been already described (Art. 540); we have only to explain its operation by the principle of induction. Let a body electrified vitreously be brought within a few feet of the knob of the cover. It attracts the resinous from the leaves into the knob, and repels the vitreous from the knob into the leaves; which are thus electrified alike, and repel each other. If the charged body is brought so near that the leaves touch the conductors, which are usually placed on the sides of the cylinder, and discharge their induced electricity to them, then they collapse. After this, they will diverge again, whether the electrified body is brought still nearer, or withdrawn; if brought nearer, they diverge by means of a new portion of vitreous, repelled from the knob; if withdrawn, they diverge by the return of resinous electricity from the knob, which is no longer neutralized by the vitreous fluid, since the latter has been discharged to the earth.

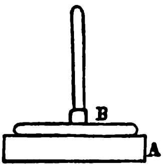
579. *Electrical condenser.*—Instruments, called by this name, are intended for the accumulation of electricity from some feeble source, until it may be rendered sensible. The most delicate is the gold-leaf condenser. Suppose the gold-leaf electroscope to have a disk, A, instead of a knob on the top (Fig. 231). Another disk, B, is furnished with an insulating handle, and between the disks is placed the thinnest possible non-conductor, as a film of varnish. Bring the finger in contact with the under-side of A, to connect it with the earth. Then bring to the upper-side of B the source of feeble electricity (as, for example, a piece of copper, after being touched to a piece of zinc), the minute quantity of fluid imparted to B, induces an equal amount of the opposite in A, drawn in from the earth. A second contact upon B repeats the action; and after this has been done a great number of times, there are condensed on the two sides of the varnish, small charges which are held in that state by induction. As yet, the gold-leaves are at rest; but on removing the finger from A, and taking up B by the insulating handle, the electricity condensed in A is set free, flows down to the leaves and repels them, thus rendering the accumulation perceptible.

FIG. 231.



580. *The electrophorus.*—This is a very simple *electrical machine* for giving the spark. It consists of a circular cake of resin in a wooden base, A (Fig. 232), and a metallic disk, B, having a glass handle. Excite the resin by fur, or flannel; set the disk B upon it, and touch the latter with the finger. The disk

FIG. 232.



now has a disguised charge of vitreous electricity, drawn in, by the resinous charge of the plate, through the finger. On lifting the disk by the handle, its charge is set free, and may be taken off in a brilliant spark. Set the disk down again, touch it, and lift it, and the same thing occurs, even hundreds or thousands of times; and, after standing for hours, is ready to operate still in the same way.

This case of inductive action seems at first perplexing, because there is no glass plate, no film of varnish, no non-conductor of any kind, between the two opposite electricities of the resin and disk. Why then do they not at once mingle, and neutralize each other? It is simply because the resin is an *excited* body, the resinous electricity having been developed upon it by friction. When we touch the finger to the disk, the vitreous that enters does meet the resinous and neutralize it for the time being; but on separating the plate and disk, the electricities also separate, as is always the case when an electric and the rubber are removed from each other after friction. Thus one charge on the resin may be made to induce any number of successive charges upon the disk.

CHAPTER IV.

ELECTRICAL LIGHT.

581. *Electrical light caused by obstruction.*—Light, we have seen, is not a constant attendant of electrical phenomena. Indeed, until noticed by Otto Guericke, it was not known to have any relation to electricity. It is seen only when the fluid is discharged in considerable quantity through an obstructing medium. Accordingly, no light is perceived when electricity flows freely through good conductors; but if such conductors suffer any interruption, as by the intervention of a space of air, or even of an imperfect conductor, then the attendant light becomes manifest. In a dark room, if a glass tube is electrified by friction, and the knuckle passed over it, faint sparks are seen; and sparks and flashes of light are copious and often brilliant about the various parts of the machine, when in vigorous operation. In these cases, the air through which the fluid darts is the obstructing medium. If the air is condensed, and offers more resistance, the light is brighter; if it is rarefied it grows fainter, until, in a perfect vacuum, its passage can not be seen at all.

Sudden compression of air, or violent action of any kind, as is well known, will develop both heat and light, and cause combustion. And since the electrical spark rends the air in passing through it, and moves with inconceivable velocity, it must produce the most sudden compression along its path. The light, the heat, and sharp sound, which attend electrical discharges, are the natural effects of such violent action. We conclude, therefore, that the light which accompanies an electrical discharge is not inherent in the fluid itself.

582. *Modifications of the light.*—The length, color, and form of the electric spark, vary with the nature and form of the conductors between which it passes, and with the quality of the medium interposed between them.

Electrical sparks are more brilliant in proportion as the substances between which they occur are better conductors. A spark received from the prime conductor upon a large metallic ball, is short, straight, and white; on a small ball it is longer, and crooked; received on the knuckle, a less perfect conductor, it is purplish or reddish; on wood, or ice, or a wet plant, or water, it is red. Moreover, a longer spark can be obtained from a small ball, attached to the prime conductor by a wire of five or six inches long, than from the prime conductor itself; and the longest and most crinkled spark is obtained, when the knob of a Leyden jar is presented to a similar brass ball attached to the prime conductor. From a point positively electrified, the fluid passes in the form of a brush or pencil of rays; a point connected with the negative side exhibits a luminous star.

A metallic chain connected with the prime conductor, becomes illuminated at the points where two links join, and at other points where the conducting powers of the metal are impaired by rust, or where roughnesses occur. If the chain has been artificially corroded by salt water, the experiment is more striking.

Some of the most pleasing experiments on electrical light are performed in rarefied air. It has been already stated, that the light is fainter as the medium is rarer; but there are also changes of color. The electrical spark, which in common air is interrupted, narrow, and white, becomes, as the rarefaction proceeds, continuous, diffused, and of a violet color, which tint it retains as long as it can be seen. If a battery is discharged through a tube several feet long, nearly exhausted of air, the whole space is filled with an intense purple light. The sparks from the machine conveyed through the same tube, exhibit flashings and tints exceedingly resembling the Aurora Borealis. If a vapor, highly rarefied, be admitted to the Torricellian vacuum, the color will be varied according to the substance. Electricity, in passing through a rare vapor of ether, causes a green light; through that of alcohol, it is red or blue

When a charged jar is placed under the receiver of an air-pump, as the exhaustion proceeds, a quivering light is seen on the glass between the top and the outer coating, indicating that a silent discharge is going on in consequence of diminished pressure of air. *Abbé Nollet's globe* is a jar so connected with a receiver, that the induced electricity, which charges the exterior, can be seen flashing through rarefied air, both in charging and discharging.

Various colors are obtained, by sending charges through different substances. An egg is bright crimson; the pith of cornstalk, orange; fluor-spar, green, and loaf-sugar, white, which lasts a few seconds.

583. Metallic conductors, if of sufficient size, transmit electricity without any luminous appearance, provided they are perfectly continuous; but if they are separated in the slightest degree, a spark will occur at every separation. On this principle, various devices are formed, by pasting a narrow band of tinfoil on glass, in the required form, and cutting it across with a penknife, where we wish sparks to appear. If an interrupted conductor of this kind be pasted round a glass tube in a spiral direction, and one end of the tube be held in the hand, and the other be presented to an electrified conductor, a brilliant line of light surrounds the tube, which has been called the spiral tube, or diamond necklace. Words, flowers, and other complicated forms, are also produced nearly in the same manner, by a proper disposition of an interrupted line of metal, on a flat piece of glass.

584. *Nature of electrical light.*—The greater density and brilliancy of the spark in condensed air, and its feebleness and diffuseness in a rarefied medium, are facts which accord well with the supposed origin, as stated in Art. 581; and the *zigzag*, or, rather, *crinkled* form of the spark when long, or when passing through condensed air, is well explained by the same theory. For the electric fluid in its passage through the air, condenses the air before it, and thus meets with a resistance which turns it off laterally; in this direction it is again condensed, and has its course again changed; and so on, until it reaches the conductor toward which it is aiming. The crinkled form of lightning is accounted for on this principle.

Electrical light is found, by optical experiments, to have in general the same nature with the light of the sun—being like this resolved into various colors by the prism, and possessing other properties, to be described under the head of Optics, which identify it with solar light.

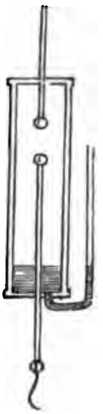
CHAPTER V.

EFFECTS OF ELECTRICAL DISCHARGES.

585. *Mechanical effects.* — Powerful electric discharges through imperfect conductors produce certain mechanical effects, such as perforating, tearing, or breaking in pieces, which are all due to the sudden and violent repulsion between the electrified particles.

A discharge through the air is supposed to perforate it. If the air, through which the spark is passed, lies partially inclosed between two bodies which are easily moved, the force by which the air is rent will drive them asunder. Thus, a little block may be driven out from the foundation of a miniature building, and the whole be toppled down. But this enlargement of inclosed air is best seen in Kinnersley's *air thermometer* (Fig. 233). As the spark passes between the knobs in the large tube, the air confined in it suffers a slight enlargement of volume, by which the water at the bottom is suddenly driven up two or three inches into the small tube. As soon as the explosion is over, the water returns again to its former level. The sound produced by a discharge is due to the sudden compression of the air, and also to the collapse which immediately succeeds. On account of the immense velocity of the electric fluid, the agitation may be considered as simultaneous through the whole line described by the spark. Hence the exceeding sharpness of the report, when the circuit is short and perfect. If the circuit is much lengthened, and especially if it is attenuated, or frequently interrupted, the discharge is more gradual, and the sharpness of the report much diminished. A few inches of wet thread, forming part of the circuit, renders the discharge almost inaudible.

Fig. 233.



586. For exhibiting the mechanical effects of electricity, a large Leyden jar is sufficient in some cases; others require the power of the battery. When the charge is passed through a thick card, or the cover of a book, a hole is torn through it, which presents the rough appearance of a bur on each side. By means of the battery, a quire of strong paper may be perforated in the same manner; and such is the velocity with which the fluid moves, that if the paper be freely suspended, not the least motion is communicated to it. (See Art. 203.) Pieces of hard wood, of loaf-sugar, of stones, and many other brittle non-conductors, are broken, or even torn asunder with violence, by a powerful charge from the battery.

The expansion of *fluids* by electricity is very remarkable, and productive of some singular results. When the charge is strong, no glass vessel can resist the sudden impulse. Beccaria inserted a drop of water between two wires in the center of a solid glass ball of two inches diameter; on passing a shock through the drop of water, the ball was dispersed with great violence. In like manner, by the sudden expansion of a small body of confined air, strongly electrified, explosions may be produced, and bodies that resist its expansion are projected with violence. Even good conductors, when minutely divided, are expanded by electricity. Thus mercury, confined in a capillary glass tube, will be expanded with a force sufficient to splinter the tube.

587. *Chemical effects.*—These are various: combustion of inflammable bodies; oxydation, fusion, and combustion of metals; separation of compounds into their elements; reunion of elements into compounds.

Ether and alcohol may be inflamed by passing the electric spark through them; nor is the effect diminished by communicating the spark by means of a piece of ice or any other cold medium. The finger may be conveniently employed to inflame these substances. Phosphorus, resin, and other solid combustible bodies, may be set on fire by the same means; gunpowder and the fulminating powders may be exploded, and a candle may be lighted. Gold leaf and fine iron wire may be burned, by a charge from the battery. Wires of lead, tin, zinc, copper, platina, silver, and gold, when subjected to the charge of a very large battery, burn with explosion, and are converted into oxides.

The same agent, moreover, is capable of reviving these oxides; that is, restoring them to the state of pure metals. By a singular contrariety of properties, water is decomposed into its gaseous elements, and the same elements are reunited to form water; and the constituent gases of atmospheric air, are, by passing a great number of electric charges through a confined portion of air, converted into nitric acid.

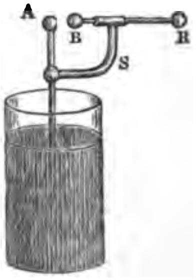
588. *Physiological effects.*—We have several times adverted incidentally to the shock experienced by the animal system, when the discharge of a jar passes through it.

A slight charge of the Leyden jar, passed through the body from one hand to the other, affects only the fingers or the wrists; a stronger charge convulses the large muscles of the arms; a still greater charge is felt in the breast, and becomes somewhat painful. The charge of a large battery is sufficient to destroy life, especially if it be received through the head. By connecting the chains, which are attached to the jar, with insulating handles, it is easy to pass shocks through any particular joint, muscle, or other part of the body, as is frequently done for medical purposes.

The charge may be passed through a great number of persons at the same time. Hundreds of individuals, by joining hands, have received the shock at once; though there is more difficulty in passing a charge of given intensity, as the number is increased.

589. *Discharging electrometers.*—These are instruments contrived for measuring the charge in the act of discharging the jar. Fig. 234 represents Lane's discharging electrometer. S is a rod of solid glass, which holds the metallic rod and balls, BR. This rod, being in a horizontal position at the height of the knob, A, can be placed at any desired distance from it. Then, if the circuit through which the charge is to be sent, terminates on one hand at A, and on the other at R, the interval of air between A and B is all which prevents discharge; and as soon as the charge is increased, till its tension is sufficient to leap that interval, the discharge will take place. The greater that space is, of course the greater the charge must be, before it will pass across. If A and B are in contact, no charge at all will collect.

Fig. 234.



590. If the spark is taken by a person from the prime conductor, the quantity is not sufficient, unless the conductor is of extraordinary size, to produce what is called the shock; a pricking sensation in the flesh where the spark strikes, and a slight spasm of the muscle, is all that is noticeable. A person may make his own body a part of the prime conductor, by standing on an *insulating stool*, that is, a stool having glass legs, and touching the conductor of the machine. This occasions no sensation at all, except what arises from the movement of the hair, in yielding to the repellency of the fluid. If another person takes the spark from him, the prick is more pungent, as the quantity is larger than in the prime conductor alone.

591. *Velocity of electricity.*—This is so great that no appreciable time is occupied in any case of discharge. When we seem to see lightning move *from* the cloud *to* the earth, we find that such a progress is imagined, not perceived; for, by a little effort, we can just as well learn to see it pass from the earth to the cloud.

About the middle of the last century experiments were tried by Watson, in London, to ascertain the length of time occupied in discharging a jar through a long circuit, which was increased to four miles. But the sparks appeared simultaneously in all parts of the line.

Wheatstone has, within a few years, devised an ingenious method of measuring the time in which electricity passes over a wire only half a mile long. The wire was so arranged that three interruptions, one near each end, and one in the middle of the wire, were brought side by side. When the discharge of a jar was transmitted, the sparks at these interruptions were seen by reflection in a swiftly revolving mirror. Any difference of time between the passage of those interruptions, even less than *one-millionth* of a second, could be easily perceived by the *displacement* of the sparks as seen in the whirling mirror. The amount of observed displacement, and the known rate of revolution of the mirror, would furnish the interval of time occupied by the electricity in passing from one interruption to the next. By a series of experiments Wheatstone arrived at the conclusion, *that, on copper wire, one-fifteenth of an inch in diameter, electricity moves at the rate of* 288,000 *miles per second*, a velocity much greater than that of light.*

Galvanic or Dynamical electricity, which is evolved not by friction, but by chemical action, moves very much slower. Its rate on iron wire, of the size usually employed for telegraph lines, is about 16,000 miles per second.

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CHAPTER VI.

ATMOSPHERIC ELECTRICITY.—THUNDER-STORMS.—MEANS OF DEFENCE.—CAUSE OF ELECTRICAL PHENOMENA.

592. *Atmospheric electricity.*—The atmosphere is always more or less electrified, sometimes positively, sometimes negatively. This fact is ascertained by several different forms of apparatus. For the lower regions, it is sufficient to elevate a *metallic rod* a few feet in length, pointed at the top, and insulated at the bottom. With the lower extremity is connected an electroscope, which indicates the presence and intensity of the electricity. For experiments on the electricity of the upper regions, a kite is employed, not unlike a boy's kite, with the string of which is intertwined a fine metallic wire. The lower end of the string is insulated by fastening it to a support of glass, or by a cord of silk. If a cloud is near the kite, the quantity of electricity conveyed by the string may be greatly increased, and even become dangerous. Cavallo received a large number of severe shocks in handling the kite-string; and Richman, of Petersburgh, was killed by a discharge of electricity which came down the rod which he had arranged for his experiments, but which was not provided with a conductor near by it, for taking off extra charges.

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* Phil. Trans., 1834, vol. ii.

The electricity of the atmosphere is most developed, when hot dry weather succeeds a series of rainy days, or the reverse; and during a single day, the air is most electrical, when dew is beginning to form before sunset, or when it begins to exhale after sunrise. In clear, steady weather, the electricity is generally vitreous; but in falling or stormy weather, it is frequently changing from vitreous to resinous, and from resinous to vitreous.

593. *Lightning and electricity identical.*—Franklin was the first to point out the resemblances between the phenomena of lightning, and those of frictional electricity. He was also the first to propose the performance of electrical experiments by means of electricity drawn from the clouds. The points of resemblance named by Franklin were these: 1. The crinkled form of the path. 2. Both take the most prominent points. 3. Both follow the same materials as conductors. 4. Both inflame combustible substances. 5. They melt metals in attenuated forms. 6. They fracture brittle bodies. 7. Both have produced blindness. 8. Both destroy animal life. 9. Both affect the magnetic needle in the same manner. In 1752, he obtained electricity from a thunder-cloud by a kite, and charged jars with it, and performed the usual electrical experiments.

594. *Thunder-storms.*—Thunder-clouds are, of all atmospheric bodies, the most highly charged with electricity; but all single, detached, or insulated clouds, are electrified in greater or less degrees, sometimes positively and sometimes negatively. When, however, the sky is completely overcast with a uniform stratum of clouds, the electricity is much feebler than in the single detached masses before mentioned. And, since fogs are only clouds near the surface of the earth, they are subject to the same conditions: a driving fog, of limited extent, is often highly electrified.

Thunder-storms occur chiefly in the hottest season of the year, and after mid-day; and are more frequent and violent in warm than in cold countries. They never occur beyond 75° of latitude—seldom beyond 65°. In the New England States, they usually come from the west, or some westerly quarter.

595. The storm itself, including every thing except the electrical appearances, is supposed to be produced in the same manner as other storms of wind and rain; and the electricity is developed by the rapid condensation of watery vapor, and by friction. Electricity is not to be regarded as the *cause*, but as a *consequence*, or *concomitant* of the storm. But the precipitation of vapor must be sudden and copious, since when the process is slow, too much of the electricity evolved would escape to allow of the requisite accumulation. Also, if a storm-cloud is of great extent, it is not likely to be highly electrified, because the opposite electricities, which may be developed in different parts of it, have opportunity to mingle and neutralize; and points of communication with the earth will here and there occur. Clouds of rapid formation, violent motion, and limited extent, are therefore most likely to be thunder-clouds.

596. When a cloud is highly charged, it operates inductively on other bodies near it; such as other clouds, or the earth. Hence, discharges will occur between them. Lightning passes frequently between two clouds; or even between two parts of the same cloud, in which opposite electricities are so rapidly developed, that they can not mingle by conduction. But, in general, the discharges of lightning take place between the electrified cloud and the earth, whose nearer part is thrown into the opposite electrical state by induction. It is supposed that, in some instances, a discharge occurs between two distant clouds by means of the earth, which constitutes an interrupted circuit between them. The crinkled form of the path of lightning is explained in the same way as that of the spark from the machine, and the thunder is caused by the simultaneous rupture and collapse of air in all parts of the line of discharge. The words *chain-lightning*, *sheet-lightning*, and *heat-lightning* are supposed not to indicate any real differences in the lightning itself, but only in the circumstances of the person who observes it. If the crinkled line of discharge is seen, it is *chain* or *fork* lightning; if only the light which proceeds from it is noticed, it is *sheet*-lightning; if, in the evening, the thunder-storm is so far distant that the cloud can not be seen, nor the thunder heard, but only the light of its discharges can be discerned in the horizon, it is frequently called *heat*-lightning.

597. *Lightning-rods.*—Franklin had no sooner satisfied himself of the identity of electricity and lightning, than, with his usual sagacity, he conceived the idea of applying the knowledge acquired of the properties of the electric fluid, so as to provide against the dangers of thunder-storms. The conducting powers of metals, and the influence of pointed bodies, to transmit the fluid, naturally suggested the structure of the lightning-rod.

The experiment was tried, and has proved completely successful; and probably no single application of scientific knowledge ever secured more celebrity to its author.

Lightning-rods are often constructed of wrought iron, about three-fourths of an inch in diameter. The parts may be made separate, but, when the rod is in its place, they should be joined together so as to fit closely, and to make a continuous surface, since the fluid experiences much resistance in passing through links and other interrupted joints. At the bottom, the rod should terminate in two or three branches, going off in a direction from the building, and descending to such a depth, that they will reach permanent moisture. At top, the rod should be several feet higher than the highest parts of the building. It is best, when practicable, to attach it to the chimney, which needs peculiar protection, both on account of its prominence, and because the products of the combustion, smoke, watery vapor, &c., are conductors of electricity. For a similar reason a kitchen chimney, being that in which the fire is kept during the season of thunder-storms, requires to be especially protected. The rod is terminated above in one or more sharp points; and as these points are liable to lose their sharpness, and have their conducting power impaired by rust, they are protected from corrosion by being covered with gold leaf or silver plate. Rods may be made of smaller size than above described; but if so, there should be a proportionally greater number. It is well to connect with the rods, and with the earth, all extended conductors upon or within the building, such as metallic coverings of roofs, water conductors, bundles of bell-wires, &c.; in order that large discharges may have opportunity to divide, and take several circuits, without doing injury at the non-conducting intervals.

598. Lightning-rods are of service, not so much in receiving a discharge when it comes, as in diminishing the number of discharges in their vicinity. They continually carry on a silent communication between the two fluids, which are attracting each other, one in the cloud, the other in the earth; so that a village, well furnished with rods, has few discharges of lightning in it. All tall pointed objects, like spires of churches, and masts of ships, exert a similar influence, though in a less degree, because not so good conductors.

During a thunder-storm, or immediately after it, if a person can be near the top of a high rod, he will sometimes hear the hissing sound of electricity escaping from it, as from a point attached to the prime conductor of a machine. In the same circumstances, if it were quite dark, he would probably see the brush or star of light on the point. The statement of Cæsar in his Commentaries, "that the points of the soldiers' darts shone with light in the night of a severe storm," probably refers to the visible escape of electricity from the weapons as from lightning-rods.

599. *Protection of the person.*—Silk dresses are sometimes worn with the view of protection, by means of the insulation they afford. They can not, however, be deemed effectual unless they completely envelop the person: for if the head and the extremities of the limbs are exposed, they will furnish so many avenues to the fluid as to render the insulation of the other parts of the system of little avail. The same remark applies to the supposed security that is obtained by sleeping on a feather bed. Were the person situated *within* the bed, so as to be entirely enveloped by the feathers, they would afford some protection; but if the person be extended on the surface of the bed, in the usual posture, with the head and feet nearly in contact with the bedstead, he would rather lose than gain by the non-conducting properties of the bed; since, being a better conductor than the bed, the charge would pass through him in preference to that. If the bedstead were of iron, its conducting quality would probably be a better protection than the insulating property of the feathers; since, by taking the charge itself, it would keep it away from the person. So, a man's garments soaked with rain have been known to save his life, being a better conductor than his body.* Animals under trees are peculiarly exposed, because the trees by their prominence are liable to be the channels of communication for the electric discharge, and the animal body, so far as it reaches, is a better conductor than the tree. Tall trees, however, situated near a dwelling-house, furnish a partial protection to the building, being both better conductors than the materials of the house, and having the advantage of superior elevation.

It is well, during a thunder-storm, to avoid every considerable mass of water, and even the streamlets that have resulted from a recent shower; for these are all excellent conductors, and the height of a human being, when connected with them, may perhaps determine the course of an electric discharge. The partial conductors, through which the lightning directs its course, when it enters a building, are usually the appendages of the walls and partitions; the most secure situation is therefore the middle of the room. It is absurd to take refuge in a cellar, or in the lowest story of a house; many instances are on record in which the basement story has been the only part of the building that has sustained severe injury.

600. *Meaning of the phrase, "struck by lightning."*—The word *strike*, which is used with reference to lightning, conveys no correct idea of the nature of the movement of electricity, or of the injury which it causes. According to the theory generally received (Du Fay's), one fluid, developed in a cloud, causes the other to be accumulated by induction in the part of the earth nearest to it. These fluids strongly attract each other; consequently, the fluid in the earth presses upward into all prominent conducting bodies toward the other; and, if those bodies are numerous, high, the best of conductors, and terminated by points, the electricity will flow off from them abundantly, and mingle with its opposite in the air above; and thus discharges are in a great degree prevented. But if these channels for silent communication are not furnished, the quantity of electricity will increase, till the strength of attraction becomes so great, that the fluid will break its way through the air, off from some prominent object, as a building or tree, and thus the union of the two electricities takes place. The building or tree, in this case is said to be *struck by lightning;* it is rent, or otherwise injured, by the great quantity of electricity which passes violently through it, in an inconceivably short space of time. So, the charge of a large battery, taken through the body in the usual way, would prostrate a person by the violence of the shock; but the same charge, if allowed to occupy a few seconds in passing by means of a point, would not be felt at all.

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* Amer. Jour. Sci., vol. v., p. 121.

601. *Nature of electricity.*—In the foregoing treatise, we have constantly used language which implies that electricity is a *substance.* But this is done, not because it is known to be a substance, but for the sake of employing the simplest method for classifying the phenomena, and presenting them clearly before the mind of the learner. There is very little, if any, positive evidence, that electricity is material. It is not known to possess either extension, impenetrability, inertia, or weight. It may consist only of a vibratory motion in the particles of some medium. *Sound* travels swiftly through the air; but it has long been known, that what thus moves at the rate of more than a thousand feet per second, is not a substance, but only a system of vibrations *in* a substance. Most philosophers of the present day believe, that *light* is not a *material*, emanating from the sun and stars through all space; but a minute and rapid *tremor*, excited by luminous bodies and propagated in all directions in a medium that fills the universe. *Heat* is probably of the same nature. And it is not *impossible*, that electricity may, in this respect, be like them. But, until a *wave-theory* can be constructed for electricity, as complete and consistent as are the *fluid-theories* which we have presented, it is expedient to employ the latter in giving a scientific exposition of electrical phenomena, without pretending to *know* that either of them is true.

602. *The two theories.*—But, regarding electricity as a fluid, which of the theories, Franklin's, or Du Fay's, has most evidence in its favor? Most of the arguments, relied on by the advocates of each theory, in the early part of this century, are found, as the science advances, to have very little weight, and therefore need not be presented here. But the fact, that both electrical states are subject to precisely the same laws of distribution on electrified bodies, points to the two-fluid theory as the most consistent, and has compelled philosophers generally to adopt it. According to the *one-fluid* theory, when a body is *negatively* electrified, there is a *deficiency* spread over the surface, while the interior retains its natural share. Now, besides the seeming absurdity, that the *deficiency* of a fluid should vary its degree of intensity on a surface by the same law of distribution as the fluid itself does, the fact militates with two principles of the same theory; viz., that the fluid is self-repellent, and that unsaturated matter attracts the fluid. The fluid in the central parts should, by its self-repellency, diffuse itself equally to the surface, and thus the deficiency should be found to exist throughout the body. Moreover, the unsaturated matter at the surface would aid in producing this diffusion, by attracting to itself some of the fluid within. Thus, the first principles of Franklin's theory conflict with each other, when we attempt to explain by it the arrangement of a negative charge on the surface of a body. The remarkable agreement between the refined experiments of Coulomb and others, and the results of the profound mathematical calculations of Poisson, has done much to give currency to the theory of Du Fay.

603. The entire subject of electricity is very comprehensive, including not only *statical electricity*, discussed in the preceding pages, but also *dynamical electricity*, or galvanism; and connecting itself with heat and magnetism, in the subjects of *thermo-electricity* and *electro-magnetism*. In many institutions of learning, it is customary to associate all these branches, except statical electricity, with the department of chemistry.

PART VII.—MAGNETISM.

604. *General definitions and remarks.*—*Magnetism* is the science which treats of the properties and effects of the magnet. The same word is also used to denote the unknown cause of magnetic phenomena; as when we speak of magnetism as being excited or developed.

Magnets are bodies, either natural or artificial, which have the property of attracting iron, and the power, when freely suspended, of taking a determinate direction in regard to the earth. The *natural magnet*, sometimes called the *loadstone*,* is an oxide of iron of a peculiar character, found occasionally in beds of iron ore. Though commonly met with in irregular masses only a few inches in diameter, yet it is sometimes found of a much larger size. One brought from Moscow to London, weighed 125 pounds, and supported more than 200 pounds of iron. The *artificial magnet* is a steel bar of regular form, in which the properties of the natural magnet are developed. The artificial is far more convenient than the natural magnet for experimental purposes.

Magnetism is the most recent of all the physical sciences, and notwithstanding the numerous discoveries achieved in it within a few years, and the remarkable precision with which its laws have been ascertained, yet it is still to be regarded as a science in its infancy, although it is rapidly progressive.

605. If a magnet be rolled in iron filings, it will attract them to itself. This effect takes place especially at two opposite points, where a much greater quantity of the filings will be collected than in any other parts of the body. The two opposite points in a magnet, where its attractive powers appear chiefly to reside, are called its *poles*. The straight line which joins the poles, is called the *axis*.

If a large sewing needle, or a small bar of steel be rubbed on the loadstone, one extremity on one pole, and the other extremity on the other, the needle or bar will itself become a magnet, capable of exhibiting all the properties of a loadstone. Without staying at present to describe more minutely the process of making artificial magnets, we will suppose ourselves provided with several magnetic needles and bars, and we may proceed with them to study the leading facts of the science of magnetism. By attaching a fine thread to the middle of a needle, and suspending it so as to move freely in a horizontal plane, or by resting it on a point, as is represented in Fig. 235, we shall have a simple and convenient apparatus for numerous experiments. The needle thus suspended will place itself in a direction nearly, though perhaps not exactly, north and south. If the needle is drawn out of the position it assumes when at rest, it will vibrate on either side of that position until it finally settles in the same line as before, one pole always returning toward the north, and the other toward the south. Hence the two poles are denominated respectively *north and south poles.* In magnets prepared for experiments, these poles are marked either by the letters N and S, or by a line drawn across the magnet near one end, which denotes that the adjacent pole is the north pole.

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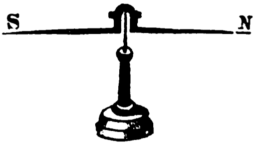
* From the Saxon *lædan*, to guide. Also spelled *lodestone*.

FIG. 235.



606. By means of the foregoing apparatus, we may ascertain that the magnet has the following general properties, viz.:

1. Mutual attraction and repulsion.
2. The power of developing magnetism in iron or steel by induction.
3. Polarity, or the power of assuming a fixed relation to the meridian of any place, called the *declination* of the needle.
4. The power of inclining itself to the horizon, at various angles, according to the locality, called the *dip* of the needle.

CHAPTER I.

MUTUAL ACTION OF MAGNETS.

607. *Action of the magnet on iron.*—When either pole of a magnet is brought near to a piece of iron, a mutual *attraction* takes place between them.

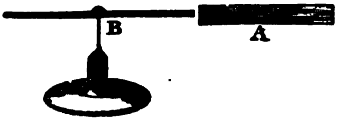
Thus, when the ends of a magnetic bar or needle are dipped into a mass of iron filings, these adhere in a cluster to each pole. A bar of soft iron, or a piece of iron wire, resting on a cork, and floating on the surface of water or mercury, may be led in any direction by bringing near to it one of the poles of a magnet. This action is moreover *reciprocal;* that is, the iron attracts the magnet with the same force that the magnet attracts the iron (Art. 12). If the two bodies be placed on separate corks and floated, they will approach each other with equal momenta; or if the iron be held fast, the magnet will move toward it.

Two other metals besides iron, namely, nickel and cobalt, are susceptible of magnetic attraction. These metals, however, exist in nature only in comparatively small quantities, and therefore by magnetic bodies, are usually intended such as are ferruginous. Even iron, in some of its combinations with other bodies, loses its magnetic properties; indeed, only a few of the numerous ores of iron are attracted by the magnet. But soft metallic iron, and some of the ores of the same metal, affect the needle even when existing in exceedingly small quantities, so that the magnet becomes a very delicate test of the presence of iron. Compass needles are sometimes said to be disturbed by the minute particles of steel left in the dial plate by the graver; and the proportion of iron in some minerals may be exactly estimated by the power they exert upon the needle.

608. *Action of magnets on each other.*—When magnets are brought near to each other, poles of the *same* name *repel*, and those of *different* names *attract* each other. Thus, the north pole of one magnet will repel the north pole of the other, and attract its south pole. The south pole of one will repel the south pole of the other, and attract its north pole. These effects, it will be perceived, are analogous to those produced by the two species of electricity; and they equally imply two species of magnetism, or two magnetic fluids (as it is convenient to call them), namely, the northern and the southern, or as they are now denominated, the *austral* and the *boreal* fluids. The fluid occupying the north end of a bar is called the *austral*, and that in the south end the *boreal*. This inversion of position and name is in order that the *earth* may be regarded as the standard magnet, exhibiting the *boreal* fluid in its *northern* parts, and the *austral* in its *southern*. Then, according to the proposition just stated, the fluids of the needle are developed in the contrary order.

609. *Magnetic induction.*—By bringing a magnet near to iron or steel, the latter is rendered *magnetic by induction*. Thus, let the north pole of a magnetic bar, A (Fig. 236), be brought near to one end of an unmagnetized bar of soft iron, B; the iron will immediately become itself a magnet, capable of attracting iron filings, having polarity when suspended, and possessing the power of communicating the same properties to other pieces of iron. It is, however, only while

FIG. 236.

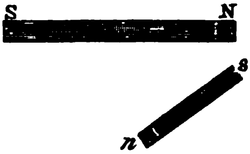


the iron remains in the vicinity of the magnet, that it is endued with these properties; for let the magnet be withdrawn, and it loses at once all the foregoing powers. This, it will be remarked, is asserted of *soft iron;* for steel and hardened iron are differently affected by induced magnetism.

On examining the kind of magnetism induced upon the two ends of the iron bar, B (Fig. 236), which we may easily do by bringing it near to the poles of the needle (Fig. 235), we shall find that the nearer end has south, and the remoter end north polarity. This effect also is analogous to that produced by electrical induction. A corresponding effect would have taken place, had the south, instead of the north pole of the magnet been presented to the bar of iron; in which case, the nearer end would have exhibited the northern, and the remoter end southern polarity; or, in general terms, when a magnetic pole is presented to iron, it induces an unlike pole on the nearer end, a like pole on the remote end; and near the center is a neutral point.

610. It is not essential to the success of these experiments, that the bars of iron which receive magnetism by induction, should be placed in a straight line with the magnet; they may be at right angles to it, or inclined at any other angle, the only essential condition being, that the end of the bar should be brought near to the pole of the magnet. Indeed, the effect is increased; that is, the magnetism of the iron bar is rendered stronger when the bar is inclined toward the magnet, as in Fig. 237, and is the strongest of all when it is placed parallel to the magnet; for it will be seen that in these two latter positions, both poles of the magnet conspire in their action upon the iron bar.

Fig. 237.



611. *Reflex influence.*—When a magnet exerts the inductive power upon a piece of iron which is near it, *its own magnetic intensity is increased.* The end of the piece of iron contiguous to the pole of the magnet, is no sooner endued with the opposite polarity, than it reacts upon the magnet, and increases its intensity; and a series of actions and reactions takes place between the two bodies, similar to what occurs in electrical induction. (Art. 566, &c.) On this account the powers of a magnet are increased by action, and impaired or even lost by long disuse. By adding, from time to time, small pieces of iron to the weight taken up by a magnet, its powers may be augmented greatly beyond their original amount. Hence, the force of attraction of the dissimilar poles of two magnets, is greater than the force of repulsion of the similar poles; because, when the poles are unlike, each contributes to enhance the power of the other; but when they are alike, the influence which they reciprocally exert, tends to make them unlike, and of course to impair their repulsive energies.

Hence, also, a strong magnet has the power of reversing the poles of a weak one. Suppose the north pole of the weaker body to be brought into contact with the north pole of the stronger, the latter will expel north polarity, and attract the south, a change which in certain cases will be permanent.

612. *Double induction.*—If the north pole of a magnetic bar be placed upon the middle of an iron bar, the two ends of the latter will each have north polarity, while the part of the bar immediately in contact with the magnet receives south polarity; and if the same north pole be placed on the center of a circular piece of iron, all parts of the circumference will be endued with north polarity, while the plate will have a south pole in the center. By cutting the plate into the form of a star, the outer extremity of each radius becomes a weak north pole when the north pole of a magnet is placed in the center of the star. If an iron bar is placed between the dissimilar poles of two magnetic bars (all being in one straight line), both of the magnets will conspire to increase the intensity of each pole of the bar, and the magnetism exhibited by the bar will be considerably stronger than if either magnet acted alone; but if the same bar be placed between the two similar poles, the opposite polarity will be imparted to each end, while the same polarity is given to the center of the bar. Thus, if the bar be placed between the north poles of two magnets, each end of the bar will become a south pole, and the center a north pole. When one end of a magnetic bar is applied to the ends of two or more wires or sewing needles, the latter arrange themselves in radii diverging from the magnetic pole. This effect is in consequence of their remoter ends becoming endued with similar polarity, and repelling each other. A like effect is observable among the filaments of iron filings, that form a tuft on the end of a magnetic bar.

613. The foregoing experiments are sufficient to show that when a piece of iron is attracted by the magnet, it is first itself converted into a magnet by the inductive influence of the magnetizing body. Each of the iron filings which compose the tuft at the pole of a magnetic bar or needle, is itself a magnet, and in consequence of being such, induces the same property in the next particle of iron, and that in the next, and so on to the last. Hence, magnetic attraction does not exist, strictly speaking, between a magnet and iron, but only between the opposite poles of magnets; for the iron must first become a magnet before it is capable of magnetic influence.

614. *Coercive force.—Soft iron* readily acquires the magnetic state, and as readily loses it; but *hardened steel* acquires it with difficulty, and retains it permanently. That quality of a body which obstructs the development of magnetism by induction, and which also hinders the magnet from returning to the natural state, when the inducing body is removed, is called the *coercive force*. In pure annealed iron, this force is reduced nearly to zero, and the iron becomes a perfect magnet the instant another magnet acts on it; and entirely loses the development as soon as the magnet is removed. But when iron is a little hardened by hammering, some coercive force is apparent; it becomes a magnet more slowly, and it more slowly returns to its unmagnetic condition.

When the iron becomes steel, by being combined with carbon, and especially when the steel is tempered to extreme hardness, the coercive force is very great. It then becomes difficult to develop magnetism in it by induction; but after the development has once taken place, it is a permanent magnet, and may by care be used as such for years.

615. If a steel bar is placed very near a strong magnet, the action of the magnet commences immediately upon the end of the bar nearest to it,—the north pole, for example, communicating south polarity to the contiguous extremity of the bar; but a sensible time is required before the north polarity is fully imparted to the remote extremity. Indeed, if the bar be a long one, it sometimes happens that the north polarity never reaches the farthest end, but stops short of it at some intermediate point. This north pole is succeeded by a second south pole, that by another north pole, and thus several alternations between the two poles occur before reaching the end of the bar.

616. The coercive force is weakened by any cause which excites a tremulous or vibratory motion among the particles of the steel. This happens when the bar is struck by a hammer, so as to produce a ringing sound, which indicates that the particles are thrown into a vibratory motion. The passage of an electric discharge through a steel bar under the influence of a magnet, overcomes the coercive force for the time being, and develops permanent magnetism. Heat produces the same effect; and hence a steel bar is conveniently magnetized by heating it to redness, placing it under a powerful inductive influence, and then hardening it by sudden cooling. The coercive force is thus neutralized by heat, till the development takes place, when it is restored, and the bar is a permanent magnet

A magnet, however, loses its virtues by the same means as, during the process of induction, were used to promote their acquisition. Accordingly, any mechanical concussion or rough usage impairs or destroys the powers of a magnet. By falling on a hard floor, or by being struck with a hammer, it is greatly injured. Heat produces a similar effect. A boiling heat weakens, and a red heat totally destroys the power of a needle.

617. *Magnetism not transferred, but only developed.*—This appears most strikingly in the remarkable fact that if a magnet be divided, even at the neutral point, where there is no sign of magnetism, the parts instantly become complete magnets, two unlike poles manifesting themselves at the place of fracture. We here meet with a striking distinction between magnetic and electric induction. When a body electrified by induction is divided into two equal parts, the individual electricities alone remain in each part respectively; but in the case of magnetic induction, both magnetisms actually exist in each half, and become apparent so soon as the bar is divided; and if each fragment is divided again, the same phenomenon is repeated, and so on indefinitely.

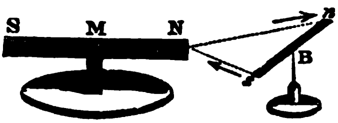
In magnetism, therefore, there is never, as in electricity, any *transfer* of properties, but only the excitation of such as were already inherent in the body acted upon. Magnetism never passes out of one body into another; nor can we ever obtain a piece of iron or steel that contains exclusively either northern or southern polarity.

618. *Relation of magnetic intensity and distance.*—The *repulsion* of like poles, and the *attraction* of unlike, each varies *inversely as the square of the distance.* This law was ascertained by Coulomb, by means of the torsion balance, in a manner similar to that adopted in investigating the law of electrical attraction. (See Art. 557.) The same law, therefore, which governs the attraction of gravitation, likewise controls electrical and magnetic attractions. It is the most extensive law of the physical world. Nor is this action at a distance prevented, or even impaired, by the interposition of other bodies not themselves magnetic.

619. *Development only at the surface.*—Experiment shows that the magnetic power developed in iron resides wholly at the surface. For a hollow globe of a given surface will produce the same effect on a needle as though it were solid throughout. In this fact we again meet with a striking analogy between magnetism and electricity, a similar property having before been shown to belong to the electric fluid. This discovery was made by Barlow, of England, to whose labors are due many important investigations in this science.

620. *Equilibrium of a needle near a magnet.*—If a small needle, free to revolve, be placed near the pole of a magnet, so that its center is in the axis of the magnet, it will place itself in the line of that axis. For suppose that SN (Fig. 238) is a large magnetic bar, and *sn* a small needle placed near the north pole of the magnet, with its center in the axis; it will be seen that the action of the pole of the magnet is such as to bring the needle into a line with the magnet. The action of the pole N upon the needle tending to give it this direction, is, since it repels *n* and attracts *s*, equal to the sum of its actions upon both poles. The pole S, by repelling *s*, and attracting *n*, tends to reverse this position, but, on account of greater distance, it acts more feebly than N.

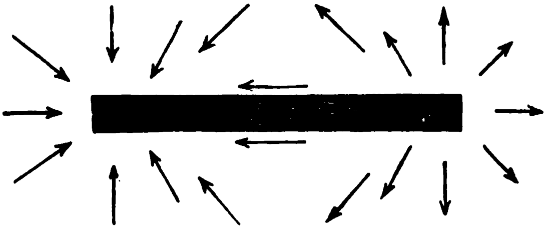
Fig. 238.



If the pivot of the needle is in a line perpendicular to the bar through its center, the needle will be in equilibrio when parallel to the bar, with its poles in contrary order. By supposing B (Fig. 238) to be placed as just stated, it will be seen that the actions of both poles of the magnet would conspire in relation to each pole of the needle; and that as these forces are equal, equilibrium will take place only when the needle is parallel to the bar.

At intermediate points the needle will assume all possible inclinations to the axis of the bar, each position being determined by the resultant of the four forces which act on the needle. The arrows in Fig. 239 indicate some of the positions which the needle takes in being carried round the magnet.

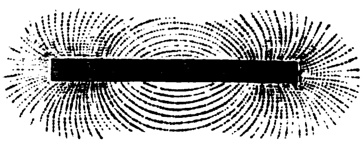
Fig. 239.



It is to be observed, that in all possible situations, the needle tends to move, as a whole, toward the bar, since the attractions always exceed the repulsions.

621. *Magnetic curves.*—All the foregoing cases are shown at once by iron filings strewn on paper or parchment, which is stretched on a frame and placed near a magnet. Let the paper be slightly jarred, while the magnet lies parallel to it, either above or below, and all the inclinations of the needle will be represented by the particles of iron arranged in curves from pole to pole. (Fig. 240.) Near the poles of the magnet, the filings stand up on the paper at various inclinations. These are the extremities of still other curves, which would be formed in all possible planes passing through the axis of the magnet, provided the filings could be suspended in the air, and the magnet placed in the midst of them. These are called *magnetic curves*. They possess several curious properties, which have been investigated by mathematicians; but we must refer the student to more extensive treatises than the present for a full development of this subject.*

FIG. 240.



622. *Theory of magnetism.*—In accordance with the facts and phenomena described in this chapter, the following theory is framed for embodying them in a system. In many particulars it resembles the theory of Du Fay in electricity. Two magnetic fluids, the austral and the boreal, reside in iron, nickel, and cobalt, ordinarily combined and neutralized, so that no magnetic properties are apparent; but capable of being developed by a magnet in the vicinity. The austral fluid of the magnet attracts the boreal of the iron, and repels its austral, but does not *separate* them beyond the atoms in which they reside. The magnet thus formed is to be conceived of as made up of innumerable magnetized particles, having their north poles turned all the same way, toward the south pole of the inducing magnet, and all their south poles the opposite way. That there is no transfer of a fluid from one particle to another, has been noticed (Art. 617). There can not, therefore, be a magnet charged with the boreal fluid alone, or with the austral fluid alone, as in electricity we can have a body charged only with the vitreous, and another only with the resinous electricity. So, also, magnetism can not be *imparted* to a body, as electricity can be; but, wherever it appears, is developed by induction, the fluids previously existing in the body in the combined and latent state.

It is not necessary that the particles should be united by cohesion into a solid bar; but a magnet can be formed by filling a brass tube with iron filings, or forming a rod of cement mixed with filings, and then subjecting it to inductive influence.

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* Journal of the Roy. Inst., Feb., 1831.

CHAPTER II.

RELATIONS OF THE MAGNET TO THE EARTH.

623. *Declination of the needle.*—When the needle is balanced horizontally, and free to revolve, it does not generally point exactly north and south; and the angle by which it deviates from the meridian is called the *declination*. A vertical circle coincident with the direction of the needle at any place is called the *magnetic meridian*. As the angle between the magnetic and the geographical meridians is generally different for different places, and also varies at different times in the same place, the word *variation* is convenient to express these *changes* in declination, though it is much used as synonymous with declination itself.

The discovery of the variation of the needle has been commonly ascribed to Columbus. His son Ferdinand states, that on the 14th of September, 1492, his father first discovered the variation, and that in consequence his crew mutinied, supposing that the needle had lost its polarity, and that they would not be able to find their way back to Europe. It appears, however, that the same phenomenon had been discovered about two hundred years before that period, though it had not become generally known to navigators.

The declination of the needle is not the same at the same time in all parts of the earth, but every part has its particular declination. For instance, if we sail from the Straits of Gibraltar to the West Indies, in proportion as we recede from Europe and approach America, the compass will point nearer and nearer due north; and when we draw near the American coast, it will point exactly north. But if we sail from Great Britain to the southern coast of Greenland, we shall find the needle deviate further and further from the north, as we approach Greenland, where the declination will not be less than 50°. In some parts of Baffin's Bay the needle points due west.

624. *Isogonic curves.*—This name is given to a system of lines imagined to be drawn through all the points of equal declination on the earth's surface. We naturally take as the standard line of the system that which connects the points of *no declination*, or the isogonic of 0°. Commencing at the north pole of dip, about Lat. 70°, Lon. 96°, it runs in a general direction E. of S., through Hudson's Bay, across Lake Erie, and the State of Pennsylvania, and enters the Atlantic Ocean near Cape Lookout, in North Carolina. Thence it passes east of the West

India Islands, and across the N. E. part of South America, pursuing its course to the south polar regions. It reappears in the eastern hemisphere, crosses New Holland, and bears rapidly westward across the Indian Ocean, and then pursues a northerly course across the Caspian Sea to the Arctic Ocean. There is also a detached line of no declination, lying in eastern Asia and the Pacific Ocean, returning into itself, and inclosing an oval area of 40° N. and S. by 30° E. and W. Between the two main lines of no declination in the Atlantic hemisphere, the declination is *westward;* in the Pacific hemisphere, outside of the oval line just described, it is *eastward.* Among the other isogonic curves there are still greater irregularities. There are at least two instances in which a curve makes a wide sweep, and then intersects its own path, while those within the loop return into themselves; as the isogonic of 8° 40′ E., which intersects in the Pacific, W. of Central America.

In the north-eastern part of the United States the declination has long been a few degrees to the west, with very slow variation, sometimes diminishing, sometimes increasing.*

625. *Annual and secular variation.*—The declination of the needle at a given place is not constant, but is subject to a small annual change, which carries it to a certain limit on one side of the meridian, when it becomes stationary for a time, and then returns, and proceeds to a certain limit on the other side of it, occupying many years in each vibration. At London, in 1580, the declination was 11¼° E.; in 1657, it was 0°; after which time, the needle continued its western movement for 157 years; in 1814, the declination was 24½° W.; since then the needle has been moving slowly eastward. The entire secular vibration will probably last more than three centuries. The average variation is 9′ 17″ annually. But like other vibrations, it is much slower toward the extremes. For instance, from 1814 to 1823 the annual variation was only 1′ 2″, while from 1692 to 1722 it was near 17′.

626. *Diurnal variation.*—The needle is also subject to a small *daily* oscillation. In the morning the north end of the needle has a motion eastward, amounting to one, two, or three minutes, when the variation to the east of its mean position is usually greater than at any other hour of the day. During winter this extreme point is attained at about 8 o'clock, but as early as 7 o'clock in the summer. After reaching this limit it gradually moves to the west, and attains its extreme position at about 3 o'clock in winter, and 1 o'clock in summer. From this time the needle again returns eastward. The whole amount of the diurnal variation rarely exceeds 12 minutes, and is commonly much less than that. These changes of declination during the day are connected with changes of *temperature*, being from May to October, inclusive, 11′ 56″, and from November to April, 5′ 11″.

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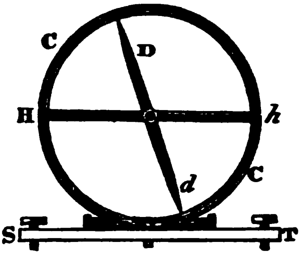
* Professor Loomis, Am. Jour., xxx, 224.

Fig. 241.



627. *Dip of the needle.*—A needle first balanced on a horizontal axis, and then magnetized and placed in the magnetic meridian, assumes a fixed relation to the horizon, one pole or the other being usually depressed below it. The angle of depression is called the *dip* of the needle. Fig. 241 represents the *dipping needle*, with its adjusting screws and spirit-level; and the depression may be read on the graduated circle. In the New England States, the dip of the north pole is as great as the figure represents, about 75°.

628. *Isoclinic curves.*—A line passing through all points where the dip of the needle is nothing, i. e. where the dipping needle is horizontal, is called the *magnetic equator of the earth.* It is an irregular curve around the earth in the region of the equator, nowhere departing from it more than 12°. At every place north of the magnetic equator the north pole of the needle descends, and south of it the south pole descends; and, in general, the greater the distance, the greater is the dip. Imagine now a system of lines, each passing through all the points of equal dip; these will be nearly parallel to the magnetic equator, which may be regarded as the standard among them. These magnetic parallels are called the *isoclinic curves;* they somewhat resemble parallels of latitude, but are inclined to them, conforming to the oblique position of the magnetic equator. The points of greatest dip, or dip of 90°, are called the *poles of dip.* There is one in the northern hemisphere, and one in the southern. The north pole of dip was found, by Capt. James C. Ross, in 1831, to be at or very near the point, 70° 14′ N.; 96° 40′ W. The south pole is not yet so well determined.

At the poles of dip the horizontal needle loses all its directive power, because the earth's magnetism tends to place it in a vertical line, and, therefore, no component of the force can operate in a horizontal plane. The isogonic lines in general converge to the two dip-poles; but, for the reason just given, they can not be traced quite to them.

The dip of the needle, like the declination, undergoes a variation, though by no means to so great an extent. In the course of 250 years, it has varied 5° at London. At present the dip in London is about 70°, and varies two or three minutes annually.

629. *Magnetic intensity of the earth.*—The force exerted by the magnetism of the earth varies in different places, being generally least in the region of the equator, and greatest in the polar regions. The ratio of intensity in different places is measured by the number of vibrations which the needle makes in a given time. In the discussion of the pendulum, it was proved (Art. 161) that gravity varies as the square of the number of vibrations. For the same reason, the magnetic force varies as the square of the number of vibrations of the needle.

630. *Isodynamic curves.*—After ascertaining, by actual observation, the intensity of the magnetic force in different parts of the earth, lines are supposed to be drawn through all those points in which the force is the same; these lines are called isodynamic curves. These also slightly resemble parallels of latitude, but are more irregular than the isoclinic lines. There is no one standard equator of minimum intensity, but there are two very irregular lines surrounding the earth in the equatorial region, in some places almost meeting each other, and in others spreading apart more than two thousand miles, on which the magnetic intensity is the same. These two are taken as the standard of comparison, because they are about the lowest which extend entirely round the globe. The intensity on them is therefore called *unity*. In the wide parts of the belt which they include—lying one in the southern Atlantic, and the other in the northern Pacific oceans—there are lines of lower intensity which return into themselves, without encompassing the earth. In approaching the polar regions, both north and south, the curves become indented like an hour-glass, and at length the indentations meet in the form of the figure 8; and at still higher latitudes, are separated into two systems, closing up around two poles of maximum intensity. Thus there are on the earth four poles of maximum intensity, two in the northern hemisphere and two in the southern. The American north pole of intensity is situated on the north shore of Lake Superior. The one on the eastern continent is in northern Siberia. The ratio of the least to the greatest intensity on the earth is about as 7 to 19.

631. *Magnetic charts.*—These are maps of a country, or of the world, having laid down upon them the systems of curves which have been described. But for the use of the navigator, only the isogonic lines, or lines of equal declination, are essential. There are large portions of the globe which have as yet been too imperfectly examined, for the several systems of curves to be accurately mapped. It must be remembered, too, that the earth is slowly but constantly undergoing magnetic changes, by which, at any given place, the declination, dip, and intensity are all essentially altered after the lapse of years. A chart, therefore, which would be accurate for the middle of the nineteenth century, will be, to some extent, incorrect at its close.

632. *Magnetic observations.*—In accordance with a suggestion of Humboldt, in 1836, systematic observations have been since made upon terrestrial magnetism, in various parts of the world, in order to deduce from them the laws of its changes. Magnetic *observatories* have been erected without any iron in their construction; and the most delicate *magnetometers* have been devised and used, for detecting minute oscillations both in the horizontal and vertical planes. By these means has been discovered a class of phenomena called *magnetic storms*, in which the needle suffers numerous and rapid disturbances, sometimes to the extent of several degrees; and it is a remarkable and interesting fact, that these disturbances occur at the same absolute time in every part of the earth.

633. *Aurora Borealis.*—This meteor is usually accompanied by a disturbance of the needle, thus affording visible indications of a magnetic storm; but the contrary is by no means generally true, that a magnetic storm is accompanied by auroral light. The connection of the aurora borealis with magnetism is manifested not only by the disturbance of the needle, but also by the fact that the streamers are parallel to the dipping needle, as is proved by their apparent convergence to that point of the sky to which the dipping-needle is directed. This convergence is the effect of perspective.

634. *Source of the earth's magnetism.*—If a needle is carried round the earth from north to south, it takes all the positions in relation to the earth's axis, which it assumes in relation to a magnetic bar, when carried round it from end to end (Art. 620). At the equator it is nearly parallel to the axis, and it inclines at larger and larger angles as the distance from the equator increases; and in the region of the poles, it is nearly in the direction of the axis. Hence the hypothesis that the earth contains, or else is itself, a magnet, having the boreal magnetism developed in its northern hemisphere, and the austral in the southern. And, to explain the double intensity-poles, and the movements of the isogonic curves, Hansteen supposed a second magnet, crossing the first at an oblique angle, and revolving very slowly about it. But all that is now known of terrestrial magnetism can not be explained in harmony with this theory. And modern discoveries in electro-magnetism and thermo-electricity furnish a clue to the hypothesis, which generally prevails at this day. Attention has been drawn to the remarkable agreement between the *isothermal* and the *isomagnetic* lines of the globe. It has already been stated (Art. 422), that the former descend in crossing the Atlantic Ocean toward America, and that there are two poles of maximum cold in the northern hemisphere. The isoclinic and the isodynamic curves also descend to lower latitudes in coming across the Atlantic; so that, at a given latitude, the degree of *cold*, the magnetic *dip*, and the magnetic *intensity*, is each considerably greater on the American, than on the European coast. This is only an instance of the general correspondence between these different systems of curves. It has likewise been noticed (Art. 626), that the needle has a movement diurnally, eastward in the morning, and westward in the middle of the day, and that it is generally much greater in the hot season than in the cold. It is obvious, therefore, that the development of magnetism in the earth is intimately connected with the temperature of its surface. Hence it is supposed that the heat received from the sun excites electric currents in the materials of the earth's surface, and these give rise to the magnetic phenomena.

635. *Magnetism can be developed in all bodies.*—Though only iron, nickel, cobalt, and perhaps manganese, can have the magnetic power developed in them by ordinary induction, yet it is a fact now well known that all substances are, in one way or another, susceptible of magnetic influence. The experiments of Faraday on this subject are numerous and important. He ascertained that some substances, when placed between the opposite poles of a pair of powerful electro-magnets, would show a development in the line of the magnets—others in a line at right angles to that. The former are called *magnetic*, or *paramagnetic* substances, the latter *diamagnetic.*

636. *Formation of permanent magnets.*—Needles and small bars may be more or less magnetized by the following methods, the reasons for which will be readily understood:

1. A feeble magnetism may be developed in a steel bar, by causing it to ring while held vertically. The earth's influence upon it, however, is stronger if it is held, not precisely vertical, but leaning in a direction parallel to the dipping needle. The inductive influence of the earth explains the fact often noticed, that rods of iron or steel, that have stood for many years in a position nearly vertical, as, for instance, lightning-rods, iron pillars, or even tongs, are found somewhat magnetic, with the north pole downward.

2. A needle may be magnetized, by simply suffering it to re-

main in contact with the pole of a strong magnet; or better, between the opposite poles of two magnets.

3. Place the needle across the opposite poles of two parallel magnets, while a bar of soft iron connects the other two poles. (Fig. 242.) Or, if a long magnet is bent into the form (Fig. 243), called the horse-shoe magnet, place the needle across the ends.

Fig. 242.

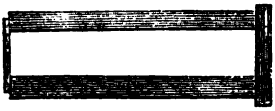


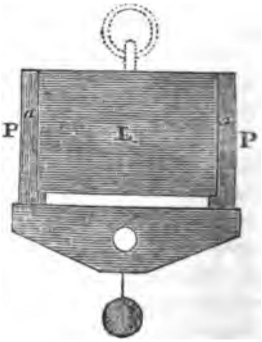
Fig. 243.

4. In order to take advantage of the earth's inductive influence, along with that of steel magnets, place the needle parallel to the dipping needle, and draw the south pole of one magnet over the lower half, and the north pole of another over the upper half, with repeated and simultaneous movements.

None of these methods, however, are of great practical value at the present day, since the galvanic circuit affords a far readier and more efficient means of magnetizing bars.

The horse-shoe magnet, sometimes called the U-magnet, is for many purposes a very convenient form, and originated in the practice of *arming* the loadstone; that is, furnishing it with two pieces of soft iron, PP (Fig. 244), which are confined by brass straps to the poles of the stone, and project below it, so that a bar and weight may be attached.

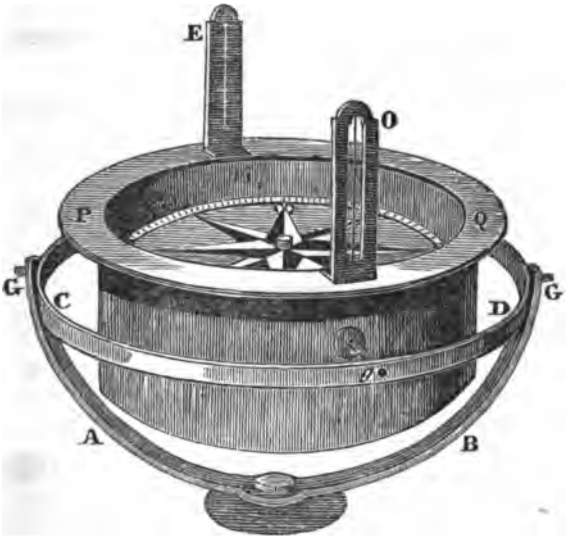
Fig. 244.



637. *The compass.*—This instrument, in its simplest form, consists of a needle, inclosed in a suitable box covered with glass. This is all that is essential when it is required merely to know the direction of the meridian, or the north and south points. But, for most purposes, the compass is furnished with a graduated circular card, divided into degrees and minutes; and in the mariner's compass, the card is also divided into thirty-two equal parts, called rhumbs. The card thus divided is fastened to the needle itself, and turns with it.

Thin, slender needles, have the greatest directive powers, and are most sensible, since they undergo less friction than those which are heavier, but due regard to strength requires them to be made of a certain degree of thickness; an increase of length is attended with an increase of directive power; but when the thickness remains the same, the weight, and consequently the friction increases in the very same ratio; no advantage, therefore, as to directive power, can be obtained by any increase of length. Moreover, needles which exceed a very moderate length, are liable to have several sets of poles, a circumstance which is attended with a great diminution of directive force. On this account, short needles, made exceedingly hard, are generally preferable.

FIG. 245.



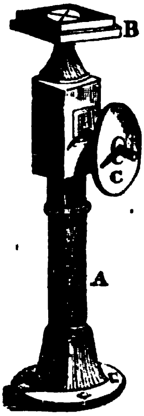
638. *Mariner's compass.*—The great importance of the mariner's compass, has made its construction an object of much attention, and the best artists have tried their skill upon it. The compass is suspended in its box in such a manner as to remain in a horizontal position, notwithstanding all the motions of the ship. This is effected by means of *gimbals.* This contrivance consists of a hoop, CD, usually of brass (Fig. 245), fastened horizontally to the box by two pivots placed opposite to each other, and constituting the axis on which the hoop turns up and down. At an equal distance from the pivots on each side, that is, at the distance of 90° from each pivot, two other pivots are attached to the ring at right angles to the former, on which the inner box that contains a card is hung. Of course when it turns on these pivots, its motion is at right angles with that of the hoop. Therefore, all the motions of which the compass-box is capable, are performed around two axes which intersect each other at right angles; consequently, the point of intersection being in both axes, will not move at all. But the needle and the attached card rest upon this point, and are connected with the compass-box in no other point. Hence they remain constantly horizontal in every position of the box.

The Azimuth compass differs from the common mariner's compass only in having sights attached, by which the bearing of any object with the meridian may be ascertained. The Surveyor's compass is a variety of the azimuth compass.

On account of dip, which increases with the distance from the equator, and is reversed by going from one hemisphere to the other, the needle needs to be loaded by a minute adjustable weight, if it is to be used in extensive voyages to the north or south. In north latitudes, the south end must be heaviest, in south latitudes, the north end.

639. *The needle astatic.*—Though magnetic intensity increases at greater distances from the equator, yet the directive power of the compass grows more feeble in approaching the poles of dip, because the horizontal component constantly diminishes, and at the poles, becomes zero. (Art. 628.) A needle in such a situation, in which the earth's magnetism has no influence to give it direction, is called *astatic.* The compass needle is astatic at the north and south poles of dip. And the dipping needle may be rendered astatic at any place, by setting its plane of rotation perpendicular to its line of dip at that place; for then there will remain no component of the magnetic force in the only plane in which the needle is at liberty to move.

FIG. 246.



640. *Local attraction of ships corrected.*—The iron used in the construction of a ship, and any iron contained in the cargo, will necessarily produce some influence on the needle of the compass, and cause errors in the observed course of the vessel. To remove this source of danger, Barlow devised the *correcting plate,* represented by C (Fig. 246). It is first ascertained by trial before the ship leaves harbor, what position the plate must occupy, in order to produce the same effect on the needle as the ship's iron does. Therefore, on the voyage, if the plate is brought to the same position again, its effect being observed, the previous effect of the iron will be known, and the error produced by it can then be corrected.

PART VIII.—ELECTRO-MAGNETISM.

641. *Galvanic electricity.*—When electricity is developed by chemical action instead of friction, it is called *Galvanic* or *Voltaic* electricity, from the names of Galvani and Volta, two Italian philosophers, who made the earliest discoveries relating to this mode of development. The great difference between the galvanic and the frictional electricity is, that the latter is in a state of great intensity, or *tension*, as it is often called, darting violently, and with the evolution of sound and light, through an obstructing substance, while the former has very low tension, flowing in a comparatively gentle stream through conductors, and unable to pass across interruptions, unless they are exceedingly small.

642. *The galvanic battery.*—Instruments, which are arranged for developing the galvanic electricity in large quantities, are called *galvanic batteries;* but these usually consist of a large number of similar parts, called *elements;* much as an electrical battery is composed of several Leyden jars.

643. Various forms of elements for batteries have been adopted; one of which, of simple construction, is presented in Fig. 247. A plate of zinc, Z, and one of copper, C, are placed in a tumbler of diluted sulphuric acid. Connected with each plate is a wire of indefinite length; and when these wires are brought into contact, a chemical action commences on the plates, and a steady stream of electricity flows from the copper plate through the wires to the plate of zinc. The electricity manifests itself in a spark at the place where the wires meet. As soon as the wires are separated, all signs of electricity cease, and are renewed again when the wires are joined. The joining of the wires, by which a communication is formed between the positive pole at the copper and the negative pole at the zinc, is called *closing the circuit;* and the separating of them is called *breaking the circuit.*

FIG. 247.

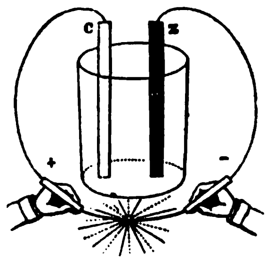
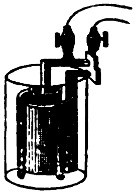
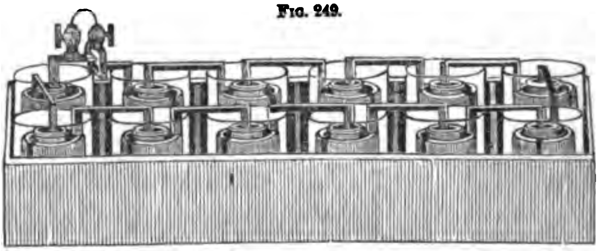


FIG. 248.



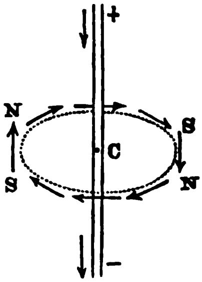
644. An element of Grove's battery is exhibited in Fig. 248, which represents a glass jar containing a hollow cylinder of amalgamated zinc and mercury, which has a narrow opening on one side from top to bottom. Within the zinc is a cylindric cup of porous earthenware, and within that a lamina of platinum is suspended from a cover. One end of the circuit-wire is in metallic communication with the zinc, and the other with the platinum, by means of the binding-screws at the top. The earthen cup is now filled with strong nitric acid, while the space outside of it, in which the zinc is placed, contains sulphuric acid diluted with ten times its weight of water. Twelve such elements are represented as combined in a battery, in Fig. 249.

FIG. 249.



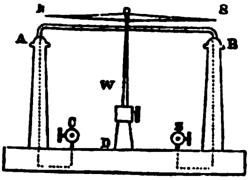
645. *Connection of galvanic electricity and magnetism.*—Wherever a free magnetic needle is placed near the wire through which the galvanic current is flowing, it is acted on by the current, and placed in the direction of a tangent to the circle whose center is in the nearest point of the circuit, and whose plane is perpendicular to the line of the current. Suppose the wire, +C− (Fig. 250), is any part of a galvanic circuit, and that the upper end connects with the positive pole of the battery, and the lower end with the negative pole, then the needle will place itself as represented, the point of the arrow in each case being the north pole. That is, if the current from the positive pole *descends*, the tangential force around the wire will place the north pole of a needle in the direction in which the hands of a watch move, as we look down upon it. If the current from the positive pole *ascends*, the north end of a needle will be thrown in the opposite direction.

FIG. 250.



646. If we obtain a clear conception of the relations in the foregoing statement, we can in every case easily determine what position a needle will take when near the circuit. For example, in Fig. 251, let the wire from the copper element be fastened by the binding-screw C, and that from the zinc by Z; so that the positive current will flow from left to right in the direction AB. If now a needle, NS, is supported *above* AB, the pole N will swing *toward* the observer; if *below*, it will swing *from* him; if the pivot, W, supports a dipping needle (which can revolve in a vertical plane), and is on *this* side of the wire, AB, the north pole will point *downward;* if on the *farther* side, it will point *upward.*

FIG. 251.



647. *Magnetic induction by the galvanic current.*—Not only will the galvanic current control the needle as above stated, but it will induce upon iron the magnetic state, with poles in the order which has just been described. For instance, if pieces of iron wire, or if iron filings, are placed about the wire, +C— (Fig. 250), while the positive current is descending, they will be found feebly magnetic, and the north poles will be in the direction N, N; but if the positive current *ascends*, the poles will all be turned in the opposite direction.

648. *The positions of the galvanic wire and the magnetic needle convertible.*—Since the needle is in a plane perpendicular to the wire, and a tangent to the circle whose center is in the wire, the same thing must be true of the wire in relation to the needle; that is, the wire is in a plane perpendicular to the needle, and a tangent to the circle whose center is in the needle. Hence, they may change places with each other, without disturbing their relations in the least. In Fig. 250, if the positive current is moving horizontally in the direction from S to N (the former direction of the needle), a needle whose center is at C will direct its north pole downward (the former direction of the positive current), and *vice versa.*

649. *Accumulation of electric actions upon the needle.*—If the galvanic circuit passes entirely round the needle, in the circumference of a circle, then, for the reason just named, *every point* of the circumference acts with equal energy to place the needle at right angles to the plane of the circle; and this indefinite number of actions may be multiplied indefinitely, by causing the wire to form a close helix around the entire length of the needle, and then a second helix around the first, and so on to any desired number of layers. And to prevent the fluid being conducted from one coil to the next one in contact with it, the whole wire must be wound with silk; and so feeble is the tension of the galvanic electricity, that it will travel through many thousands of feet of wire, rather than make its way through these thin coverings of silk.

Fig. 252 represents the manner in which the horse-shoe electro-magnet is prepared. P and N are the ends of the wire from the poles of the battery, terminating in two cups of mercury. The helix which surrounds the horse-shoe of soft iron, has the extremity W in the cup attached to the positive pole, and *w* in that connected with the negative pole. As represented in the figure, the circuit is closed; it is broken by lifting either W or *w* out of the mercury. When the circuit is closed, the tangential forces at all points of the entire helix conspire to develop magnetic poles in the same direction in the bar; and according to the law stated in Art. 645, the north pole is at the right hand, toward N, whereas if W had passed the *back* side of the iron, and the whole coil had thus been wound in the opposite direction, the north pole would have been on the left, nearest to P. A bar of iron, called the armature, passes across the poles, from which a weight is suspended. In order to make the magnet very powerful, the helix is to be wound close, and in successive layers, so as to make as many turns as possible round the iron. Thousands of pounds may be sustained in this manner, when the circuit is closed; but, by raising W from the mercury, and thus breaking the circuit, the whole force is lost in a moment. Thus easily may an electro-magnet be made to act with great energy at one instant, and to become powerless at the next.

FIG. 252.

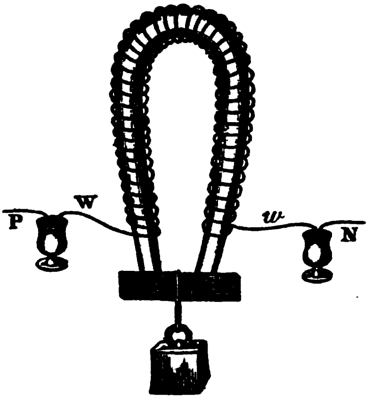
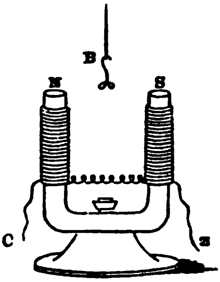


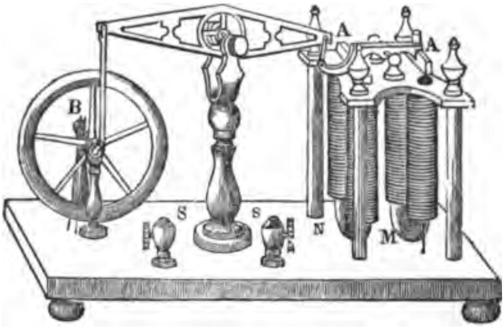
FIG. 253.



650. It was by means of an electro-magnet, mounted as in Fig. 253, that Faraday examined the magnetic and dia-magnetic character of different substances, as stated in Art. 635. By a thread, AB, having at the lower end a stirrup or holder, a short bar of any material is suspended between the poles N and S of the electro-magnet. In some cases, the extremities of the bar are turned toward the poles, with more or less energy; in others, a repulsion seems to exist between the body and the poles, so that the bar places itself at right angles to the line joining N and S. The former were called by Faraday, *magnetic*, the latter, *diamagnetic* substances. (See Art. 635.)

651. *Electro-magnetism as a motive power.*—It is abundantly proved by experiment, that machinery may be moved by the electro-magnetic force; and many models of machines, both stationary and locomotive, have been constructed and successfully operated; but as yet this power proves to be far more expensive than steam. Fig. 254 is a representation of one of the electro-magnetic machines devised by Dr. Page. Two electro-magnets, of the horse-shoe or U form, are fastened in a vertical position at M and N, having their four poles just in the upper surface of the table. The two armatures, AA, are attached to a lever, so as to be brought alternately into contact with the poles of each magnet. This is done by closing and breaking the galvanic circuit, which is carried through the insulated wires that are coiled around the magnet, and connected with a battery by the binding-screws, SS. When the circuit around M is closed, that magnet attracts the armature, A, and draws away from N its armature, the latter magnet not being in communication with the battery, and therefore having no magnetic force. But the next instant, the circuit of N is closed, and that of M is broken, so that the armature above N is attracted to it, and raises the one above M. This alternating motion is readily communicated to the working-beam, and thence by a crank to a fly-wheel, as represented in the figure. The arrangement for closing and breaking the circuits is at B, where a cam on the axle throws back a lever, which operates to close one of the circuits. After half a revolution, the other is closed in a similar manner. Each lever is brought back to its place by a spring, and thus each circuit remains broken, except when the cam passes one of the levers.

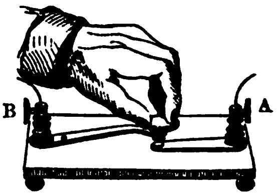
Fig. 254.



652. *The electro-magnetic telegraph.*—The fact that the galvanic current deflects a magnetic needle in its vicinity was discovered by Oersted of Denmark, in 1820. Sturgeon of England, in 1825, first formed the electro-magnet, by forming a loose coil of wire around soft iron bent in the U form. In 1830, Henry, now secretary of the Smithsonian Institution, increased the power of the electro-magnet, by coiling a great length of insulated wire upon it, till it could sustain many hundreds of pounds; and also proved by experiment the practicability of employing the electro-magnet with a battery of many elements, to produce mechanical effects at a distance, for telegraphic purposes. In 1837, Morse, of the United States, invented the best form of electro-magnetic telegraph, which has now been adopted in most countries of the world. By Morse's telegraph, a person in one part of the country may cause permanent marks to be recorded in another part, hundreds of miles from him. For this purpose he must have a battery of many elements, called an *intensity battery*, and a conducting circuit extending from one pole of it to the distant place where the record is to be made, and thence back again to the other pole of the battery. It was at first considered necessary to extend two wires from one place to the other—one to *carry*, and the other to *return*, the galvanic current. But the valuable discovery was made by Steinheil of Germany, in 1837, that if a current is conveyed to a distance by a wire, it may be *returned* through the earth.

653. *Transmission of the electro-magnetic power.*—Let us suppose that an operator in Washington desires to give motion to a recording apparatus in Boston. Near him is an intensity battery, from one pole of which a wire passes to the earth, and is soldered to a broad plate of copper buried in the ground. From the other pole a wire is extended to the binding-screw A (Fig. 255), on the table of the operator. Another wire attached to the binding-screw B, continues to Boston, and *there* passes in a series of coils around an electro-magnet of the U form, and on leaving the magnet descends into the earth, and unites with a plate of copper. Between the two copper plates, one in Washington, and the other in Boston, the earth itself serves as a conductor for the galvanic current. The circuit is now complete, except the minute break under the fingers of the operator (Fig. 255). But if he presses down the metallic key, the circuit is closed, and electricity flows from the battery through the instrument AB, thence to Boston, around the horse-shoe magnet into the earth, and back to the other pole of the battery, all in a small fraction of a

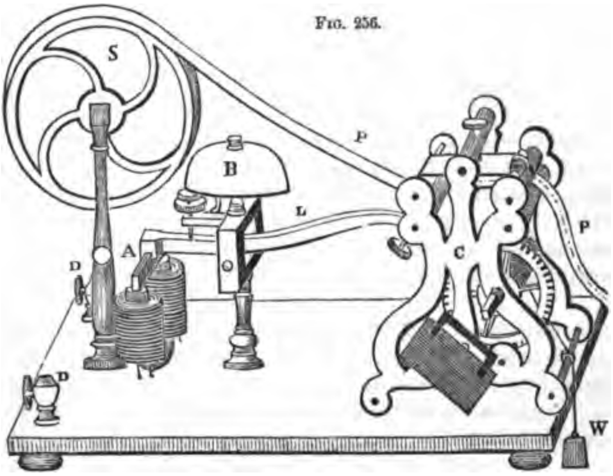
FIG. 255.



second (Art. 591). And while it circulates through the helix of the magnet in Boston, it induces the magnetic power, and causes it to attract the armature with which it is furnished. But if the operator ceases to press on the key, a spring throws it up, and breaks the circuit, and instantly the magnet in Boston loses its power to hold down the armature, which is at once withdrawn by the force of a spring connected with it. Thus an individual in Washington can control the movements of the armature at Boston as perfectly as if it were under his hand.

654. *The recording instrument.*—Let us next see what is accomplished by the movement of the armature in Boston. Fig. 256 represents the recording instrument of Morse's telegraph. A, the armature just spoken of, is situated above the poles of the electro-magnet, and attached to one end of the lever L, the other extremity of which carries the *style*, a blunt metallic point, for recording the movements. Whenever the circuit from Washington is closed, A is attracted to the magnet, and the style is pressed upward against a strip of paper, PP, which is carried over a roller at that point by the action of clock-work. The roller has a small groove around it at the place where the style bears, so that it indents the paper into the groove. If the operator strikes the key with an impulse, and instantly removes his finger, the style merely indents a dot in the paper; if he continues the pressure a moment, the paper is drawn along, and the indentation becomes a line. An alphabet is formed of dots and short lines variously combined, which the operator learns to use with rapidity in spelling the words of a message, and the record is permanently stamped on the

Fig. 256.



paper. Those who are accustomed to the work of sending and receiving messages, soon acquire the power of distinguishing the letters by the ear, and can therefore understand a communication without looking at the record. W is the weight which gives motion to the clock; this motion draws along the strip of paper, PP, over and under successive rollers from the reel, S. The first stroke upon the key at Washington causes a bell, B, to ring, by which it is known that a message is to be sent. DD are the binding-screws, by which the wire from Washington, and that from the copper-plate in Boston, are connected with the two extremities of the helix around the magnet.

The same circuit will of course serve to convey messages back from Boston to Washington, provided the operator at Boston has a battery, and the one at Washington a recording machine.

655. *Action of galvanic currents on each other.*—It was discovered about the year 1822, that if the galvanic current flows through two parallel portions of wire, they attract each other, when the currents have the same direction, but repel each other, when the currents move in opposite directions. Hence, if the current passes through a loose coil, whose spires are free to move, each spire attracts the next, and the whole coil is *shortened* by means of this attraction. Also, if the end of one coil is brought toward the end of another, they will attract, provided the circulation is in the same direction, but repel, if the two circulations are in opposite directions. From these observed phenomena, Ampère, of France, constructed his ingenious theory, that all magnetic phenomena result from the attraction or repulsion of galvanic currents, circulating in the magnets as in coils or bundles of coils, which extend from one pole to the other.

656. *Magneto-electric phenomena.*—In accordance with the theory of Ampère, it might be expected, not only that an electric current would act on another electric current, and that an electric current would give motion to a magnet, and induce the magnetic state on iron, as we have seen, but also that a magnet might excite an electric current. This is likewise found to be true in fact. If a hollow coil, prepared for the electro-magnet, is connected with a *galvanometer* (a delicate instrument for measuring the strength of the galvanic current), and a bar-magnet be quickly thrust within the helix, and then withdrawn, the needle of the galvanometer will be deflected; and if the other pole be inserted, the needle will be deflected in the opposite direction. The name, *magneto-electricity*, has been given to the currents when developed by the magnet.

657. Those phenomena which Faraday at first ascribed to magnetism of *motion*, or magnetism of *rotation*, belong to this class of actions. Copper and zinc are the best substances with which to perform these simple experiments. If a circular disk of copper, loaded on one edge, vibrates on a horizontal axis, the vibrations will cease much sooner when the poles of powerful magnets are placed almost in contact with the disk. If a horseshoe magnet is revolved round a vertical axis, a copper disk, delicately suspended just above it, will also revolve in the same direction. If a copper disk revolves swiftly in a horizontal plane, a needle over it will be made to revolve; or if a needle is revolving, the disk of copper placed near it, above or below, will reduce it to rest much sooner than if it were obstructed only by friction and the air. Faraday discovered that all such phenomena are due to electric currents momentarily excited in the copper, or other substance, by the magnet.

658. *Thermo-electricity.*—When electric currents are excited, as they may be, by differences of temperature, the name *thermo-electricity* is given to the development. As in the other modes of excitation, much depends on the substance used. Bismuth and antimony are the metals which most readily furnish thermo-electric currents, though all the metals and many other substances will do it in some degree. The following is an elementary experiment in thermo-electricity. Let a short bar of bismuth stand perpendicular to the horizon, while a copper wire makes a small circuit from the top to the bottom of it, each end of the wire being soldered to the bismuth. If now a magnetic needle be placed within this circuit, having its north pole turned to the bar of bismuth, and a lamp be placed below the latter so as to heat it, the needle gives evidence that a positive current flows downward through the bismuth, and round through the copper circuit; for the north end of the magnet turns to the west, and the amount of its deflection depends on the difference of the temperatures of the ends of the bar. When they become equally heated, the needle resumes its first position in obedience to the earth's inductive influence. If the upper end of the bismuth were heated, while the lower end remains cold, the deflection of the needle is the other way.

659. The earth's inductive influence on the magnet is considered as arising from the thermo-electric currents which pass around the globe from east to west as its parts are brought successively under the sun by rotation upon the axis. It will be perceived, by recurring to the law for the position of the needle near an electric current (Art. 645), that if the current moves westward, that end of the needle, which is called its *north pole*, should be directed northward, as it is. (See Art. 634, for the connection of the isomagnetic and the isothermal lines of the earth.)

PART IX.—OPTICS.

CHAPTER I.

MOTION AND INTENSITY OF LIGHT.

660. *Definitions.*—*Optics* is the name of that branch of Natural Philosophy, which treats of light and vision. *Light*, according to the theory most generally received at the present day, consists of exceedingly minute and rapid vibrations in a medium or ether which fills space; which vibrations, on reaching the retina of the eye, cause vision, as the vibrations of the air cause hearing, when they inpinge on the tympanum of the ear.

Bodies, which of themselves are able to produce vibrations in the ether surrounding them, are said to *emit* light, and are called *self-luminous*, or simply *luminous;* those, which only *reflect* light, are called *non-luminous*. Most bodies are of the latter class. A *ray* of light is a line, along which light is propagated; a *beam* is made up of many parallel rays; a *pencil* is composed of rays either diverging or converging; and is not unfrequently applied to those which are parallel.

A substance, through which light is transmitted, is called a *medium;* if objects are clearly seen through the medium, it is called *transparent;* if seen faintly, *semi-transparent;* if only light is discerned through a medium, but not the objects from which it comes, it is called *translucent;* substances which transmit no light are called *opaque*.

661. *Light moves in straight lines.*—So long as the medium continues uniform, the line of each ray is perfectly straight. For an object can not be seen through a bent tube; and if three disks have each a minute aperture through it, a ray can not pass through the three, except when they are exactly in a straight line. The shadow which is projected through space from an opaque body proves the same thing; for the edges of the shadow, taken in the direction of the rays, are all straight lines.

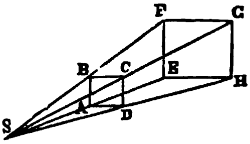
From every point of a luminous surface light emanates in all possible directions, when not prevented by the interposition of an opaque body. Thus, a candle is seen by night at the distance of one or two miles; and within that limit, no space so small as the pupil of the eye, is destitute of rays from the candle. A point, from which light emanates is called a *radiant.* If light from a radiant falls perpendicularly on a circular disk, the pencil is a cone; if on a square disk, it is a square pyramid, &c., the illuminated surface, in each case being the base, and the radiant the vertex.

662. *The velocity of light.*—It has been ascertained by several independent methods, that light moves at the rate of 192,500 *miles per second.* One method is by means of the eclipses of Jupiter's satellites. To render this mode intelligible to those who have not studied astronomy, it may be premised, that the planet Jupiter is attended by four moons which revolve about their primary, as our moon revolves about the earth. These small bodies are observed, by the telescope, to undergo frequent eclipses by falling into the shadow which the planet casts in a direction opposite to the sun. The exact moment when the satellite passes into the shadow, or comes out of it, as would be seen by a spectator at the mean distance of the earth from the sun, is calculated by astronomers. But sometimes the earth and Jupiter are on the same side, and sometimes on opposite sides of the sun; consequently, the earth is, in the former case, the whole diameter of its orbit, or about one hundred and ninety millions of miles nearer to Jupiter than in the latter. Now it is found by observation, that an eclipse of one of the satellites is seen about sixteen minutes and a half sooner when the earth is nearest to Jupiter, than when it is most remote from it, and consequently, the light must occupy this time in passing through the diameter of the earth's orbit, and must therefore travel at the rate of about 192,000 miles per second.

663. Another method of estimating the velocity of light, wholly independent of the preceding, is derived from what is called the *aberration of the fixed stars.* The full explanation of this method must be referred to astronomy; but it may be understood in general, that the apparent place of a fixed star is altered by the motion of its light being combined with the motion of the earth in its orbit. It will be remarked that the place of a luminous object is determined by the direction in which its light meets the eye. But the direction of the impulse of light on the eye is modified by the motion of the observer himself, and the object appears forward of its true place. The stars, for this reason, appear slightly displaced in the direction in which the earth is moving; and the velocity of the earth being known, that of light may be computed in the same manner as we determine one component, when the resultant and the other component are known. The velocity of light has been determined also by direct experiment, in a manner somewhat analogous to that employed by Wheatstone for ascertaining the velocity of electricity.

664. *Loss of intensity by distance.*—The *intensity* of light varies *inversely as the square of the distance.* In Fig. 257, suppose light to radiate from S, through the rectangle ABCD, and fall on EFGH, parallel to ABCD. As SAE, SBF, &c., are straight lines, SAB, SEF are similar, as also ABCD, EFGH; therefore, ABCD : EFGH :: $AB^2 : EF^2 :: SA^2 : SE^2$. But the same quantity of light, diffused over ABCD and EFGH, will be more intense, as the surface is smaller. Hence, the intensity of light at E : intensity at A :: ABCD : EFGH :: $SA^2 : SE^2$. This demonstration is applicable to every kind of emanation in straight lines from a center.

FIG. 257.



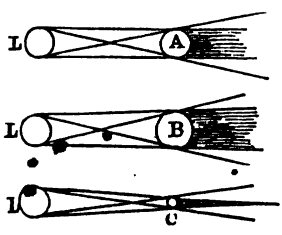
665. *Brightness the same at all distances.*—The *brightness* of an object is the quantity of light which it sheds, as compared with the apparent area from which it comes. Now the *quantity* (or intensity), as has just been shown, varies inversely as the square of the distance. The apparent *area* of a given surface also diminishes in the same ratio, as we recede from it. Hence, the brightness is constant. For illustration, if we remove to *three* times the distance from a luminous body, we receive into the eye nine times less light, but the body also appears nine times smaller, so that the relation of light to apparent area remains unaltered.

666. *Loss of intensity by the obstruction of a medium.*—In a uniform medium, while the distance increases *arithmetically*, the intensity diminishes *geometrically.* Imagine the medium to be divided by parallel planes into strata of equal thickness; and suppose the first stratum to diminish the intensity by $\frac{1}{n}$ of the whole. Then the intensity of the light which reaches the second stratum is $1-\frac{1}{n}=\frac{n-1}{n}$. But on account of the uniformity of the medium, every stratum would produce the same effect, that is, it would transmit to the next, $\frac{n-1}{n}$ of that which falls upon it. Therefore, $\frac{n-1}{n}$ of $\frac{n-1}{n}=\frac{(n-1)^2}{n^2}$, leaves the second stratum, $\frac{(n-1)^3}{n^3}$, the third, and so on, in a geometrical series. For example, if a piece of colored glass is $1\frac{1}{4}$ inch thick, and each quarter of an inch absorbs $\frac{3}{5}$ of the light which falls upon it, then about *one-hundredth* of what enters the first surface will escape from the last. For $\left(\frac{2}{5}\right)^5 = .01$ nearly.

667. *Photometers.*—These are instruments designed for the measurement of relative intensities of light. By the eye alone we have not the power of determining how many times more intense one light is than another, though we can judge with tolerable accuracy when two surfaces are *equally* illuminated. Photometers are, therefore, generally constructed on the plan of determining the ratio of intensities of two lights, by means of our ability to decide when they illuminate two surfaces equally. It is sufficient to mention Rumford's method *by shadows.* Let the two unequal lights be so placed that the two shadows of an opaque body cast by them shall fall side by side on a white screen. If one shadow appears more luminous than the other, remove to a greater distance the light which illuminates it (or bring the other nearer), until the shadows appear of the same degree of illumination. Then measure the distances from the lights to the screen, and the intensities of the lights will be *directly as the squares of the distances.* For the light at the greater distance, since it illuminates the screen equally with the other, must gain as much by intensity as it loses by distance; that is, in the ratio of the square of the distance.

668. *Shadows.*—When a luminous body shines on one which is opaque, the space beyond the latter, from which the light is excluded, is called a *shadow.* The same word, as commonly used, denotes only the *section* of a shadow made by a surface which crosses it. Shadows are either *total* or *partial.* If tangents are drawn on all the corresponding sides of the two bodies, the space inclosed by them beyond the opaque body, is the total shadow; if other tangents are drawn, crossing each other between the bodies, the space between the total shadow and the latter system of tangents is the partial shadow, or *penumbra.* In case the bodies are spheres, as in Fig. 258, the total shadow will be a cylinder, or conical frustum, each of infinite length, or a complete cone, according to the relative size of the spheres. But, in every case, the penumbra and inclosed total shadow will form an increasing frustum. It is obvious that the shade of the penumbra grows grad-

Fig. 258.



ually deeper from the outer surface to the total shadow within it.

Every shadow cast by the sun has a penumbra bordering it, which gives to the shadow a blurred edge; and the more remote the sectional shadow is from the opaque body which casts it, the broader will be the blur on the edge.

CHAPTER II.

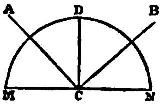
REFLECTION OF LIGHT.

669. *Radiant and specular reflection.*—Light is said to be *reflected*, when, on meeting a surface, it is turned back into the same medium. In ordinary cases of reflection, the light is diffused in all directions, and it is by means of the light thus scattered from a body that it becomes visible, when it sheds no light of its own. This is called *radiant reflection.* It is produced by unpolished surfaces. But when a surface is highly polished, a beam of light falling on it is reflected in some particular direction; and, if the eye is placed in this reflected beam, it is not the reflecting surface which is seen, but the original object, apparently in a new position. This is called *specular reflection.* It is, however, generally accompanied by some degree of radiant reflection, since the reflector itself is commonly visible in all directions. Ordinary mirrors are not suitable for accurate experiments on reflection, because light is modified by the glass through which it passes. The *speculum* is therefore used, which is a reflector made of solid metal, and accurately ground to any required form, either *plane*, *convex*, or *concave.* The word *mirror* is, however, much used in optics, for every kind of reflector.

Optical experiments are usually performed on a beam of light admitted through an aperture into a darkened room; the direction of the beam being regulated by an adjustable mirror placed outside. An instrument consisting of a plane speculum moved by a clock, in such a manner that the reflected sunbeam shall remain stationary at all hours of the day, is called a *heliostat.*

670. *The law of reflection.*—When a ray of light is incident on a mirror, the angle between it and a perpendicular to the surface at the point of incidence, is called the *angle of incidence;* and the angle between the reflected ray and the same perpendicular, is called the *angle of reflection.* The law of reflection found to be universally true is the following: *the angles of incidence and reflection are in the same plane, and are equal to each other.* This is well shown by attaching a small mirror to the center of a graduated semicircular board, perpendicular to its plane. Let Fig. 259 represent the semicircle, two or three feet in diameter, graduated from D both ways to M and N; let the small mirror be at C, with its plane perpendicular to CD; then a ray from the heliostat, as AC, passing the edge at a particular degree, will be seen after reflection to pass the corresponding degree in the other quadrant. By revolving the semicircle, any angle of incidence may be tried, and the two rays are always found to be in the same plane with CD, and equally inclined to it. As the mirror revolves, the reflected ray revolves twice as fast; for ACD is increased or diminished by the angle through which the mirror turns; therefore DCB is also increased or diminished by the same; hence ACB, the angle between the two rays, is increased or diminished by the sum of both, or twice the same angle.

FIG. 259.



It follows from the law of reflection, that a ray which falls on a mirror perpendicularly, retraces its own path after reflection. It is obvious, also, that the complements of the angles of incidence and reflection are equal, i. e. ACM = BCN. The law of reflection is applicable to curved as well as to plane mirrors; the radius of curvature at any point being the perpendicular with which the incident and reflected rays make equal angles.

Radiant reflection forms no exception to the foregoing law, though the incident rays are in one and the same direction, and the reflected rays are scattered every way. For the minute cavities and prominences which constitute the roughness of the general surface are bounded by small surfaces lying at all inclinations; and each one reflecting the rays which meet it in accordance with the law, those rays are necessarily thrown off in all possible directions.

671. *Relation of rays preserved by the plane mirror.*—1. If the incident rays are *parallel*, the reflected rays are parallel also. Let RS (Fig. 260) be the mirror, and AB, CD, the incident, BG, DH, the reflected rays. Then CDR = ABR; but CDR = HDS, and ABR = GBS; $\therefore$ HDS = GBS, and hence BG and DH are parallel. But if the angles of incidence are not in the same plane, still the reflected rays are parallel. Let AB, BG (Fig. 261) be the incident and reflected rays in one plane; CD, DH, in another; and let BE, DF be the perpendiculars. Since AB, BE are respectively parallel to CD, DF, the planes ABE,

FIG. 260.

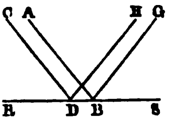
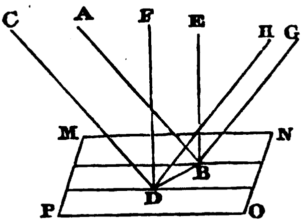


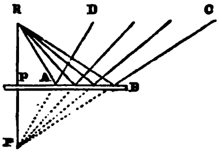
FIG. 261.



CDF are parallel; also the angles of incidence, ABE, CDF, are equal, and therefore the angles of reflection, EBG, FDH, are equal. Hence, BG, DH (being in the planes ABE and CDF) are in parallel planes, and make equal angles with the parallels BE, DF, on the same side of them. Therefore they are themselves parallel.

2. *Diverging or converging rays* continue to diverge or converge after reflection at the same angle as before. Let RAB (Fig. 262) be a pencil of *diverging* rays, incident upon PB; draw the perpendicular RF, and produce the reflected rays back till they intersect it. The triangles APR, APF are in all respects equal, since AP is common, the angles at P are right angles, and the angles at A are each equal to DAB; hence, the angles PRA, PFA are equal. In the same manner the angles PRB, PFB are proved equal. Therefore, their differences, ARB, AFB, which are the angles of divergence before and after reflection, are also equal. The same reasoning applies to the pencil DACB, *converging* to F before reflection, and to R after reflection. The plane mirror, therefore, never changes the relation of the rays to each other.

FIG. 262.

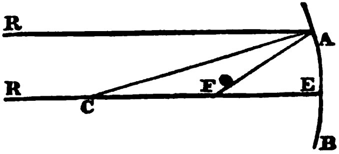


Since RP = PF, it appears that the real radiant, R, and the apparent or virtual radiant, F, are equally distant from the mirror, and in the same perpendicular to its surface; the same thing is true of the apparent or *virtual focus*, F, to which the converging rays were tending before reflection, and the real focus, R, in which they are collected after reflection.

672. *Spherical mirrors.*—In the practical use of spherical mirrors, it is found that the light must strike the surface very nearly at right angles; hence, in the following statements, the mirror is supposed to be an indefinitely small part of the whole spherical surface, and the rays nearly coincident with the axis, or central radius.

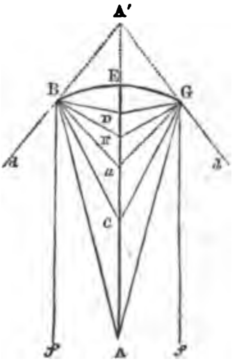
673. *Converging effect of a concave mirror.*—1. *Parallel* rays are converged to the *middle point* between the center and surface, which is hence called the *focus of parallel rays.* Let RA, RE (Fig. 263), be parallel rays incident upon the concave mirror AEB, whose center is C. The ray RE, passing through the center C, and therefore falling perpendicularly on the mirror at E, will be reflected in the direction EC. Having joined CA and made the angle CAF = CAR, the ray RA will be reflected in the direction AF. Then

FIG. 263.



because RA and RE are parallel, the angle RAC = ACE; ∴ CAF = ACE, and AF = FC; and as A is indefinitely near to E, EF = FC. Hence, the focus of parallel rays, also called the *principal focus*, is at the middle point between C and E.

FIG. 264.



2. *Diverging* rays, falling on a given concave mirror, are reflected *converging*, *parallel*, *or less diverging*, according to the degree of divergency in the original pencil. Let C (Fig. 264) be the center of concavity, and F the focus of parallel rays. Then, rays diverging from any point, A, beyond C, will be converged to some point, a, between C and F, since the angles of incidence and reflection are less than those for parallel rays. Rays diverging from C are reflected back to C; those from points between C and F are converged to points beyond C; from F they become parallel; and from points between F and the mirror, as D, they *diverge* after reflection, but at a less angle than before, and seem to flow from A′. To prove, in the last case, that the angle of divergence, A′, after reflection, is less than the angle D, the divergence before reflection, observe that the half angle A′DG = DGf, which is greater than DGC, or dGC, while the half angle DA′d is less than the exterior dGC. Much more, then, is DA′d less than A′DG.

3. *Converging* rays are made to *converge more*. The rays dB, dG, converging to A′, are reflected to D, nearer the mirror than F is. And it has been shown that the angle D is larger than A′, hence the convergency is increased.

From the three foregoing cases, it appears that the *concave* mirror always tends to produce *convergency;* since, when it does not actually produce it, it diminishes divergency.

674. *Conjugate foci.*—When light radiates from A, it is reflected to a; when it radiates from a, it meets at A. Any two such interchangeable points are called *conjugate foci*. If the radius of the mirror, and the distance of one focus from the mirror, are given, the distance of its conjugate focus may be determined. Let the radius $= r$; the distance AE $= m$; and $a\mathrm{E} = n$. The angle CGA = ECG − EAG; and CGa = EaG − ECG; but CGA = CGa; ∴ ECG − EAG = EaG − ECG; and 2ECG = EAG + EaG. Now EG, being a small arc, may be regarded as a straight line perpendicular to EA, subtending each of the angles at a, C, and A. In such a case, the angles (being small) are reciprocally as the distances. Therefore we may substitute, for the angles in the last equation, the reciprocals of the distances, r, m, and n; and we have,

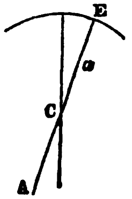
$$\left(2\frac{1}{r}=\right)\frac{2}{r}=\frac{1}{m}+\frac{1}{n};$$

from which equation we determine m, when r and n are given, or n, when r and m are given. For

$$m=\frac{rn}{2n-r};\text{ and }n=\frac{rm}{2m-r}.$$

If A is not on the axis of the mirror, as in Fig. 265, let a line be drawn through A and C, meeting the mirror in E; this is called a *secondary* axis, and the light radiating from A will be reflected to *a* on the same secondary axis, for AE is perpendicular to the mirror, and will be reflected directly back; and if AE and CE are given, *a*E may be found as before.

Fig. 265.



675. *Diverging effect of a convex mirror.*—

1. *Parallel* rays are reflected diverging from the *middle point* between the center and surface. Let C (Fig. 266) be the center of convexity of the mirror MN, and draw the radii, CM, CD, CN, producing them in front of the mirror; these are perpendicular to the surface. The ray AD will be reflected back; AM will be reflected in MB, making BME = AME; and AN in NB, making BNE = ANE. Produce the reflected rays back of the mirror, and they will meet at F, midway from C to D; for FCM = AME, and FMC = BME; therefore the triangle FCM is isosceles, and CF = FM, and as M is very near D, CF = FD. Hence the rays, after reflection, diverge as if they radiated from a point in the middle of CD, which is the imaginary radiant.

Fig. 266.

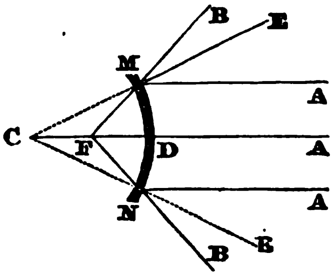
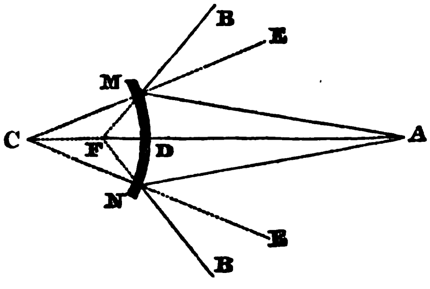


Fig. 267.



2. *Diverging* rays have their divergency increased. In Fig. 267, let the divergent rays, AM, AN, be reflected in MB, NB; these produced intersect in F, their imaginary radiant. MAD is the half angle of divergency of the incident rays, and DFM of the reflected rays. Now DFM

is greater than FMC, or BME, or AME. But AME is greater than the interior and opposite, MAD; much more, then, is DFM greater than MAD, that is, the divergency is increased.

3. *Convergent* rays are at least rendered less convergent, and may become parallel or divergent, according to the degree of previous convergency. The two first effects are shown by Figs. 266 and 267, reversing the order of the rays. And it is easy to perceive that rays converging to C, will diverge from C after reflection; if to a point more distant than C, they will diverge afterward from a point between C and F (Fig. 266), and *vice versa.*

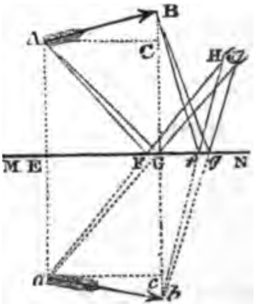
The general effect, therefore, of a *convex* mirror, is to produce *divergency.*

A and F (Fig. 267) are called conjugate foci, being interchangeable points; for rays from A move after reflection as though from F, and rays converging to F are by reflection converged to A. Conjugate foci, in the case of the convex mirror, are in the same axis either principal or secondary, as they are in the concave mirror, and for the same reason, viz., that every axis is perpendicular to the surface.

676. *Images by reflection.*—An optical image consists of a collection of focal points, from which light either really or apparently radiates. When rays are converged to a focus they do not stop, but cross, and diverge again, as if originally emanating from the focal point. A collection of such points, arranged in order, constitutes a *real image.* When rays are reflected diverging, they proceed *as though* they emanated from a point behind the mirror. A collection of such imaginary radiants forms an *apparent* or *virtual image.* The images formed by *plane* and *convex* mirrors are always apparent; those formed by *concave* mirrors may be of either kind.

677. *Images by plane mirrors.*—When an object is before a plane mirror, its image is at the *same distance behind* it, of the *same magnitude,* and *equally inclined* to it. Let MN (Fig. 268) be a plane mirror, and AB an object before it, and let the position of the object be such that the reflected rays may enter an eye placed at H. From A and B let fall upon the mirror the perpendiculars Aa, Bb, bisected in E and G. Now, since the rays from A will, after reflection, radiate as if from a (Art. 671), and those from B, as if from b, and the same of all other points, therefore the image and object are equally distant from the mirror. AC, ac, parallel to the mirror, are equal; as BG = bG, and AE = aE,

Fig. 268.



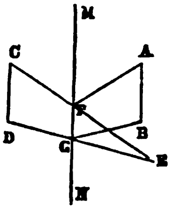
therefore, by subtraction, $BC = bc$; also the right angles at C and c are equal. Therefore $AB = ab$, and $BAC = bac$; that is, the object and image are of equal size, and equally inclined to the mirror.

678. The object and image have to each other twice the inclination that each has to the mirror. Hence, in a mirror inclined 45° to the horizon, a horizontal surface appears vertical, and one which is vertical appears horizontal.

All the three dimensions of the object and image are respectively *equal*, as shown above, but one of them is *inverted* in position, namely, that dimension which is perpendicular to the mirror. Hence, a person and his image face in opposite directions; and trees seen in a lake have their tops downwards. Those dimensions which are parallel to the mirror are not inverted. In consequence of the inversion of *one* dimension alone, the object and its image are not *similar*, but *symmetrical* forms; and one could not coincide with the other if brought to occupy the same space. The image of a *right* hand is a *left* hand, and all relations of right and left are reversed. It is for this reason that a printed page, seen in a mirror, is like the type with which it was printed.

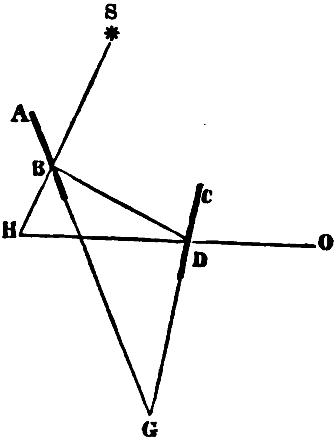
679. *The breadth of mirror requisite for seeing an object.*—If an object is parallel to a mirror, the breadth of mirror occupied by the image is to the diameter of the object as the reflected ray to the sum of the incident and reflected rays. Let AB (Fig. 269) be the diameter of the object, CD that of the image, and FG that of the space occupied on the mirror; then, $FG : CD :: EF : EC$. But $CD = AB$, and $CF = AF$; $\therefore FG : AB :: EF : AF + FE$. If the eye is brought nearer the mirror, the space on the mirror occupied by the image is diminished, because EF has to $AF + FE$ a less ratio than before. The same effect is produced by removing the object from the mirror. The length of mirror necessary for a person to see himself is equal to half his height, because in that case, $EF : AF + FE :: 1 : 2$, which ratio will not be altered by change of distance.

FIG. 269.



680. *Displacement of image by two reflections.*—If an image is seen by light reflected from two mirrors in a plane perpendicular to their common section, its angular deviation from the object is equal to twice the inclination of the mirrors. Let AB, CD (Fig. 270) be two plane mirrors inclined at the angle AGC. If an eye at H sees the star S in the direction O, the angle $SHO = 2AGC$. For $HBG = ABS = GBD$; $\therefore HBD =$

FIG. 270.



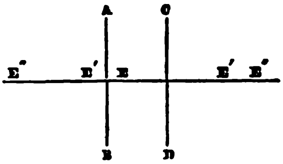
2GBD. In like manner, BDO = 2BDC. But SHO = BDO − HBD = 2BDC − 2GBD = 2BGD; ∴ SHO = 2BGD.

This principle is employed in the construction of *Hadley's quadrant*, and the *sextant*, used for measuring the altitudes of heavenly bodies at sea. The angles measured are twice as great as the arc passed over by the index which carries the revolving mirror; hence, in the quadrant, an arc of 45° is graduated into 90°; and, in the sextant, an arc of 60° is graduated into 120°.

681. *Multiplied images by two mirrors.*—

1. *Parallel* mirrors. The series of images is *indefinite* in number, and arranged *in a straight line*, perpendicular to the mirrors. The object E between the parallel mirrors, AB, CD (Fig. 271), has an image at E′, as far behind AB as E is in front of it, and in the same perpendicular. The rays reflected by AB diverge, as though they emanated from E′; hence, E′ may be regarded as an object before CD, whose image is at E″, as far behind it. Again, E″ may be considered as an object before AB, and so on indefinitely. Another series exists in the same line, by beginning with E′, the first image behind CD. As light is absorbed and scattered by each reflection, these images grow fainter, and at length disappear. Articles of jewelry are sometimes apparently multiplied and extended over a large surface, by lining the cases with parallel mirrors.

FIG. 271.

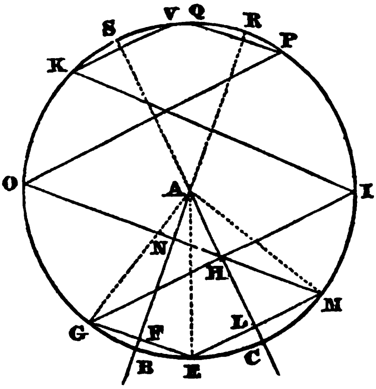


The multiplied images of a small bright object, sometimes seen in a looking-glass, are produced by repeated reflections between the front and the silvered covering on the back side. At each internal impact on the first surface some light escapes, and shows us an image, while another portion is reflected to the back, and thence forward again. The image of a lamp viewed very obliquely in a mirror is sometimes repeated eight or ten times; and a planet, or bright star, when seen in a looking-glass, will be accompanied by three or four faint images, caused in the same way.

2. *Inclined* mirrors. In this case, the images are *limited* in number, and arranged *in the circumference of a circle*, whose

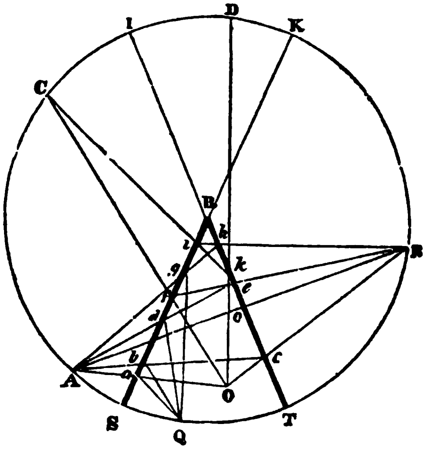
center is in the common section of the planes of the mirrors, and whose radius is the distance of the object from that point. Let AB, AC (Fig. 272) be the mirrors, and E the object. Draw EG perpendicular to AB, and make EF = FG, then will G be the first image: in the same way, find I, the image of G by AC; K, the image of I; and V, that of K. Then begin with the mirror AC, and find, as before, M, O, P, Q, the successive images by the two mirrors. No image of V or Q can be formed, because they are behind both mirrors. All these images are in the circumference of a circle, whose radius is EA; for EF, FA, and angle at F, are respectively equal to GF, FA, and angle at F; ∴ EA = GA; and in the same way it may be proved, that EA = AM, AI, &c. If the edges at A be separated, making the inclination of the mirrors less and less, the number of images will increase, and the circumference approach a straight line, so that ultimately we shall have the case described in (1), in which the mirrors are parallel.

FIG. 272.



682. Fig. 273 will assist to understand how each image is seen by a pencil of light which passes back and forth between the mirrors, until it reaches the eye. If the eye is at O, and the object at Q, and its images at A, B, C, D, each image is of course seen by a pencil which comes from the mirror to the eye, as if it originated in that image. Therefore, draw a line from any image as D, to the eye, and from its intersection with the mirror draw a line to the preceding image; from the intersection of that line with the other mirror, a line to the image next preceding, and so on back to Q; the whole path of the pencil will then be

FIG. 273.



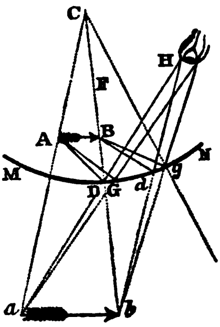
traced. Thus A is seen by Q*a*, *a*O; B, by Q*b*, *bc*, *c*O; C, by Q*d*, *de*, *ef*, *f*O; and D, by Q*g*, *gh*, *hi*, *ik*, *k*O.

683. *The kaleidoscope.*—This instrument, when carefully constructed, beautifully exhibits the phenomenon of multiplied reflection by inclined mirrors. It consists of a tube containing two long, narrow, metallic mirrors, inclined at a suitable angle; and is used by placing the objects (fragments of colored glass, etc.), at one end, and applying the eye to the other. In order that there may be perfect symmetry in the figure made up of the objects and their successive images, the angle of the mirrors should be of such size, that it can be exactly contained an even number of times in 360°. The best inclination is 30°; and the field of view is then composed of 12 sectors. It is also essential, that the small objects, forming the picture, should lie at the least possible distance beyond the mirrors.*

684. *Images by concave mirrors.*—The concave mirror forms various images, either *real* or *apparent*, either *greater* or *less* than the object, either *erect* or *inverted*, according to the place of the object.

1. The object *between the mirror and its principal focus.* By Art. 673, (2), rays which diverge from a point between the mirror and its principal focus, continue to diverge after reflection, but in a less degree. Let C be the center, and F the principal focus of the mirror MN (Fig. 274), and AB, the object. Draw the axes, CA, CB, and produce them behind the mirror. The pencil from A will be reflected to the eye at H, radiating as from *a*, in the same axis; likewise, those from B, as from *b*. Therefore, the image is *apparent*, since rays do not actually flow from it; *erect*, as the axes do not cross each other between the object and image; *enlarged*, because it subtends the angle of the axes at a greater distance than the object does. As the object approaches, and finally reaches the principal focus, the reflected rays approach parallelism, and the image departs from the mirror, till it is at an infinite distance, and is viewed as a heavenly body.

Fig. 274.



2. Object *between the principal focus and the center.* As soon as the object passes the principal focus, the rays of each pencil begin to converge; and each radiant of the object has its conjugate focus in the same axis beyond the center (Art. 673). For example, the pencil ADG (Fig. 275) is converged to *a* in the axis AC*a*,

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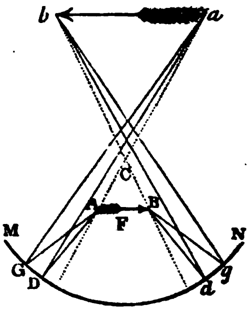
* Edinburgh Encyclopædia, art., *Kaleidoscope.*

and Bdg, to b, in the axis BCb. Therefore, the image of AB is ab beyond the center; and if an observer is beyond ab, the rays, after crossing at the image, will reach him, as though they originated in ab; or if a screen is placed at ab, the light which is collected in the focal points, will be thrown in all directions by radiant reflection from the screen. Hence, the image is *real;* it is also *inverted*, because the axes cross between the conjugate foci; and it is *enlarged*, since it subtends the angle of the axes at a greater distance than the object does. That aC is greater than AC, is proved by joining CG, which bisects the angle AGa, and therefore divides Aa so that AC : Ca : : AG : Ga. When the object reaches the center, the image is there also, but inverted in position, since rays which proceed from the right of C, are reflected to the left of it.

FIG. 275.



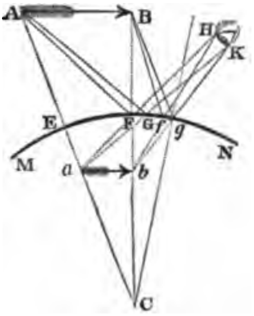
3. Object *beyond the center.* This is the reverse of (2), the conjugate foci having changed places; ab, therefore, being the object, AB is its image, *real*, *inverted*, *diminished.* As the object removes to infinity, the image proceeds only to the principal focus F.

685. These cases are shown experimentally by placing a lamp close to the mirror, and then carrying it along the axis to a considerable distance away. While the lamp moves from the mirror to the principal focus, its image behind the mirror recedes from its surface to infinity; we may then regard it as being either at an infinite distance behind, or an infinite distance in front, since the rays of every pencil are parallel. After the lamp passes the principal focus, the image appears in the air at a great distance in front, and of great size, and they both reach the center together, where they pass each other; and, as the lamp is carried to great distances, the image, growing less and less, approaches the principal focus, and is there reduced to a point. The only part of the infinite line of the axis before and behind, in which no image can appear, is the small distance between the mirror and its principal focus.

If a person looks at *himself*, so long as he is between the mirror and the principal focus, he sees his image behind the mirror and enlarged. But when he is between the principal focus and center, the image is *real*, and behind him; the converging rays of the pencils, however, enter his eyes, and give an indistinct view of his image as if at the mirror. When he reaches the center, the pupil of the eye is seen covering the entire mirror, because rays from the center are perpendicular, and return to it from all parts of the surface. Beyond the center, he sees the real image in the air before him, distinct and inverted.

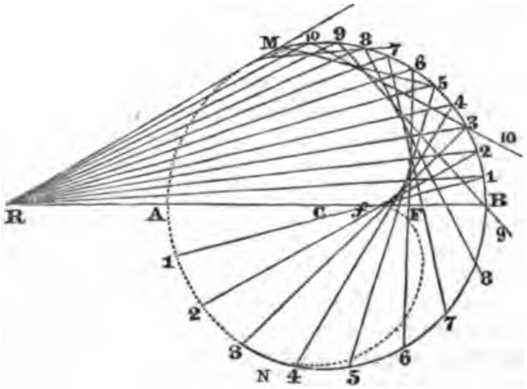
686. *Images by the convex mirror.*—The convex mirror affords no variety of cases, because diverging rays, which fall upon it, are made to diverge still more by reflection. In Fig. 276, the pencil from A is reflected, as if radiating from *a* in the same axis AC, and that from B, as from *b* in the axis BC; and these apparent radiants are always nearer the surface than the middle point between it and C (Art. 675). The image is therefore *apparent;* it is *erect*, since the axes do not cross between the object and image; and it is *diminished*, as it subtends the angle of the axes at a less distance than the object.

FIG. 276.



687. *Caustics by reflection.*—These are luminous curved surfaces, formed by the intersections of rays reflected from a hemispherical concave mirror. The name *caustic* is given from the circumstance that *heat*, as well as light, is concentrated in the focal points which compose it. In Fig. 277, MN represents a section of the mirror, and M*f*N of the caustic; the point *f*, where all the sections of the caustic through the axis meet each other, is called the *cusp*. Of the rays from R, 1, 1, meets the axis at the conjugate focus *f*, between F, the principal focus, and C, the center; 2, 2, intersects 1, 1, near *f*, and cuts the axis a little further from C; 3, 3, intersects both 1, 1, and 2, 2; and thus the curve from *f* to M is composed of

FIG. 277.



successive intersections of the reflected rays, and every ray is a tangent to the curve. Rays on the other half, BN, form a similar curve, *f*N. The curve is luminous, because it consists of the foci of all the pencils reflected from the hemisphere; for example, all the rays between 7 and 8 have their focus in the curve nearly on the line R4; all those from the arc 8, 9, near R5, &c.; the curve, therefore, when received on a surface lying in the plane of the section, is seen as a delicate line of great brilliancy; and all the space on the convex side is somewhat illuminated by the tangent rays, which have crossed each other in the curve. The caustic is well seen on the surface of milk, the light being reflected from the edge of the bowl which contains it.

If the rays, instead of flowing from R, were parallel, as those from a point of the sun, a caustic very similar to this would be formed, having its cusp at F, the focus of parallel rays.

If parallel or divergent light falls on a convex hemispherical mirror, there will be *apparent* caustics behind the mirror; that is, the light will be reflected as if it radiated from points arranged in such curves.

688. *Spherical aberration of mirrors.*—It has already been mentioned (Art. 672), that the statements in this chapter relating to focal points and images, as produced by spherical mirrors, are true only when the mirror is a very small part of the whole spherical surface. In Art. 687 we have seen the effect of using a large part of the spherical surface—viz., the rays neither converge *to*, nor diverge *from* a single point, but a series of points arranged in a curve. This general effect is called the *spherical aberration* of a mirror; since the deviation of the rays is due to the spherical curvature. The deviation, as we have seen, is quite apparent in a hemisphere, or any considerable portion of one; but it exists in some minute degree in any spherical mirror, unless infinitely small compared with the hemisphere.

But there are curves which will reflect without aberration. Let a concave mirror be ground to the form of a paraboloid, and rays parallel to its axis will be converged to the focus without aberration. For, at any point on such a mirror, a line parallel to the axis, and a line drawn to the focus, make equal angles with the tangent, and therefore, equal angles with the perpendicular to the surface. And rays, parallel to the axis of a convex paraboloid, will diverge as if from its focus, on the same account. Again, if a radiant is placed at the focus of a concave parabolic mirror, the reflected rays will be parallel to the axis, and will illuminate at a great distance in that direction. Such a mirror, with a lamp in its focus, is sometimes used in light-houses. If a concave mirror is ellipsoidal, light, emanating from one focus, is collected without aberration to the other, because lines from the foci to any point of the curve make equal angles with the tangent at that point.

Since heat is reflected according to the same law as light, a concave mirror is a burning-glass. When it faces the sun, the light and heat together are concentrated to a small image of the sun at the principal focus. And, if no heat were lost by the reflection, the intensity at the focus would be to that of the direct rays, as the area of the mirror to the area of the sun's image. Burning mirrors have sometimes been constructed on a large scale, by giving a concave arrangement to hundreds of plane mirrors.

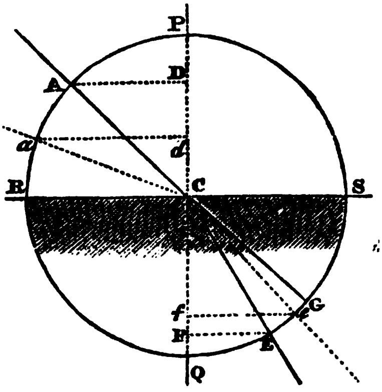
CHAPTER III.

REFRACTION OF LIGHT.

689. *Division of the incident beam.*—When light falls on an *opaque* body, we have noticed that it is arrested, and a shadow formed beyond. Of the light thus arrested, a portion is reflected, and another portion lost, which is said to be absorbed by the body. When light meets a *transparent* body, a part is still reflected, and a small portion absorbed, but, in general, the greater part is transmitted. The ratio of intensities in the reflected and transmitted beams varies with the angle of incidence, but little being reflected at small angles of incidence, and almost the whole at angles near 90°.

690. *Refraction.*—The transmitted beam suffers important changes, one of which is a change in *direction*. This change is called *refraction*, and takes place at the surface of a new medium. In Fig. 278, AC, incident upon RS, the surface of a different medium, is turned at C into another line, as CE, which is called the *refracted ray*. The angle ACP, between the incident ray and the perpendicular to the surface, is called the *angle of incidence;* the angle ECQ, between the refracted ray and the perpendicular, is called the *angle of refraction;* the angle GCE, between the directions of the two rays, is the *angle of deviation.*

FIG. 278.



It is a general fact, to which there are only a few exceptions, that a ray of light in passing out of a rarer into a denser medium is refracted *toward* the perpendicular to the surface; and in passing out of a denser into a rarer medium, it is refracted *from* the perpendicular. But the chemical constitution of bodies, as well as their density, sometimes affects their refracting power. Thus, inflammable bodies, as sulphur, amber, and essential oils, have a very great refracting power in comparison with other bodies; and in a given instance, a ray of light in passing out of one of these substances into another of greater density might not be turned toward, but from, the perpendicular. In the optical use of the words, therefore, *denser* is understood to mean, *of greater refractive power;* and *rarer* signifies, *of less refractive power.* In Fig. 278, the medium below RS is of greater refractive power than that above.

We see an example of refraction in the bent appearance of an oar in the water, the light of the part immersed (by which it is visible) being turned *from* the perpendicular as it passes from water into air, and causing it to appear higher than its true place. In the same manner, the bottom of a river appears elevated, and diminishes the apparent depth of the stream. The following ancient experiment illustrates the same principle: If a small piece of silver be placed in the bottom of a bowl, and the eye be withdrawn until the piece of silver disappears, on filling up the bowl with water, the silver comes into view again.

691. *Law of refraction.*—The law, which is found by observation to hold true in all cases of common refraction, is this: the angles of incidence and refraction are *in the same plane*, and their *sines*, for given media, *have a constant ratio.* For example, in Fig. 278, if AC is refracted to E, then aC will be refracted to e, so that AD : EF : : ad : ef; and if the rays pass out in a contrary direction, the ratio is also constant, being the reciprocal of the former, viz., EF : AD : : ef : ad.

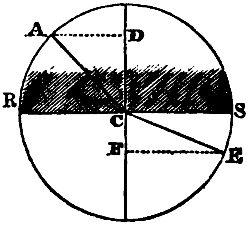
A ray perpendicular to the surface, passing in either direction, is not refracted; for, according to the law, if the sine of one angle is zero, the sine of the other must be zero also. Whichever way light passes, when air is one of the media, suppose the sine of the smaller angle, i. e. the angle in the denser medium, to be 1, then the sine of the larger angle for water is about 1.33; and for glass, it is near 1.5. The number, in each case, expresses the constant ratio of the sines for the given media, and is called the *index of refraction*, and is employed as the measure of refractive power. The following table gives the index of refraction for a few substances.

692. *Table of refractive powers.*—

	Index of Refraction.
Chromate of lead	2.974
Red silver-ore	2.564
Diamond	2.439
Phosphorus	2.224
Sulphur (melted)	2.148
Glass (composed of lead two parts, flint one)	1.830
Sapphire, and other precious gems	1.800
Sulphuret of carbon	1.768
Oil of cassia	1.641
Quartz, or rock-crystal	1.548
Amber	1.547
Crown-glass	1.530
Oil of olives	1.470
Alum	1.457
Fluor spar	1.434
Mineral acids	1.410
Alcohol	1.372
Water	1.336
Ice	1.309
Tabasheer	1.111

693. *Limit of transmission from a denser to a rarer medium.*—As a consequence of the law of refraction, there is a limit beyond which a ray can not escape from a denser medium. Let AC (Fig. 279) be the ray incident upon the rarer medium RS. It will be refracted from the perpendicular DF into the direction CE, so that AD is to EF in a constant ratio. (Art. 691.) If we increase the angle ACD, the angle FCE will also increase, till the lines CE and FE coincide with the radius CS. But if beyond this position of the ray AC, the angle ACD is still further increased, it is manifest its sine is also increased; and consequently, in order that the ratio may be constant, the sine of refraction EF must also increase, which is impossible, since it is already by hypothesis equal to the radius CS. Hence it follows, that whenever the angle of incidence is greater than that at which the sine of the angle of refraction becomes equal to radius, the ray can not be refracted consistently with the constant ratio of the sines.

FIG. 279.

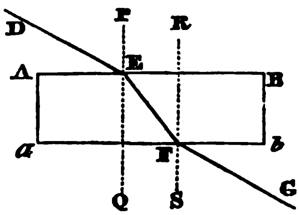


This is found to be the case by experiment; and at the angle thus indicated, all the incident rays are *reflected* from the inner surface of the denser medium, having a reflection more brilliant than what can be produced from any metallic surface. This is called *total reflection.* If n = the index of refraction, the limit at which refraction ceases, and total reflection begins, is found by the proportion, $n : 1 :: \text{rad.} : \text{sine of the limit}$. If the refractive power is greater, the limit is smaller; for, by the above proportion, n varies inversely as sine of limit. For *water*, it is 48° 28′; for *glass* (crown), 40° 49′; for *diamond*, 24° 12′.

694. *Transmission through plane surfaces.*—

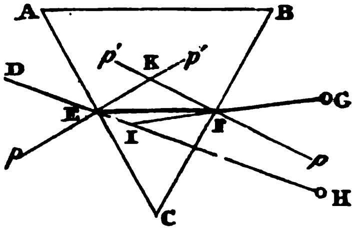
1. A medium bounded by *parallel* planes. In this case the *incident* and *emergent* rays are *parallel*. Let DE (Fig. 280) enter the medium AB*ba* at E, and leave it at F, and let PQ, RS be the perpendiculars at E, F. The first angle of refraction, QEF, and the second angle of incidence, EFR, are equal, being alternate; therefore, DEP = SFG, since their sines have a constant ratio to those of QEF, EFR. Hence, if the incident rays are produced, the angles of deviation are equal; but DEF is supplement to the first angle of deviation, and EFG of the second. Therefore, as DEF and EFG are alternate and equal, DE is parallel to FG.

FIG. 280.



2. A medium bounded by *inclined* planes, called a *prism*. The transmitted ray is turned *from the refracting angle*. Let ABC (Fig. 281) be the cross-section of a glass prism, and AC, BC the inclined sides of it, through which the light is transmitted. C is called the *refracting angle*, and AB the *base*. As prisms are usually constructed and mounted, either A, B, or C, may be the refracting angle; but it is not essential that any of the faces should meet at an edge, as the effect on the light depends only on the inclination. In ordinary directions of the ray, the two refractions, one on entering, the other on leaving the prism, conspire to increase the deviation of the ray from its original direction. DE is first bent *toward* Ep', making the deviation HEF; at F, it is turned *from* Fp, making a second deviation, equal to EFI, the same way. The sum of the two deviations, IEF + EFI = GIH, the total deviation away from the refracting angle, C.

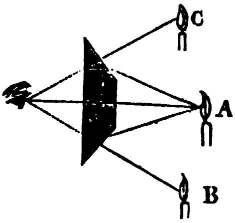
FIG. 281.



To an eye at G, the radiant D is seen in the direction GFI.

695. *The multiplying glass.*—A piece of glass, ground with one side plane, and the other, in any number of plane facets on a convex surface, is called a *multiplying glass.* Each facet, along with the opposite plane surface, forms a prism; and, if a radiant A, is placed in the axis (a perpendicular through the center of the plane surface), the pencils, falling on the several facets, will be turned from the edge, and may by two refractions at the opposite surfaces, be brought to an eye placed also in the axis, and thus as many images will be seen as there are facets. Fig. 282 exhibits the effect of three such facets.

FIG. 282.



696. *Prism used for measuring refractive power.*—The following theorem may be used for determining the refractive power of a substance, after first forming it into a prism of small angle. *If the angle of deviation be divided by the refracting angle of the prism, and the quotient be added to unity, the sum is the index of refraction.* In proving this, it is assumed, that all the angles are very small, so that they vary as their sines. Let $n=$ the index of refraction, then (Fig. 281),

$$p'\mathrm{EI}(=\mathrm{DE}p):p'\mathrm{EF}::n:1,\ \therefore\ \mathrm{FEI}:p'\mathrm{EF}::n-1:1;$$
$$\text{also, } p'\mathrm{FI}(=\mathrm{GF}p):p'\mathrm{FE}::n:1,\ \therefore\ \mathrm{EFI}:p'\mathrm{FE}::n-1:1;$$
$$\therefore\ \mathrm{FEI}+\mathrm{EFI}:p'\mathrm{EF}+p'\mathrm{FE}::n-1:1,$$
$$\therefore\ \mathrm{FIH}:p'\mathrm{KF}::n-1:1.$$

But p'KF and ACB are equal, each being the supplement of EKF. Therefore,

$$\mathrm{FIH}:\mathrm{ACB}::n-1:1;$$
$$\therefore\ n-1=\frac{\mathrm{FIH}}{\mathrm{ACB}};\ \therefore\ n=\frac{\mathrm{FIH}}{\mathrm{ACB}}+1.$$

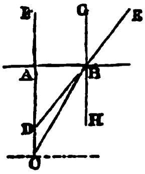
Now, in glass, $\frac{\mathrm{FIH}}{\mathrm{ACB}}$ is found by trial to be very near ½; $\therefore\ n=1.5$ nearly.

In order to find the index of refraction for any solid substance, grind it into a prism, whose sides are nearly parallel, and carefully measure their inclination. Then measure the displacement of a distant object seen through it at right angles to its surface. For example, the faces of a transparent mineral incline 1° 10′; and when held before the eye, it displaces a distant object 50′; ∴ the index of refraction $=1+\frac{5}{7}=1.714$.

697. *A pencil of rays through one surface of a medium.*—When *parallel* rays pass into another medium, through a plane surface, they remain parallel. For the perpendiculars being parallel, the angles of incidence are equal, and therefore the angles of refraction are equal also, and the refracted rays parallel. But a pencil of *diverging* rays is made to diverge *less*, when it *enters a denser medium.* For the outer rays make the largest angles of incidence, and are therefore most refracted toward the perpendiculars, and thus toward parallelism with each other. And when *diverging* rays *enter a rarer medium*, they diverge *more;* because the outside rays make the largest angles of incidence, and therefore the largest angles of refraction, by which means they spread more from each other.

The last case is illustrated, when we look perpendicularly into water, and see its depth apparently diminished by about one-fourth of the whole. Let AB (Fig. 283) be the surface, and C a point at the bottom, from which a pencil comes to the eye. Let CF, the axis of the pencil, be perpendicular to AB, and CBE, an oblique ray of the pencil. The angle C = CBH = angle of incidence; and ADB = GBE = angle of refraction. Now, in the triangle BDC, BC : BD (: : AC : AD nearly) : : sin D : sin C : : sine of refraction : sine of incidence : : 1.34 : 1. Hence the apparent depth is one-fourth less than the real depth.

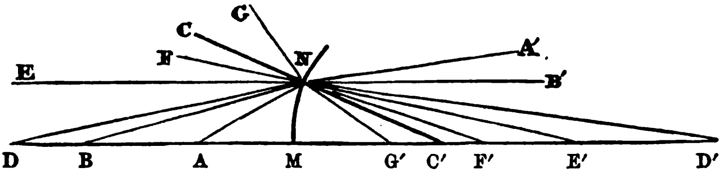
FIG. 283.



698. *Transmission through spherical surfaces.*—

1. The *convex* surface of a denser medium *tends to converge rays.* In Fig. 284, suppose the medium on the right of MN to be more dense than on the left; and let C′ be the center of convexity, and C′D, C′C, two radii. As rays are bent *toward* the perpendiculars (radii), in entering a denser medium, and as the perpendiculars themselves converge to C′, the general effect of such a surface is to produce convergency. The pencil, AM, AN, is merely made less divergent, MD′, NA′; BM, BN, become parallel, MD′, NB′; DM, DN, convergent to D′; the parallel rays, DM, EN, convergent to E′; the convergent pencil, DM, FN, more convergent to F′; but DM, CN, which converge equally with the radii, are not changed; and DM, GN, which converge more than the radii, converge less than before, to G′. The two last cases, which are exceptions to the general effect, rarely occur in the practical use of lenses.

FIG. 284.

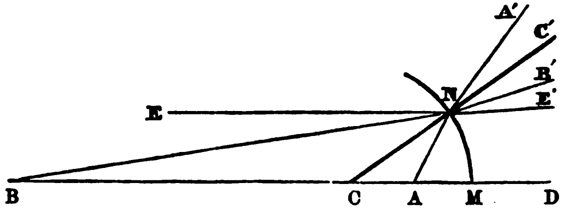


If we trace in the opposite direction the rays, A′, B′, D′, &c.,

comparing each with D′D, we find, in this case also, that the convex surface tends to converge the rays, by bending them *from* the diverging radii, C′D, C′C.

2. The *concave* surface of a denser medium *tends to diverge rays.* In Fig. 285, let the medium on the right of MN be more

FIG. 285.

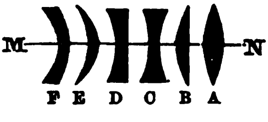


dense than on the left; and let CC′, CM, be the radii of concavity. As the radii *diverge* in the direction in which the light moves, the rays, being bent *toward* them, will generally be made to diverge also. Hence, parallel rays, BM, EN, are diverged, MD, NE′; and diverging rays, BM, BN, are diverged more, MD, NB′. If, however, rays diverge as much as the radii, or more, they remain unchanged in direction, or diverge less, a case which rarely occurs.

If the rays are traced in the opposite direction, the tendency in general to produce divergency, appears from the fact, that the perpendiculars are now *converging* lines, and the rays are refracted *from* them.

699. *Lenses.*—A *lens* is a circular piece of glass, whose surfaces are plane or spherical, and the spherical surface, either convex or concave. The usual varieties are shown in Fig. 286.

FIG. 286.



A *double convex* lens (A) consists of two spherical segments having a common base.

A *plano-convex lens* (B) is a lens having one of its sides convex and the other plane, being simply a segment of a sphere.

A *double concave lens* (C) is a solid bounded by two concave spherical surfaces, which may be either equally or unequally concave.

A *plano-concave lens* (D) is a lens one of whose surfaces is plane and the other concave.

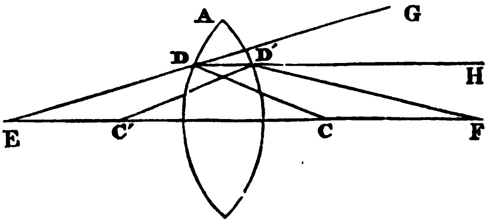
A *meniscus* (E) is a lens one of whose surfaces is convex and the other concave, but the concavity being less than the convexity, it takes the form of a crescent, and has the effect of a convex lens whose convexity is equal to the difference between the sphericities of the two sides.

A *concavo-convex lens* (F) is a lens, one of whose surfaces is convex and the other concave, the concavity exceeding the convexity, and the lens being therefore equivalent to a concave lens, whose sphericity is equal to the difference between the sphericities of the two sides.

A line (MN) passing through a lens, perpendicular to its opposite surfaces, is called the *axis*. The axis usually, though not necessarily, passes through the center of the figure.

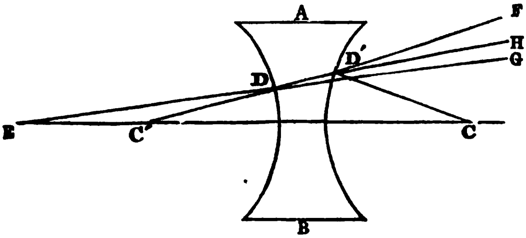
700. *The convex lens.*—Whether double-convex or plano-convex, its general effect is to converge light. It has been shown (Art. 698), that the convex surface of a denser medium tends to converge rays, whichever way they pass through it. Therefore, if E (Fig. 287) is a radiant, while EC′C follows the axis without change of direction, the oblique ray ED is first refracted *toward* DC, and then from C′D′ produced, and both actions conspire to converge it to the axis. The rays are represented as meeting in the focus F. Whether the rays are *actually* converged, depends on their previous relation to each other. If the lens is *plano-convex*, the plane surface has usually but little effect in converging the light; but by Art. 697, it may be shown, that its action will usually conspire with that of the convex surface.

Fig. 287.



701. The *concave lens.*—This lens, whether double-concave, or plano-concave, tends to produce *divergency*. This is evident from what has been shown in Art. 698. The ray ED (Fig. 288), in entering the denser medium is first refracted *toward* C′D produced, and on leaving the medium at D′, is refracted *from* D′C; and is thus twice refracted from the ray EC, which being in the axis, is not refracted at all. If the lens is *plano-concave*,

Fig. 288.



the effect of the plane surface may, or may not, conspire with that of the concave surface.

702. *The optic center of a lens.*—There is, in the axis of every lens, a point called its *optic center*, so situated that every ray that passes through it, has its incident and emergent rays parallel. Let R, r (Fig. 289) be the centers from which the surfaces of a given lens are described, and REr their axis. Draw any two of their radii, RA, *ra*, parallel to each other, and join A*a*; the point E, where this line intersects the axis, will be the point above described, and any ray, as Q*q*, passing through A, will have the incident ray, QA, parallel to the emergent ray *aq*. For since the triangles REA, *r*E*a*, are similar, RA : *ra* : : RE : *r*E, ∴ RA + *ra* : *ra* : : RE + *r*E : *r*E. And, as the three first terms of this proportion are invariable, the last *r*E must also be invariable. Hence it follows, that to whatever points in the surface of the lens, the parallel radii RA, *ra* are drawn, the line A*a* will always cut the axis R*r* in the same point E. If we now suppose the ray A*a* to pass both ways out of the lens, it will be refracted equally and in contrary directions; because RA, *ra*, being perpendiculars to the surface at A and *a*, the angles of incidence of the ray A*a* or *a*A, will be equal. Consequently, AQ will be parallel to *aq*. When the thickness of the lens is inconsiderable, and when a ray falls nearly perpendicularly upon it, the part of the ray through E, viz. QAE*aq*, may be taken as a straight line, passing through the center E of the lens; for the perpendicular distance between AQ, *aq* diminishes, both with the thickness of the lens and with the obliquity of the ray to the axis. The same may be proved of the other forms of lens.

Fig. 289.

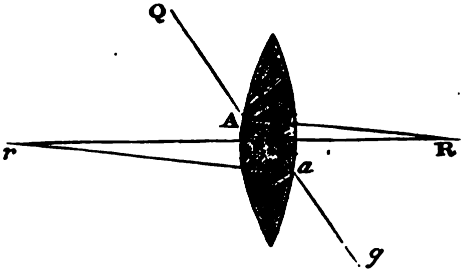
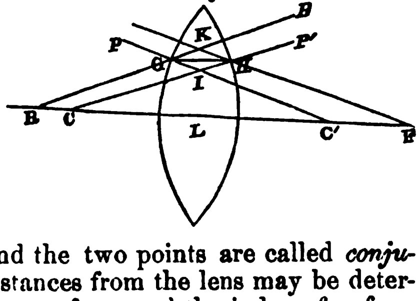


Fig. 290.



703. *Conjugate foci.*—If the rays from R (Fig. 290) are collected at F, then rays emanating from F will be returned to R; and the two points are called *conjugate foci*. Their relative distances from the lens may be determined when the radii of the surfaces and the index of refraction are known. Let n be the index of refraction, and assume, what is practically true, that the angles of incidence and refraction are so small that their ratio is the same as the ratio of their sines. Then

$$\text{RGP}\,(=\text{KGI}) : \text{IGH} :: n : 1;$$

$$\therefore\quad \text{KGH} : \text{IGH} :: n - 1 : 1;$$

in like manner $\quad \text{KHG} : \text{IHG} :: n - 1 : 1;$

$$\therefore\ \text{KGH} + \text{KHG} : \text{IGH} + \text{IHG} :: n - 1 : 1.$$

But $\quad \text{KGH} + \text{KHG} = \text{BKF} = \text{R} + \text{F};$

and $\quad \text{IGH} + \text{IHG} = \text{GIC} = \text{C} + \text{C}';$

naming the acute angles at R, C, C′, F, by those letters respectively.

$$\therefore\ \text{R} + \text{F} : \text{C} + \text{C}' :: n - 1 : 1.$$

Now, the lens being thin, and the angles R, C, C′, and F very small, the same perpendicular to the axis, at L, the center of the lens, may be considered as subtending all those angles. Hence, each angle is as the reciprocal of its distance from L. Let $\text{RL} = p$; $\text{FL} = q$; $\text{CL} = r$; and $\text{C}'\text{L} = r'$. Then the equation above becomes

$$\frac{1}{p} + \frac{1}{q} : \frac{1}{r} + \frac{1}{r'} :: n - 1 : 1;$$

which expresses in general the relation of the conjugate foci. To adapt it to crown-glass, call $n = \frac{3}{2}$, and we have

$$\frac{1}{p} + \frac{1}{q} : \frac{1}{r} + \frac{1}{r'} :: 1 : 2.$$

704. To find the *principal* focus (focus of parallel rays) for crown-glass, let one of the focal distances, as p, become infinite, and call the other F; then $\frac{1}{p} = 0$, and we have

$$\frac{1}{\text{F}} : \frac{1}{r} + \frac{1}{r'} :: 1 : 2;\ \text{or}\ \text{F} = \frac{2rr'}{r + r'};$$

that is, the distance of the principal focus of a glass lens is equal to *twice the product of the radii of curvature divided by their sum.*

If the convexities are equal, the formula is reduced to

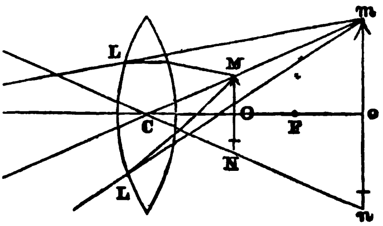
$$\frac{1}{p} + \frac{1}{q} : \frac{2}{r} :: 1 : 2;\ \text{or}\ \frac{1}{p} + \frac{1}{q} = \frac{1}{r};$$

and the distance of the principal focus now becomes $\text{F} = r$. The principal focus of a glass lens is at the center of convexity, when both sides are equally convex.

The foregoing formulæ are readily adapted to the other forms of lens. When a surface is plane, its radius is infinite, and $\frac{1}{r}$, or $\frac{1}{r'}=0$. When concave, its center is thrown upon the same side as the surface, and its radius is to be called negative. And if the focal distance, as given by the formula, becomes negative, it is understood to be on the same side as the radiant; that is, the focus is a virtual radiant.

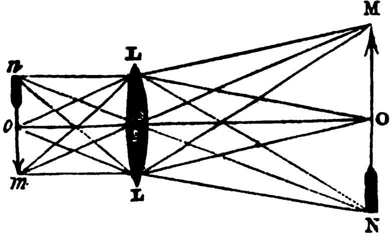
705. *Images by convex lenses.*—The *convex* lens forms a variety of images, whose character and position depend on the place of the object. If it is at the *principal focus*, the rays of every pencil pass out parallel, and seem to come from an infinite distance. If the object is *nearer* than the principal focus, the rays of each pencil diverge less than before, and therefore seem to radiate from points further back; the image is therefore *apparent*. Let MN (Fig. 291) be the object nearer than the principal focus, F. Then the pencil from M will, after refraction, diverge as from *m* in CM produced, and so of every point; hence *mn* is the image. It is *erect*, because the axes of the pencils do not cross between the object and image; and it is *enlarged*, because it subtends the angle MCN at a greater distance than the object.

FIG. 291.



But if the object is *further* from the lens than the principal focus, the rays of each pencil converge to a point in the axis of that pencil produced through the lens; and thus light is collected in focal points, which consequently become actual radiants. The last case is illustrated by Fig. 292, in which MN is the object, and *mn* the image. A cone of rays from O covers the lens, LL, and is converged again into the axis at *o*, the conjugate focus of O, and there cross, and proceed as from a radiant. The cone of rays from M is converged to *m* in the axis M*m*' of that cone, which is a straight line through the optic center (Art. 702); and so from every point of the object. Though the rays of every radiant *converge* from the lens to the conjugate focus of that radiant, yet the axes of the pencils *diverge* from each other, having all crossed at the

FIG. 292.

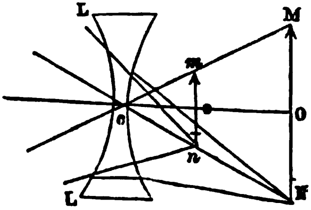


optic center. The image is therefore *inverted*, as are all real images, in whatever way produced.

706. The formula for conjugate foci shows that if p is increased, q is diminished; therefore the further MN is removed from the lens, the nearer *mn* approaches to it; but the nearest position is the principal focus, which it reaches, when the object is at an infinite distance. As the object and image subtend equal angles at the optic center, and are parallel, or nearly parallel with each other, their *diameters* are proportioned to their *distances* from the lens. But the area of the lens has no effect on the size of the image, since change of area does not alter the relation of the axes, but only the size of the luminous cones, and thus the quantity of light in each pencil.

707. *Images by concave lenses.*—As the rays of each pencil are diverged more after passing through the lens than before, the image is *apparent*, and is situated between the lens and the object. Let MN (Fig. 293) be the object; the cone of rays from N will, after refraction, diverge more, as from *n*, in the same axis CN; and all other pencils will be affected in a similar manner, and form an apparent image *mn*. It will be *erect*, since the axes do not cross between; and *diminished*, being nearer the angle C, which is subtended by both object and image.

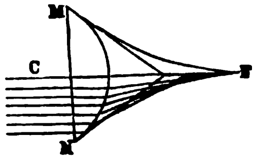
Fig. 293.



It is noticeable, that the *concave mirror* and the *convex lens* are analogous in their effects, forming images on both sides, both real and apparent, both erect and inverted, both larger and smaller than the object; while the *convex mirror* and the *concave lens* also resemble each other, producing images always on one side, always apparent, always erect, always smaller than the object.

708. *Caustics by refraction.*—If the convex surface of a lens is a considerable part of a hemisphere, the rays more distant from the axis will be so much more refracted than others, as to cross them and meet the axis at nearer points, thus forming caustics by refraction. Fig. 294 shows this effect in the case of parallel rays; those near the axis intersecting it at the principal focus F, and the intersections of remoter rays being nearer and nearer to the lens, so that the whole converging pencil assumes a form resembling a cone with concave sides.

Fig. 294.



709. *Spherical aberration of lenses.*—The production of caustics is an extreme case of what is called spherical aberration. Unless the lens is of small angular breadth, a pencil whose rays originated in one point of an object is not converged accurately to one point of the image, but the outer rays are refracted too much, and make their focus nearer the lens than that of the central rays, as represented in Fig. 295. If F is the focus of the central rays, and *f* of the extreme ones, other rays of the same beam are collected in intermediate points, and *f*F is called the *longitudinal spherical aberration*; and GH, the breadth covered by the pencil at the focus of central rays, is called the *lateral spherical aberration.*

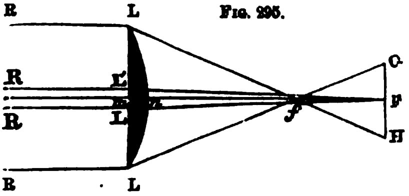


FIG. 295.

Such a lens can not form a distinct picture of any object; because perfect distinctness requires that all rays from any one point of the object should be collected to one point in the image. If, for example, the beam, whose outside rays are RL, RL, comes from a point of the moon's disk, that point will not be perfectly represented by F, because a part of its light covers the circle, whose diameter is GH, thus overlapping the space representing adjacent points of the moon. And if that point had been on the edge of the moon's disk, F could not form part of a well-defined edge of the image, since a blur, whose breadth is FG, would extend outside of it.

710. As spherical lenses refract too much those rays which pass through the outer parts, it is obvious that, to destroy aberration, a lens is required whose curvature diminishes toward the edges. Accordingly, forms for *ellipsoidal* lenses have been calculated, which in theory will completely remove this species of aberration. But no curved solids can be so accurately ground as those whose curvature is uniform in all planes, that is, the spherical. Hence, in practice it is found better to *reduce* the aberration as much as possible by spherical lenses, than to attempt an entire *removal* of it by other forms which can not be well made.

In a plano-convex lens, whose plane surface is toward the object, the spherical aberration is 4.5; that is (Fig. 295), *f*F = 4.5 times *mn*, the thickness of the lens. But the same lens, with its convex side toward the object, is far better, its aberration being only 1.17. In a double convex lens of equal curvatures, the aberration is 1.67; if the radii of curvature are as 1 : 6, and the most convex side is toward the object, the aberration is only 1.07. By placing two plano-convex lenses near each other, the aberration may be still more reduced.

711. *Atmospheric refraction.*—The atmosphere may be regarded as a transparent spherical shell, whose density increases from its upper surface to the earth. The radii of the earth produced are the perpendiculars of all the laminæ of the air; and rays of light coming from the vacuum beyond, if oblique, are bent *gradually* toward these perpendiculars; and therefore heavenly bodies appear more elevated than they really are. The greatest elevation by refraction takes place at the horizon, where it is about half a degree.

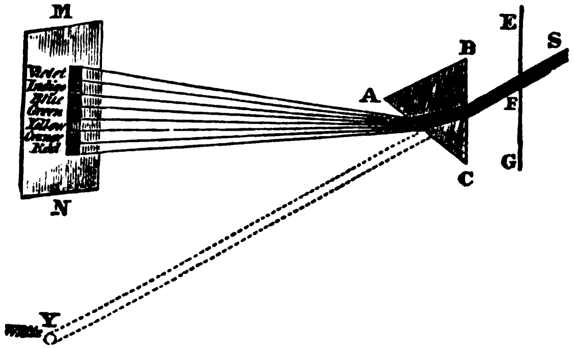
712. *Mirage.*—This phenomenon, called also *looming*, consists of the formation of one or more images of a distant object, caused by horizontal strata of air of very different densities. Ships at sea are sometimes seen when beyond the horizon, and their images occasionally assume distorted forms, contracted or elongated in a vertical direction. These effects are generally ascribed to *extraordinary refraction* in horizontal strata, whose difference of density is unusually great. But many cases of mirage seem to be instances of *total reflection* from a highly rarefied stratum resting on the earth. These occur frequently on extended sandy plains, as those of Egypt. When the surface becomes heated, distant villages, on more elevated ground, are seen accompanied by their images inverted below them, as in water. As the traveler advances, what appeared to be an expanse of water retires before him. By placing alcohol upon water in a glass vessel, and allowing them time to mingle a little at their common surface, the phenomena of mirage may be artificially represented.

CHAPTER IV.

DECOMPOSITION AND DISPERSION OF LIGHT.

713. *The prismatic spectrum.*—One change which light suffers in passing into another medium, called refraction, has been considered. We come to notice another, called *decomposition*, or the separation of light into colors. For this purpose, the glass prism is generally employed. It is so mounted on a jointed stand, that it can be placed in any desired position across the beam from the heliostat. The beam, as already noticed, is bent away from the refracting angle, both in entering and leaving the prism, and deviates several degrees from its former direction. If received on a white screen, it no longer forms a circular white spot, as before the prism was interposed, but a lengthened colored band, having straight parallel sides, and semicircular ends; this is the *prismatic spectrum*. Its colors are usually regarded as seven in number—*red*, *orange*, *yellow*, *green*, *blue*, *indigo*, *violet*. The red is invariably nearest to the original direction of the beam, and the violet the most remote, as shown in Fig. 296; and it is because the elements of white light are unequally refrangible, that they become separated, by transmission through a refracting body. The spectrum is properly regarded as consisting of circles of innumerable shades of color, overlapping each other from end to end. Instead of Newton's division into *seven* colors, many choose to consider all the varieties of tint as caused by the combination of *three* primitive colors, *red*, *yellow*, and *blue*, varying in their proportions throughout the entire spectrum. The number *seven*, as perhaps any other particular number, must be regarded as arbitrary. Red, yellow, and blue, are frequently called *simple* colors, and the others, *compound* colors.

FIG. 296.

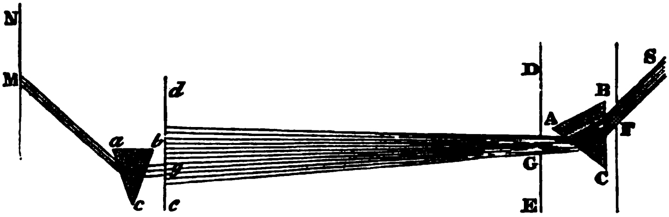


The spectrum contains other elements besides those which affect the eye. We might expect to find *heat* in it, as well as in every sunbeam; but the greatest heat lies outside of the visible spectrum at the red extremity, showing that heat is less refrangible than light. On the other hand, the chemical agency is greatest near the violet extremity, where the light is faintest.

Light from other sources than the sun is susceptible of decomposition by the prism; but the spectrum, though resembling the solar spectrum, usually differs in the proportion of the colors.

714. If a pencil of any decomposed beam be passed through an aperture as represented in Fig. 297, and then be received upon a second prism, it is refracted at the same angle as if it had been in combination with all the other colors, but it is not decomposed. Whether the green of the spectrum be a compound of yellow and blue, or not, it is at least not susceptible of decomposition by the prism; and the same is true of every color of the spectrum.

FIG. 297.

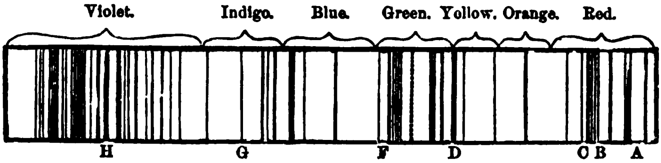


715. *Colors of the spectrum recombined.*—It may be shown, in several ways, that if all the colors of the spectrum be combined, they will *reproduce white light.* One method is by transmitting the beam successively through two prisms whose refracting angles are on opposite sides. By the first prism, the colors are separated at a certain angle of deviation, and then fall on the second, which tends to produce the same deviation in the opposite direction, by which means all the colors are brought upon the same ground, and the illuminated spot is white and circular, as if no prism had been interposed. Or the colors may be received on a series of small plane mirrors, which admit of such adjustment, as to reflect all the beams upon one spot. Or finally, the several colors can, by different methods, be passed so rapidly before the eye, that their visual impressions shall be united in one; in which case the illuminated surface appears white.

716. *Complementary colors.*—If certain colors of the spectrum are combined in a compound color, and the others in another, these two are called *complementary colors*, because, when united, they will produce white. For example, if *green*, *blue*, and *yellow*, are combined, they will produce green, differing slightly from that of the spectrum; the remaining colors, *red*, *orange*, *indigo*, and *violet*, compose a kind of purple, unlike any color of the spectrum. But these particular shades of *green* and *purple*, if mingled, will make perfectly white light, and are therefore complementary colors.

717. *Fixed lines of the spectrum.*—When the greatest care is taken to form the spectrum by refracting laterally a single lamina of light, admitted through a narrow crevice, so that there shall be no overlapping of spectra, and also to use a prism of uniform density, the phenomenon of *fixed lines* presents itself. The celebrated Fraunhofer made the remarkable discovery, that the colors of the spectrum are not continuously shaded off into each other, but interrupted by dark, and in many instances, by perfectly black lines, crossing the spectrum in the direction of the division lines between colors. He enumerated about 500 of them, very unequally distributed, some of which are more easily seen and very sharply defined; and hence used as a means of measurement in the comparison of the refractive power of media. In Fig. 298, the letters A, B, &c., mark those which are used as lines of reference. The lines of *solar* light, whether direct or reflected, are always invariable. But light from other sources, as that from particular stars, and that from the combustion of particular substances, has in every case its own system of lines, by which it may be identified.

FIG. 298.



718. *Dispersion of light.*—Decomposition of light refers to the *fact* of a separation of colors; *dispersion*, rather to the *measure* or *degree* of that separation. The *dispersive power* of a medium indicates the amount of separation which it produces, compared with the amount of refraction. For example, if a substance, in refracting a beam of light 1° 51′ from its course, separates the violet from the red by 4′, then its dispersive power is $\frac{4}{111}$=.036. The following table gives the dispersive power of a few substances much used in optics:

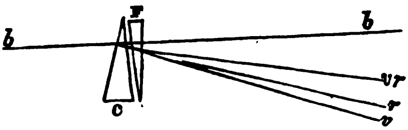
	Dispersive power.		Dispersive power.
Oil of Cassia,........	0.139	Plate-Glass,..........	0.032
Sulphuret of Carbon,..	0.130	Sulphuric Acid,......	0.031
Oil of Bitter Almonds,.	0.079	Alcohol,............	0.029
Flint-Glass...........	0.052	Rock-Crystal,.......	0.026
Muriatic Acid,.......	0.043	Blue Sapphire,.......	0.026
Diamond,...........	0.038	Fluor-spar,..........	0.022
Crown-Glass (green),. .	0.036		

The discovery that different substances produce different degrees of dispersion, is due to Dollond, who soon applied it to the removal of a serious difficulty in the construction of optical instruments.

719. *Chromatic aberration of lenses.*—This is a deviation of light from a focal point, occasioned by the different refrangibility of the colors. If the surface of a lens be covered, except a narrow ring near the edge, and a sunbeam be transmitted through the ring, the chromatic aberration becomes very apparent; for the most refrangible color, violet, comes to its focus nearest, and then the other colors in order, the focus of red being most remote. Since the distinctness of an image depends on the accurate meeting of rays of the same pencil in one point, it is clear that discoloration and indistinctness are caused by the separation of colors.

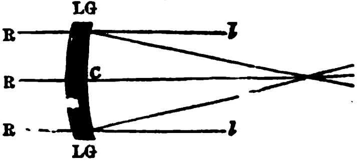
720. *Achromatism.*—In order to refract light, and still keep the colors united, it is necessary that, after the beam has been refracted, and thus separated, a substance of greater dispersive power should be used, which may bring the colors together again, by refracting the beam only a part of the distance back to its original direction. For instance, suppose two prisms, one of crown-glass and one of flint-glass, each ground to such a refracting angle as to separate the violet from the red ray by 4′. In order for this, the crown-glass, whose dispersive power is .036, must refract the beam 1° 51′; for $\frac{4'}{1^\circ\ 51'} = .036$; and the flint-glass, whose dispersive power is .052, must refract only 1° 7′; for $\frac{4'}{1^\circ\ 7'} = .052$. Place these two prisms together, base to edge, as in Fig. 299, C being the crown-glass and F the flint-glass. Then C will refract the beam, *bb*, downward 1° 51′, and the violet, *v*, 4′ more than the red, *r*; F will refract this decomposed beam upward 1° 7′, and the violet 4′ more than the red, which will just bring them together at *vr*. Thus the colors are united again, and yet the beam is refracted downward 1° 51′ − 1° 7′ = 44′, from its original direction.

Fig. 299.



721. *Achromatic lens.*—If two prisms can thus produce achromatism, the same may be effected by lenses; for a convex lens of crown-glass may converge the rays of a pencil, and then a concave lens of flint-glass may diminish that convergency sufficiently to unite the colors. This requires that the *principal focal distances* of the two lenses should be directly *as their dispersive powers*. A lens thus constructed of two lenses of different materials and opposite curvatures, so adapted as to produce an image free from chromatic aberration, is called an *achromatic lens*. Fig. 300 shows such a combination.

Fig. 300.



It is assumed in the foregoing discussion, that when the red and violet are united, all the intermediate colors will be united also. It is found that this is not strictly true, but that different substances separate two given colors of the spectrum by intervals which are not exactly proportional to the whole length of the spectrum. This departure from a constant ratio in the distances of the several colors, as dispersed by different media, is called the *irrationality of dispersion.* In consequence of it, there will exist some slight discoloration in the image, after uniting the extreme colors. It is found better in practice, to fit the curvatures of the lenses, for uniting those rays which most powerfully affect the eye.

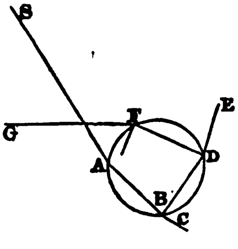
CHAPTER V.

RAINBOW AND HALO.

722. *The rainbow.*—This phenomenon, when exhibited most perfectly, consists of two colored circular arches, projected on falling rain, on which the sun is shining from the opposite part of the heavens. They are called the *inner* or *primary* bow, and the *outer* or *secondary* bow. Each contains all the colors of the spectrum, arranged in contrary order; in the primary, red is outermost; in the secondary, violet is outermost. The primary bow is narrower and brighter than the secondary, and, when of unusual brightness, is accompanied by *supernumerary* bows, as they are called; that is, narrow red arches just within it, or overlapping the violet; sometimes three or four supernumeraries can be traced for a short distance. The common center of the bows is in a line drawn from the sun through the eye of the spectator.

723. *Action of a transparent sphere on light.*—It will aid in understanding the manner in which the bow is formed, to notice the experiments which first led to a correct theory respecting it. Let a hollow sphere of glass be filled with water and placed in the sunlight, and then let the directions and conditions of the most luminous pencils, which emerge from it, be observed. Fig. 301 exhibits the general result. The entire hemisphere exposed to the rays is of course penetrated by them; but a narrow pencil, SA, about 60° distant from the ray which passes through the center, is converged to B, where some light escapes, but a large

FIG. 301.

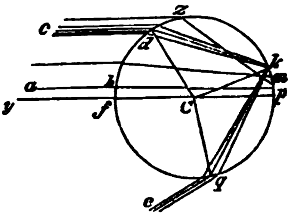


part is reflected to D; at that point a division occurs again, and the emerging pencil, DE, consists of decomposed light, each color of which can be seen at a great distance. The part reflected at D divides again at and near F, but the emergent portion diverges, and has none of the intensity of DE. But if a pencil, which strikes the drop about 10° outside of SA (not represented in the figure), were to be traced, we should find that a part is reflected near B, a part of that again reflected near D, and then, on reaching a point near F, the emerging portion is not only decomposed, but retains its intensity to a great distance. The pencil, DE, which has been *once* reflected at B, is the one concerned in the production of the primary bow. The pencil which leaves the drop near F, after it has been *twice* reflected, is one of those which constitute the secondary bow.

In general, the light which enters a drop of rain, and leaves it again, either not reflected at all, or reflected one or more times, is scattered in various directions, and brings to the eye of an observer no impression of intense light. It is only such rays as are reflected and transmitted in circumstances to be contiguous to each other, and to continue parallel after leaving the drop, which can produce the bright colors of the rainbow.

724. *Course of rays in the primary bow.*—Let *fzpq* (Fig. 302) be the section of a drop of rain, *fp* a diameter, *ab*, *cd*, &c., parallel rays of the sun's light, falling upon the drop. Now *yf*, a ray coinciding with the diameter, would suffer no refraction; and *ab*, a ray near to *yf*, would suffer only a very small inclination toward the radius, so as to meet the remoter surface of the drop very near to *p*; but the rays which lie further from *yf*, being inclined toward the radius in a greater angle, would be more and more refracted as they were further removed from the diameter.

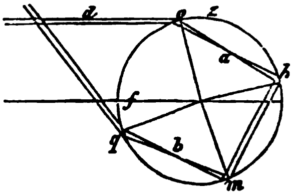
Fig. 302.



And it is found, by a simple calculation on the course of the rays, that those which enter beyond the limit of about 60°, cross more or less of those entering nearer the axis, the furthest one of all at 90°, being refracted almost to *p*. Hence all the rays falling on the quadrant *fz*, meet the circumference within the arc *kp*. But when a varying quantity is approaching its limit, or is beginning to depart from it, its changes are nearly insensible. Thus, a large number of rays near *cd*, on both sides of it, meet very near *k*, the limit of the arc *pk*. Consequently, a greater number of rays will be reflected from that point than from any other in the arc. Now were these rays to return in the same lines, they would emerge parallel in the lines near *cd*; but if, instead of returning back in the quadrant *fz*, they are reflected on the other side of the radius, they make the same angles with the radius, and therefore with each other, as the incident rays do, and consequently meet the curve at the same inclination on the other side of the axis, and emerge parallel. Hence it appears that there is a particular point in the section of the drop, where the rays of the sun's light *accumulate*, and then diverge, so that, on emerging, those of a given color form a compact pencil of parallel rays. It is found by calculation that the angle which the incident and emergent rays make with each other is, for the *red* rays, 42° 2′, and for the *violet* rays, 40° 17′, and for other colors, between these limits. Calculation shows, also, that these are the *greatest* deviations possible for rays once reflected; since all rays on the quadrant *fz*, whether nearer or further than the pencil *cd*, at 60°, emerge with smaller deviations.

725. *Course of rays in the secondary bow.*—There is also an accumulation at a certain limit, for the light which emerges after suffering *two* reflections. If, as before, we calculate the course of the rays which fall on the quadrant *fz*, we shall find that those, *de* (Fig. 303), which enter at about 71°, 72°, from the axis, after crossing each other in the drop at *a*, are reflected at *h* into parallel lines, and, consequently, after a second reflection at *m*, have their relations to each other and the radii exactly reversed. Hence, they cross a second time at *b*, and emerge parallel at *q*. Such a pencil, entering above the axis, will, on emerging, ascend and cross its own path, outside of the drop, the violet rays intersecting *de* at an angle of 54° 9′, the red at 50° 59′, and the other colors in order between. That the emergent pencil may *descend* to the observer, the incident pencil must enter *below* the axis, and come out above it. These rays, entering at the distance of 71° and 72° from the axis, are the only ones which, after *two* reflections, emerge compact and parallel, and give a bright color at a great distance. All rays which enter nearer the axis, and also those which enter more remote, make, after two reflections, larger angles of deviation, and diverge from each other.

FIG. 303.



726. *Axis of the bows.*—Let ABDGI (Fig. 304) represent the path of the pencil of red light in the primary bow. If AB and IG are produced to meet in K, the angle K is the deviation, 42° 2′, of the incident and emergent red rays. Suppose the spectator at I, and let a line from the sun be drawn through his position to T; it is sensibly parallel to AB, and therefore the angles I and K are equal. As T is opposite to the sun, the red color is seen at the distance of 42° 2′, on the sky, from the point opposite to the sun; and so the angular distance of each color from T equals the angle which the ray of that color makes with the incident ray. In like manner, in the secondary bow, if IT (Fig. 305) be drawn through the sun and the eye of the observer, it is parallel to AB, and the angular distance of the colored ray from T, is equal to K, the deviation of the incident and emergent rays. IT is called the *axis* of the bows, for a reason which is explained in the next article.

FIG. 304.

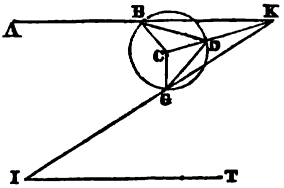
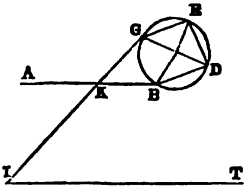
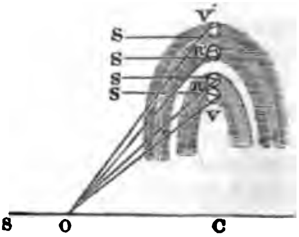


FIG. 305.



727. *Circular form of the bows.*—Let SOC (Fig. 306) be a straight line passing from the sun, through the observer's place at O, to the opposite point of the sky; and let VO, RO be the extreme rays, which after *one* reflection bring colors to the eye at O, and R′O, V′O, those which exhibit colors after *two* reflections; then (according to Arts. 724, 725), VOC = 40° 17′, ROC = 42° 2′, R′OC = 50° 59′, V′OC = 54° 9′. Now, if we suppose the whole system of lines, SV′O, SVO, to revolve about SOC, as an axis, the relations of the rays to the drops, and to each other will not be at all changed; and the same colors will describe the same lines, whatever positions those lines may occupy in the revolution. The emergent rays, therefore, all describe the surfaces of cones, whose common vertex is in the eye at O; and the colors, as seen on the cloud, are the circumferences of their bases.

FIG. 306.



In a given position of the observer, the extent of the arches depends on the elevation of the sun. When on the horizon, the bows are semicircles; but less as the sun is higher, because their center is depressed by the same angle. If rain is near, however, the bows may sometimes be traced down upon the landscape, so as to add considerably to the length of the branches; and, on a mountain, the bows have been seen as entire circles.

728. *Colors of the two bows in reversed order.*—The reason for the inversion of colors in the two bows may be seen in the fact, that, in the primary bow, the rays which descend to the observer's eye, must emerge from the lower or inner quadrant of the drop, and be bent *upward* (*outward*) from the radius produced; while, in the secondary, they must emerge from the upper, or outer, quadrant, and be bent from the radius *downward*. The ray VO (Fig. 306), of the primary, being supposed a violet ray, is the most refrangible, and, therefore, all other rays from that drop fall below it, and do not reach the eye. To bring other colors to O, drops must be selected higher up; hence, violet is the color seen nearest the axis. In the secondary bow, if V′O is the violet ray, the other colors, being bent in a less degree from the radius of the drop, lie above V′O; and, therefore, in order that other colors may reach O, they must emerge from lower drops, i. e. drops nearer the axis. Hence, violet is the outer color of the secondary bow.

729. *Rainbows, the colored borders of illuminated segments of the sky.*—The primary bow is to be regarded as the *outer edge* of that part of the sky, from which rays can come to the eye, after suffering but *one* reflection in drops of rain; and the secondary bow is the *inner edge* of that part from which light, after being *twice* reflected, can reach the eye.

It is found by calculation, that in case of one reflection, the incident and emergent rays can make no inclinations with each other, greater than 42° 2′ for red light, and 40° 17′, for violet; but the inclinations may be less, in any degree, down to 0°. Therefore, all light, once reflected, comes to the eye from *within* the primary bow.

But the angles, 50° 59′ and 54° 9′, are, by calculation, the *least* deviations of red and violet light from the incident rays, after *two* reflections. But the deviations may be greater than these limits, up to 180°. Therefore, rays twice reflected can come to the eye from any part of the sky, except between the secondary bow and its center.

It appears then, that from the zone lying between the two bows, no light, reflected by drops internally, either once or twice, can possibly reach the eye. Observation confirms this; when the bows are bright, the rain within the primary is more luminous than elsewhere; and outside of the secondary bow, there is more illumination than between the two bows, where the cloud is perceptibly darkest.

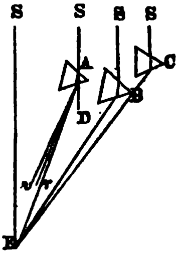
730. *The tertiary bow.*—A *tertiary* bow, or a bow formed by light three times reflected in drops of rain, is on the same side of the sky with the sun, and distant about 40° 40′ from it. The incident rays, which form it, enter the drops about 77°

from their axis, and emerge on the back side. But this order of bow is so very faint from repeated reflections, and so unfavorably situated, that it is very rarely seen.

731. *The common halo.*—This, as usually seen, is a white or colored circle of about 22° radius, formed around the sun or moon. It might, without impropriety, be termed the *frost-bow*, since it is known to be formed by light refracted by crystals of ice suspended in the air. It is formed, when the sun or moon shines through an atmosphere somewhat hazy. About the sun, it is a white ring, with its inner edge red, and somewhat sharply defined, while its outer edge gradually shades off into the light of the sky. Around the moon, it differs only in showing little or no color on the inner edge.

732. The phenomenon is produced by light passing through crystals of ice, whose cross-section is an equilateral triangle. Let the eye be at E (Fig. 307), and the sun in the direction ES. Let SA, SB, &c., be rays striking upon such crystals as may happen to lie in a position to refract the light toward SE as an axis. Each crystal is a prism of 60°, which turns the ray from the refracting edge, on entering; and again, on leaving, it is bent still more, and decomposition has taken place. The color, which would come from each one to the eye E, would depend on its angular distance from ES, and the position of its refracting angle. The angle of deviation for A, is EAD = SEA; for B, it is SEB, and so on. It is found by calculation, that the least deviation for red light is 21° 45′; the least for orange must be a little greater, because it is a little more refrangible, and so on, for the colors in order. The greatest deviation for the rays generally is about 43° 13′. All light, therefore, which can be transmitted by such crystals, must come to the observer from points somewhere between these two limits, 21° 45′ and 43° 13′ from the sun. But by far the greater part of it, as ascertained by calculation, passes through near the least limit.

FIG. 307.



733. *Its circular form.*—What takes place on one side of ES, may occur on every side; or, in other words, we may suppose the figure revolved about ES, as an axis, and then the transmitted light will appear in a ring about the sun S. The inner edge of the ring is red, since that color deviates least; just outside of the red, the orange mingles with it; beyond that, are the red, orange, and yellow combined; and so on, till, at the minimum angle for violet, all the colors will exist (though not in equal proportions), and the violet will be scarcely distinguishable from white. Beyond this narrow colored band, the halo is white, growing more and more faint, so that its outer limit is not discernible at all.

734. *The halo, a bright border of an illuminated zone.*—As in the rainbow, so in the halo, the visible band of colors is only the border of a large illuminated space on the sky. The ordinary halo, therefore, is the bright inner border of a zone, which is more than 20° wide. The whole zone, except the inner edge, is too faint to be noticed, though it is perceptibly more luminous than the space between the halo and the luminary.

735. *Frequency of the halo.*—The halo is less brilliant and beautiful, but far more frequent, than the rainbow. Scarcely a week passes during the whole year, in which the phenomenon does not occur. In summer, the crystals are three or four miles high, above the limit of perpetual congelation. As the rainbow is sometimes seen in dewdrops on the ground, so the frostbow, just after sunrise, has been noticed in the crystals which fringe the grass.*

736. *The mock sun.*—The mock sun, or sun-dog, is a short arc of the halo, occasionally seen at 22° distance, on the right and left of the sun, when near the horizon. The crystals, which are concerned in producing the mock sun, are supposed to have the form of *spiculæ*, or three-sided *needles*; these, if suspended in the air in a vertical position, could refract the light only in directions nearly horizontal, and therefore present only the right and left sides of the halo.

In high latitudes, other and complex forms of halo are frequent, depending for their formation on the prevalence of crystals of other angles than 60°.

THE ANGULAR SIZE OF RAINBOWS AND HALO, DETERMINED BY THE CALCULUS.

737. *Primary bow.*—To find the angle of maximum deviation. In Fig. 304, let x = angle of incidence; y = angle of refraction; z = angle of deviation; n = index of refraction. Then, in the quadrilateral BDGK, $DBK = DGK = x - y$; angle at $D = 360 - 2y$; $\therefore K = z = 4y - 2x$;

$$\therefore \frac{dz}{dx} = \frac{4dy}{dx} - 2 = 0.$$

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* Amer. Journ. Sci., vol. xlix. p. 78.

But $\sin x = n \sin y$;

$$\therefore \cos x\, dx = n \cos y\, dy, \text{ and } \frac{dy}{dx} = \frac{\cos x}{n \cos y}.$$

By substitution, $\dfrac{4 \cos x}{n \cos y} = 2$.

$$\therefore 2 \cos x = n \cos y; \text{ and } 4 \cos^2 x = n^2 \cos^2 y.$$

But $\sin^2 x = n^2 \sin^2 y$;

$$\therefore 3 \cos^2 x + 1 = n^2; \text{ since } \sin^2 + \cos^2 = 1.$$

$$\therefore \cos x = \sqrt{\frac{n^2 - 1}{3}}.$$

If 1.33 and 1.55, the values of n for extreme red and violet, be used in this formula, we obtain x, and therefore y and z, for the limiting angles of the primary bow.

738. *Secondary bow.*—To find the angle of minimum deviation. Using the same notation as before, we have in the pentagon GEDBK (Fig. 305), $G = B = 180 - x + y$; $E = D = 2y$; $\therefore K = z = 180 + 2x - 6y$;

$$\therefore \frac{dz}{dx} = 2 - \frac{6dy}{dx} = 0.$$

$$\therefore \frac{6 \cos x}{n \cos y} = 2; \text{ and } 3 \cos x = n \cos y;$$

$$\therefore 9 \cos^2 x = n^2 \cos^2 y;$$

but $\sin^2 x = n^2 \sin^2 y$;

$$\therefore 8 \cos^2 x + 1 = n^2;$$

$$\therefore \cos x = \sqrt{\frac{n^2 - 1}{8}};$$

which, as before, will furnish z for each limiting color of the secondary bow.

739. *Halo.*—To find the angle of minimum deviation in the common halo. Instead of Fig. 307, use Fig. 281, where all the angles are marked by letters. Let DE be the ray from the sun, and FG the emergent ray. Let $DEp = x$; $KEF = y$; $KFE = x'$; $GFp = y'$; $I = z = x - y + y' - x'$. Now, $y + x' = p'KF = C = 60°$.

$$\therefore z = x + y' - C.$$

$$\sin x = n \sin y,$$

and $\sin y' = n \sin x'$;

$$\therefore x = \sin^{-1}(n \sin y),$$

and $y' = \sin^{-1}(n \sin x') = \sin^{-1}\{n \sin (C - y)\}$.

By substitution, $z = \sin^{-1}(n \sin y) + \sin^{-1}\{n \sin (C - y)\} - C$.

Therefore z is a function of y; and, by differentiating, we have

$$\frac{dz}{dy}=\frac{n\cos y}{\sqrt{1-n^2\sin^2 y}}-\frac{n\cos(C-y)}{\sqrt{1-n^2\sin^2(C-y)}}=0.$$

$$\therefore\ \frac{n^2\cos^2 y}{1-n^2\sin^2 y}=\frac{n^2\cos^2(C-y)}{1-n^2\sin^2(C-y)};$$

$$\therefore\ \frac{1-\sin^2 y}{1-n^2\sin^2 y}=\frac{1-\sin^2(C-y)}{1-n^2\sin^2(C-y)};$$

$$\therefore\ (n^2-1)\sin^2 y=(n^2-1)\sin^2(C-y);$$

$$\therefore\ y=C-y,\text{ and } y=\tfrac{1}{2}C;$$

$$\text{and } x'=\tfrac{1}{2}C.$$

Hence, the minimum deviation occurs, when the ray within the crystal is equally inclined to the sides. Knowing n, the index of refraction for ice, x, and its equal, y', can be obtained, and then z, the deviation required.

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CHAPTER VI.

COLOR, BY REFLECTION, INFLECTION, STRIATION OF SURFACE, AND THIN PLATES.

740. *Natural colors of bodies.*—The colors which bodies exhibit, when seen in ordinary white light, are owing to the fact that they decompose light, by absorbing or transmitting some colors and reflecting the others. We say that a body *has* a certain color, whereas it only *reflects* that color; a flower is called red, because it reflects only or principally red light; another, yellow, because it reflects yellow light, &c. A white surface is one which reflects all colors in their due proportion; and such a surface, placed in the spectrum, assumes each color perfectly, since it is capable of reflecting all. A substance which reflects no light, or but very little, is black. What peculiarity of constitution that is, which causes a substance to reflect a certain color, and to absorb others, is unknown.

Very few objects have a color which exactly corresponds to any color of the spectrum. This is found to result from the fact that most bodies, while they reflect some one color chiefly, reflect the others in some degree. A red flower reflects the red light abundantly, and perhaps some rays of all the other colors with the red. Hence there may be as many shades of red as there can be different proportions of other colors intermingled with it. The same is true of each color of the spectrum. Thus there is an infinite variety of tints in natural objects. These facts are readily established by using the prism to decompose the light which bodies reflect.

741. *Inflection, or diffraction of light.*—This phenomenon consists of delicate *colored fringes bordering the edges of shadows*, when the light comes from a luminous point or line.

For the purpose of experiments on this subject, a beam of light is admitted into a dark room, through a very small aperture, as a pin-hole made in sheet-lead; or, what is better, a convex lens is placed in the window-shutter, which brings the rays to a focus, and affords a divergent pencil of light. If we introduce into this pencil any opaque body, as a knife-blade, for example, and observe the shadow which it casts on a white screen, we shall observe on both sides of the shadow *fringes of colored light*, the different colors succeeding each other in the order of the spectrum, from violet to red. Three or four series can usually be discerned, the one nearest to the shadow being the most complete and distinct, and the remoter ones having fewer and fainter colors. The phenomenon is independent of the density or thickness of the body which casts the shadow. The light, in passing by the edge or back of a razor, by a block of marble or a bubble of air in glass, is in each case affected in the same way. But if the body is very narrow, as, for example, a fine wire, a modification arises from the light which passes the opposite side; for now fringes appear *within* the shadow, and at a certain distance of the screen, the central line of the shadow is the most luminous part of it.

742. If, in the foregoing experiments, we use light of one color alone, instead of white light, then the fringes are only of that color, separated from each other by lines which are comparatively dark; and, on measuring the breadths and distances of fringes of different colors, those of *red* light are found to be widest, those of *violet*, narrowest, and the other colors have breadths according to their order. This explains why, in the case of white light, the several colors appear in a series, with the red outermost; for each element of the white light forms its own system of fringes, but the systems do not coincide—the wider ones project beyond the narrower, and thus become separately visible.

If the screen is removed from the body, the distance of a given color from the edge of the shadow becomes greater, but not in proportion to the distance of the screen from the body, which proves that the color is not propagated in a straight line, but in a curve. These curves are found to be *hyperbolas*, having their concavity on the side next the shadow, and are in fact a species of caustics.

743. The phenomena of inflection are exhibited in a more interesting manner, when we view with a magnifying glass a pencil of light, after it has passed through a small aperture. For instance, in the cone already described as radiating from the focus of a lens in a dark room, let a plate of lead be interposed, having a pin-hole pierced through it, and let the slender pencil of light which passes through the pin-hole fall on the magnifier. The aperture will be seen as a luminous circle surrounded by several rings, each consisting of a prismatic series. These are, in truth, the fringes formed by the edge of the circular puncture, but they are modified by the circumstance that the opposite edges are so near to each other. If, now, the plate be removed, and another interposed having *two* pin-holes, within one-eighth of an inch of each other, besides the colored rings round each, there is the additional phenomenon of long lines crossing the space between the apertures; the lines are nearly straight, and alternately luminous and dark, and varying in color, according to their distance from the central one. These lines are wholly due to the overlapping of two pencils of light, for on covering one of the apertures, they entirely disappear. By combining circular apertures and narrow slits in various patterns in the screen of lead, very brilliant and beautiful effects are produced.

744. *Why inflection is not always noticed in looking by the edges of bodies.*—It must be understood, that light is *always* inflected when it passes by the edges of bodies; but that it is rarely observed, because, as light comes from various sources at once, the colors of each pencil are overlapped and reduced to whiteness by those of all the others. By using care to admit into the eye only isolated pencils of light, some cases of inflection may be observed which require no apparatus. If a person, standing at some distance from a window, holds close to his eye a book or other object having a straight edge, and passes it along so as to come into apparent coincidence with the sash-sticks of the window, he will notice, when the edge of the book and the stick are very nearly in a range, that the latter is bordered with colors, the violet extremity of the spectrum being on the side next to the book, and the red extremity on the other side. Again, the effect produced, when light passes through a narrow aperture, may be seen by looking at a distant lamp through the space between the bars of a pocket-rule, or between any two straight edges brought almost into contact. On each side of the lamp are seen several images of it, growing fainter with increased distance, and finely colored. An experiment still more interesting is to look at a distant lamp through the net-work of a bird's feather. There are several series of colored images, having a fixed arrangement in relation to the disposition of the minute apertures in the feather; for the system of images revolves just as the feather itself is revolved.

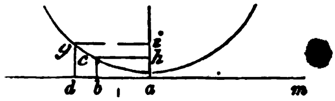
745. *Striated surfaces.*—If the surface of any substance is ruled with fine parallel grooves, 2000 or more to the inch, it will reflect bright colors when placed in the sunbeam. *Mother-of-pearl*, and many kinds of sea-shell exhibit colors on account of delicate striæ on their surface. These are the edges of thin laminæ, which compose the shell, and which crop out on the surface, in fine and nearly parallel lines. It may be known that the color arises from such a cause, if, when the substance is impressed on fine cement, its colors are communicated to the cement. Indeed, it was in this way that Dr. Wollaston accidentally discovered the true cause of such colors. The changeable hues in the plumage of some birds, and the wings of some insects, are owing to a striated structure of their surfaces. But the metals can be made to furnish the most brillant spectra, by stamping them with steel dies, which have been first ruled by a diamond, with lines from 2000 to 10,000 per inch, and then hardened. Gilt buttons and other articles for dress are sometimes prepared in this manner, and are called *iris ornaments.* The color in a given case depends on the distance between the grooves, and the obliquity of the beam of light. Hence, the same surface, uniformly striated, may reflect all the colors, and every color many times, by a mere change in its inclination to the beam of light.

746. *Thin laminæ.*—Any transparent substance, when reduced in thickness to a few millionths of an inch, reflects brilliant colors, which vary with every change of thickness. Examples are seen in the thin laminæ of air, occupying cracks in glass and ice, and the interstices between plates of mica, also in thin films of oil on water, and alcohol on glass, but most remarkably in soapy water blown into very thin bubbles.

If a lens of slight convexity is laid on a plane lens, and the two are pressed together by a screw, and viewed by reflected light, rings of color are seen arranged around the point of contact. The smallest rings are broadest and most brilliant, and each one contains the colors of the spectrum in their order, from violet on the inner edge, to red on the outer. But the larger rings not only become narrower and paler, but contain fewer colors; yet the succession is always in the same order as above. Increased pressure causes the rings to dilate, while new ones start up at the center, and enlarge also, until the center becomes black, after which no new rings are formed. These are commonly called Newton's rings, because Sir Isaac Newton first investigated their phenomena.

747. A given color appears in a circle around the point of contact, because equal thicknesses are thus arranged. If the diameters of the successive rings of any one color be carefully measured, their squares are found to be as the odd numbers, 1, 3, 5, 7; and hence the thicknesses of the laminæ of air at the repetitions of the same color, are as the same numbers. For, let Fig. 308 represent a section of the spherical and plane surfaces in contact at *a*. Let *ab*, *ad*, be the radii of two rings at their brightest points. Suppose *ai*, perpendicular to *mn*, to be produced till it meets the opposite point of the circle of which *ag* is an arc, and call that point *f*; then *af* is the diameter of the sphere of which the lens is a segment. Let *be*, *dg* be parallel to *ai*, and *eh*, *gi*, to *mn*, then we have

FIG. 308.



$$(eh)^2 : (gi)^2 :: ah \times hf : ai \times if.$$

But the distances between the two lenses being exceedingly small in comparison with the diameter *af*, *hf* and *if* may be taken as equal to *af*, whence, by substitution,

$$(eh)^2 : (gi)^2 :: ah \times af : ai \times af :: ah : ai :: be : dg.$$

748. *Thickness of laminæ for Newton's rings.*—The absolute thickness, *be*, *gd*, &c., can also be obtained, *af* being known, since

$$af : ae :: ac : ah \text{ or } be;$$

for in so short arcs, the chord may be considered equal to the sine, that is, the radius of the ring. The conclusions of Sir Isaac Newton (who was the earliest investigator of the colors of thin laminæ), were, that when air was between the lenses, all the rings ranged between the thickness of *half a millionth* of an inch, and 72 *millionths;* if water was used, the limits were $\frac{3}{8}$ *of a millionth*, and 58 *millionths*. Below the smaller limit, the medium appears black, or no color is reflected; above the highest limit, the medium appears white, all colors being reflected together. When water is substituted for air, all the rings contract in diameter, indicating that a particular order of color requires less thickness of water than of air; the thicknesses for different media were found by Newton to be in the inverse ratio of the indices of refraction.

749. *Relation of rings by reflection and by transmission.*—If the eye is placed beyond the lenses, the transmitted light also is seen to be arranged in very faint rings, the brightest portions being at the same thicknesses, as the darkest ones by reflection; and these thicknesses are as the even numbers, 2, 4, 6, &c. The center, when black by reflection, is white by transmission, and where red appears on one side, blue is seen on the other; and, in like manner, each color by reflection answers to its *complementary color* (or that which added to it would make white), by transmission.

750. *Newton's rings by a monochromatic lamp.*—The number of reflected rings seen in common light is not usually greater than from *five* to *ten*. The number is thus small, because as the outer rings grow narrower, the different colors overlap each other, and produce whiteness. But if a light of only one color falls on the lenses, the number may be multiplied to several hundreds; the rings are alternately of that color and black, growing more and more narrow at greater distances, till they can be traced only by a microscope. A good light for such a purpose is the flame of an alcohol lamp, whose wick has been soaked in strong brine, and dried.

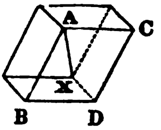
CHAPTER VII.

DOUBLE REFRACTION AND POLARIZATION.

751. *Double refraction.*—There are many transparent substances, particularly those of a crystalline structure, which, instead of refracting a beam of light in the ordinary mode, *divide it into two beams.* This effect is called *double refraction*, and substances which produce it are called *doubly-refracting substances.*

This phenomenon was first observed in a crystal of carbonate of lime, denominated *Iceland spar.* This substance may be seen in every cabinet of minerals, presenting the figure of a rhomb. It is a solid, bounded by six rhomboidal faces. It is colorless and highly transparent, and distinguished for its beauty in mineralogical collections; but its most remarkable property is that of rendering letters, or any other small objects placed behind it, *double.* Double refraction is, however, not confined to this substance; it takes place in numerous transparent crystals. It also occurs in a variety of other bodies, which are more or less transparent, where there is any disposition toward a regular arrangement of the particles,

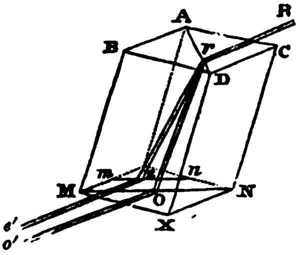
Fig. 302.



such as hair, quills, and the like, and in all bodies when in a state of unequal dilatation or compression.

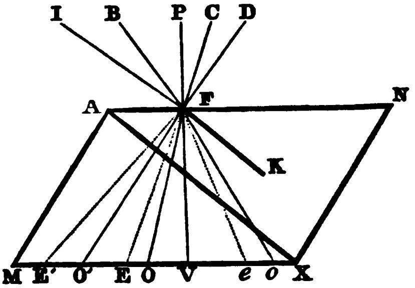
If a rhomb of Iceland spar, represented in Fig. 310, be placed above a black line drawn on white paper, and viewed with the eye at R, the line will appear double, as *mn*, MN; or if we cause a pencil of light R*r*, to fall upon the surface of the rhomb, it will be separated into two pencils. *r*O, *r*E, which will emerge from the rhomb at points O and E in directions O*o*′, E*e*′, parallel to R*r*. The pencil R*r* has therefore suffered *double refraction* in passing through the rhomb.

FIG. 310.



752. *Ordinary and extraordinary rays.*—To illustrate the manner in which these effects take place, let AMXN (Fig. 311), be a section of the Iceland crystal (Fig. 309), made by a plane passing through the diagonal AX. Let PV be a perpendicular to the upper surface of the crystal at any point F. Then rays of light DF, CF, incident in the same plane, will each have two refracted rays, FO, FO′, and FE, FE′. On measuring the several angles, it will be found that the two rays FO, FO′, follow the ordinary law of refraction, making the sines of the angles of refraction as the sines of those of incidence. Each of these, therefore, is called the *ordinary ray*. But the other rays FE, FE′, do not conform to that law, but make angles with the perpendicular, that are sometimes greater and sometimes less than would be required by it, and are thus separated from the ordinary rays. Hence, each of these is called the *extraordinary ray*. In the case of the Iceland crystal, while at a perpendicular incidence the ordinary ray undergoes no refraction, the extraordinary ray is turned from the perpendicular 6° 12′; and at angles of 10°, 20°, 30°, &c., while the ordinary ray suffers a regular refraction according to the law of the sines, the extraordinary ray suffers a refraction sometimes greater than that, and sometimes less, according to its relations to AX.

FIG. 311.



753. *Positive and negative axis.*—The line which connects the two obtuse angles of the rhomboid, as AX (Fig. 311), is called the *optic axis*, since when a ray enters a crystal in the direction of this line, either coincident with it or parallel to it, such a ray suffers no double refraction. Thus the ray IF, which meets the crystal in such a direction as to be refracted into the line FK, parallel to AX, is not separated into two rays, but merely undergoes ordinary refraction. When the extraordinary ray makes with the axis FK, or AX, a larger angle than the ordinary ray, as in Fig. 311, the crystal is said to have a *negative* or *repulsive* axis; while crystals of a *positive* or *attractive* axis are those in which the extraordinary ray makes a smaller angle with the axis than the ordinary ray does. Any plane, like that of Fig. 311, passing through the optic axis, and every plane parallel to this, is called the *principal section*. Whenever the principal section contains the ordinary ray, it also contains the extraordinary ray, which is not the case with planes inclined to that.

An axis of double refraction, however, is not like the axis of the earth, a *fixed line*, within the rhomb or crystal. It is only a *fixed direction;* for if we divide, as we may do, the rhomb ABC (Fig. 309), into two or more rhombs, each of these separate rhombs will have its axis of double refraction; but when these rhombs are again put together, their axes will all be parallel to AX. Every line, therefore, within the rhomb parallel to AX, is an axis of double refraction, or optic axis; but as these lines have all one and the same direction in space, the crystal is still said to have only one optic axis.

754. *Two or more axes.*—A great number of crystals have *two* axes of double refraction, or two directions inclined to each other, along which the double refraction is nothing. In crystals with one axis, the axis has the same position, whatever be the color of the pencil of light which is used; but in crystals with two axes, the axes change their position according to the color of the light employed, so that the inclination of the two axes varies with differently colored rays.

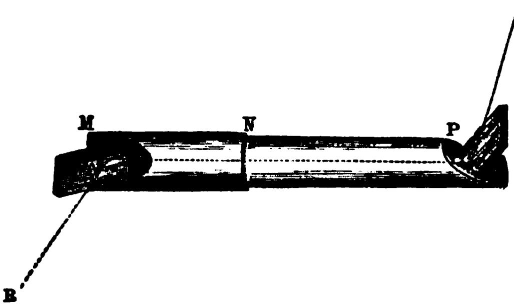
Until recently it was supposed that the number of optic axes never exceeds two; but Dr. Brewster has lately discovered an example of a mineral (*analcime*) which has an indefinite number of axes of double refraction, in the direction of which, light suffers no separation, although when passing through the body in any other direction, it undergoes double refraction.

A cylinder of glass, first heated red hot, and then rolled on a plate of metal until it is cold, acquires a permanent doubly-refracting structure. If, instead of heating the glass cylinder, we had placed it in a vessel, and surrounded it with boiling oil or boiling water, it would have acquired the same doubly-refracting structure, when the heat had reached the axis; but this structure is only transient, as it disappears when the cylinder is uniformly heated. Analogous structures may be produced by pressure, and by the induration of soft solids, such as animal jellies, isinglass, &c.

755. *Polarization of light.*—This name is given to a change which may be produced in light, such that it has different properties on different sides. *Common light*, as, for instance, a direct sunbeam, has the same relation to space on all sides. If it falls on a piece of glass at a given angle, it will suffer reflection equally well in every plane, as we turn the glass round, and so of refraction, or any change we may attempt. But if a beam could be reflected upward, but could not be reflected to the right, it would be called, not *common*, but *polarized* light.

756. *Polarization by reflection.*—The first method of polarizing to be described, is *by reflection.* Let two tubes, MN and NP (Fig. 312), be fitted together in such a manner that one can be revolved upon the other; and to the end of each let there be attached a plate of dark-colored glass, A and C, capable of reflecting only from the first surface. These plates are hinged so as to be adjusted at any angle with the axis of the tube. Let the plane of each glass incline to the axis of the tube at an angle of 33°, and let the beam RA make an incidence of 57°, the complement of 33°, on A; then it will, after reflection, pass along the axis of the tube, and make the same angle of incidence on C. If now the tube NP be revolved, the second reflected ray will vary its intensity, according to the angle between the two planes of incidence on A and C. The beam AC is *polarized light;* the glass A, which has produced the polarization, is called the *polarizing plate;* the glass C, which shows, by the effects of its revolution, that AC is polarized, is the *analyzing plate;* and the whole instrument, constructed as here

FIG. 312.



represented, or in any other manner for the same purpose, is called *a polariscope.*

757. The changes on the ray CE are as follows. When the tube NP is placed so that the plane of incidence on C is coincident with the former plane of incidence, RAC, whether CE is reflected forward or backward in that plane, the intensity at E will be the same as if AC had been a beam of common light. If NP is revolved, E will begin to grow fainter, and reach its minimum of intensity when the planes, RAC and ACE, are at right angles, which is the position indicated in the figure. Continuing the revolution, we find the intensity increasing through the second quadrant of revolution, and reaching its maximum, when the two planes of incidence again coincide, 180° from the first position. The next half revolution repeats these changes in the same order.

758. The angle of 57° is called the polarizing angle for glass, not because glass will not polarize at other angles of incidence, but because at all other angles it polarizes the light in a less degree; and this is indicated by the fact that, in revolving the analyzing plate, there is less change of intensity, and the light at E is not so faint. Different substances have different polarizing angles, and these are found to be so connected with the degree of refractive power, that by a knowledge of the index of refraction for any substance, its polarizing angle can be calculated, and *vice versa.* Hence the refractive power of opaque bodies may be determined. No substance *entirely* polarizes the light incident upon it, even at the angle of polarization. Complete polarization of the ray AC would be indicated by the entire extinction of CE, at two opposite points of its revolution. On the other hand, every substance polarizes, in *some* degree, the light which it reflects. The polarization produced by reflection from the metals is very slight.

759. *Polarization by a bundle of plates.*—Light may also be polarized by *transmission* through a bundle of laminæ of a transparent substance, at an angle of incidence equal to its polarizing angle. Let a pile of twenty or thirty plates of transparent glass, no matter how thin, be placed in the same position as the reflector A, in Fig. 312, and a beam of light be sent through them to C. When C is revolved, the beam will undergo the same changes as before, with this difference, that the places of greatest and least intensity will be reversed. If RAC is the plane of incidence in which the original beam is transmitted through the bundle, then the light reflected from C in that plane, either forward or backward, will be the most feeble, and it will be brightest at E, and 180° from E.

760. *Polarization by crystals.*—The third and most perfect method of polarizing light, is by *transmission through certain crystals. Some* crystals, which produce single refraction, polarize the transmitted ray; and *every* doubly-refracting crystal polarizes both the ordinary and the extraordinary ray. If a thin plate be cut from a crystal of tourmaline, by planes parallel to its axis, the beam transmitted through it is polarized, and, when received on the analyzing plate, will alternately become bright and faint, as the tube of the analyzer is revolved. And, if a beam is passed through a doubly-refracting crystal, and the two parts fall on the analyzing plate, they will come to their points of greatest and least brightness at alternate quadrants; indeed, when one ray is brightest, the other is entirely extinguished. Therefore, the two rays which emerge from a doubly-refracting crystal are polarized *completely*, and in planes at right angles with each other.

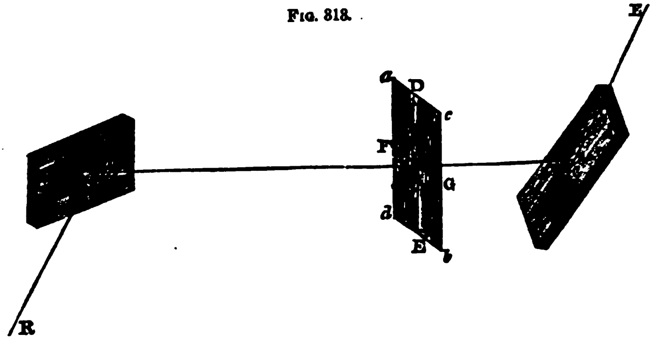
761. *Every polarizer an analyzer.*—We have seen that light is polarized by reflection from glass at an incidence of 57°, and analyzed by another plate at the same angle of incidence. This is but an instance of what is always true, that every method of polarizing light may be used to analyze, i. e. to test its polarization. Hence, a bundle of thin plates of glass may take the place of the analyzer, C, as well as of the polarizer, A. For, on turning it round, though the transmitted beam remains in the same place, yet it will, at the alternate quadrants, brighten to its maximum, and fade to its minimum of intensity.

So, again, if light has passed through a tourmaline, and is received on a second, whose crystalline axis is parallel to that of the former, the ray will proceed through that also; but if the second is turned in its own plane, the transmitted ray grows faint, and nearly disappears at the moment when the two axes are at 90° of inclination, and this alternation continues at each 90° of the whole revolution.

Finally, place a double-refractor at each end of the polariscope, and let a beam pass through them and fall on a screen. The first crystal will polarize each ray, and the second will doubly refract and also analyze each, exhibiting a very interesting series of changes. In general, four rays will emerge from the second crystal, producing four luminous spots on the screen. But, on revolving the tube, not only do the rays commence a revolution round each other, but two of them increase in brightness, and the other two at the same time diminish as fast, till two alone are visible, at their greatest intensity. At the end of the second quadrant, the spots before invisible are at their maximum of brightness, and the others are extinguished. This alternation continues as long as the crystal is revolved. In the middle of each quadrant, the four are of equal brightness.

762. *Color by polarized light.*—The phenomena of *color* produced by polarized light, are beautiful, and of great interest. Let a very thin plate of some doubly-refracting crystal, as a thin film of mica, be placed perpendicularly across the axis of the polariscope (Fig. 313), and let the analyzed ray, CE, fall on a screen. When the principal section of the crystal, ED,

FIG. 313.



coincides with the first plane of reflection, RAC, or is perpendicular to it, all the phenomena are the same as if no crystal was interposed. But let the film be revolved 45° in its own plane, till *ab* or *cd* is in the plane RAC; then, instead of the dark spot at E, a brilliant color appears. That color may be any tint of the spectrum, according to the thickness of the interposed film. If now the revolution of the crystal is continued, the color fades out at the end of the next 45°, reappears at 90°, and so on. But if *ac* be fixed in the plane RAC, and the analyzing plate be revolved, a different series presents itself. A brilliant color is observed at E; during the first 45°, this color fades, and during the next 45°, its *complement* appears and brightens to its maximum. The original color is restored at 180°, and the complementary color, at 270°.

The most interesting form of this experiment is seen when the light is polarized and analyzed by means of double-refractors; since the polarization is more perfect, and the two pairs of oppositely polarized rays are on the screen at once. When two of the images are of a certain color, the other two have the complementary color.

763. *Systems of colored rings.*—Systems of irised bands and rings may also be produced by the polariscope. Let a plate be cut from a doubly-refracting crystal of one axis, by planes perpendicular to that axis; and place it between the polarizer and analyzer. If now a pencil of sufficient divergency is transmitted, a system of colored circles will be formed, resembling Newton's rings between lenses. If a polariscope is formed of two tourmalines, and the crystal laid between them, and the whole combination, less than half an inch thick, is brought close to the eye, the pencil of light will consist of rays of various obliquity, and the rings may be seen beautifully projected on the sky. Or the ring systems may be projected on a screen by a polariscope furnished with concentrating lenses. Fig. 314 presents the system as seen through Iceland spar, when the planes of reflection in the polariscope are at right angles. Two dark diameters cross the system and interrupt the rings. If the planes of reflection are coincident, the system is in every respect complementary to the other. (Fig. 315.) The colors of the rings are all reversed, and the crossing bands are white. If double-refractors of two axes are used instead of the spar, compound systems are formed, of various forms and great beauty.

FIG. 314.

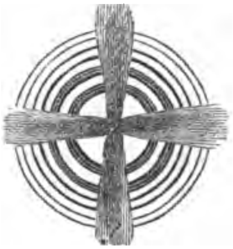
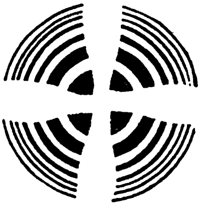


FIG. 315.



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CHAPTER VIII.

NATURE OF LIGHT. WAVE THEORY.

764. *The two theories.*—Light may be regarded as consisting of material particles emanating from luminous bodies, or as a system of vibrations excited by luminous bodies, and propagated through a medium filling space. The former is called the *corpuscular* or *emission theory*, and was advocated by Newton; the latter, the *undulatory*, or *wave theory*, adopted by Huygens, afterward revived by Young, and now received by philosophers generally, as alone adequate to explain consistently the more recently discovered properties of light.

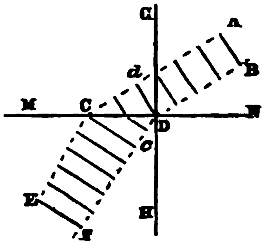
765. *The wave theory.*—This supposes that a delicate and elastic *ether* fills all space, even that apparently occupied by other matter, and that bodies called luminous possess the power of exciting vibrations in the ether, which vibrations are communicated in all directions at the rate of 192,500 miles per second. It also assumes, that the density and elasticity of the ether is different in different media, and that in some substances of crystalline structure, the elasticity varies according to direction in the same medium.

The mode of vibration is not *longitudinal*, like that of air in conveying sound, but *transverse* or perpendicular to the line of progress; and in common, or unpolarized light, it is transverse *in all directions*, within an inconceivably short space of time. This peculiar mode of oscillation was not at first contemplated by the advocates of the theory; but the phenomena of polarization require this as an essential condition.

766. Another postulate of the wave theory is, that difference of *color* is due to difference of *rate* in the undulations of the ether, just as *pitch* in sound depends on the rate of aerial vibrations. *Violet* is the color produced by the most rapid vibrations, corresponding to a *high tone;* and *red*, by the slowest, corresponding to a *low tone;* and other colors are produced by intermediate rates. The velocity of high and low tones in the air is the same: in like manner, all colors are known to travel through space with the same velocity. Hence, the most rapid undulations imply the shortest waves. The *shortest* waves, therefore, affect the eye with the sensation of *violet*, and the *longest* with that of *red*. Again, *white* light is to the eye what harmony is to the ear, the resultant effect of several rates of vibration combined.

767. *Reflection and refraction on the wave theory.*—When the waves of light reach the surface of a new medium, the ether within it being generally in a different state of elasticity and density, a system of waves will be propagated backward in the former medium, and another onward in the new medium. The reflected system will make the same angle with the perpendicular as the incident system, analogous to the reflection of waves of water and of sound. But the system which enters the medium, will change its direction, according to its velocity in the medium; and the velocity depends on the elasticity of the ether. In media of greater refractive power, the elasticity is considered to be less than in those of less refractive power; and the waves are therefore propagated more slowly in the former than in the latter. Let AB, *d*D (Fig. 316) represent the parallel waves of a beam falling on MN, the surface of a denser medium. The side of the wave which enters first

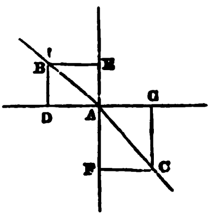
FIG. 316.



at D, advances more slowly than the side still moving in the rarer medium. Suppose D to reach c, while d is going to C; then the wave, now wholly within the medium, lies in the position Cc, and advances in a line perpendicular to Cc, so long as it continues in the medium. Thus the light is refracted *toward* the perpendicular GH, in entering a denser medium. In a similar manner, it is shown, that EFCc, in entering a rarer medium, is refracted *from* the perpendicular, since the side Cd emerges first, and then gains velocity over the side cD.

768. It appears, therefore, that the wave theory requires us to suppose light to move more slowly in denser media. But, in the emission theory, it is necessary to suppose it to move more swiftly. For the bending of the path of a particle of light toward the perpendicular must be attributed to the attraction exerted by the medium on the particle. Suppose, then, that a particle of light moves along BA (Fig. 317), and enters a denser medium. Let the velocity, BA, be resolved into BE, EA; the latter will be increased by the attraction of the medium; the former will not be changed. Make AG = BE or DA, and AF greater than AE; then AC represents the direction and velocity of the ray after entering the medium. But as AF is greater than EA, while AG = DA, ∴ AC is greater than BA. On the other hand, if a ray, CA, is entering a rarer medium, the attraction of the denser draws it *backward*, and renders the component AE less than FA; and hence the velocity AB, in the rarer medium, is less than CA, the velocity in the denser. The two theories are thus in conflict on the question, whether light *gains* or *loses* velocity in entering a more refractive medium. Several direct tests have been applied in order to determine this; and they all unite in proving that light moves more slowly in substances which refract toward the perpendicular.

Fig. 317.



It appears that the shortest waves are most retarded in entering a denser medium, and the longest least retarded; because the violet is most refrangible, and red the least refrangible.

769. *Interference.*—Many interesting phenomena are explained on the principle of interference of waves. As two systems of water-waves may increase or diminish their height by being combined, and as sounds, when blended, may produce various results, and even destroy each other, so may two pencils of light either augment or diminish each other's brightness, and even produce darkness.

Any one may try for himself the following experiment. Prick two very small holes, quite near each other, through paper, and holding the paper close to the eye, look through the holes at any small bright spot, such as occurs in a crack of glass, when the sun shines upon it; then will the bright spot be seen striped across with parallel black lines, which will be further apart as the holes are closer together. The two pencils of light, through the two apertures, overlap on the retina of the eye, and cause bright and dark lines by interference. Where like phases meet, the lines are bright; where opposite phases meet there is *no* light, and the lines are black.

But there are other forms of experiment, which admit of accurate measurement, so that the exact length of wave for each color may be determined.

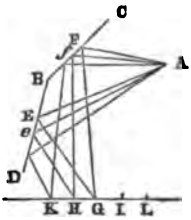
770. *By thin plates.*—Let light of any one color, as yellow, fall on the lenses which exhibit Newton's rings. A system of waves is reflected from the first surface of the thin stratum of air, and another system from the second, and these two come to the eye together. Suppose, at a given point, the thickness of air is such that the waves of the second system meet those from the first, phase for phase, in exact concert; at that point is seen a brighter yellow than if there was but one reflecting surface. But, at a contiguous point, the thickness may be such that the two systems disagree by half a wave, bringing opposite phases together; in which case all motion is destroyed, and the point is black. The former is one point of a yellow circle, the latter of a black circle, each around the point of contact. It is obvious, that at the smallest bright ring, the second system must be just *one* wave-length behind the other; at the second, *two* waves behind, &c.; and, in general, luminous circles appear where the two systems differ by an exact number of whole waves, and dark circles where they differ by half a wave, or any whole number and a half. The exact measurement of the thicknesses of air at any point (Art. 748), has led to the determination of the length of waves of each color.

771. If orange or red light is used, instead of yellow, the rings are a little enlarged, and therefore the waves for those colors are longer; but if green, blue, indigo, and violet are each tried separately, the rings grow smaller in each case; and it is inferred that the lengths of waves are less in the same order, and in the ratio of the thicknesses.

The reason becomes obvious why, in white light, the rings are few in number, and consist of a series of different colors, without any black circles between. As rings of different colors are of different sizes, when separate, so when all colors are used together they will be arranged side by side, and some will be likely to fall where the black circles between others would occur. Again, as all the rings grow narrower at greater distances, they crowd upon and overlap each other, and produce white light. Hence, a full prismatic series occurs only near the center, and after *five* or *ten* repetitions, growing less and less perfect, white light covers the whole surface.

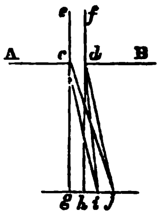
772. *By plane mirrors.*—If two plane reflectors, inclined at a very obtuse angle, receive light from a minute radiant, and reflect it to one spot on a screen, the reflected pencils will interfere, and produce bright and dark lines. Suppose light of one color, as violet, flows from a radiant point A (Fig. 318); let mirrors BC and BD reflect it to the screen KL. F and E may be so selected, that AF + FG = AE + EG, the sum of incident and reflected rays, on the two mirrors. Then G will be luminous, because the same phase of wave will occur at the same distance from the radiant A. But if H be so situated, that A*f* + *f*H differs *half a violet wave* from A*e* + *e*H, then H will be a dark point, because opposite phases meet there. A similar point, I, may lie on the other side of G. Again, there are two points, K and L, one on each side of G, to each of which the whole path of light by one mirror shall exceed the whole by the other, by just *one* violet wave; those points are bright. Thus, there is a series of bright and dark points on the screen; or a series of bright and dark *lines*, if a line of light is substituted for the radiant point. Other colors will give bands a little wider, indicating longer waves. And white light, producing all these results at once, will give a repetition of the prismatic series.

Fig. 318.



773. *By inflection.*—One of the forms of inflection is explained as follows. Through an opaque screen, AB (Fig. 319), let there be a very narrow aperture, *cd*, by which is admitted the beam of light, *efgh*, of some one color, and emanating from a single point. That part of the aperture near *d* may be regarded as a luminous center, from which emanate waves in all directions, but more feebly in lines more oblique from *dh*; and the same is true of the other part of the aperture near *c*. Let *i* be a point on one side of the beam, so situated that the distances *di* and *ci* shall differ by half a wave of the color employed; then, as opposite phases meet there, *i* will be a dark point. Let *j* be a point still further removed from the beam, where *cj* − *dj* equals the length of a wave, then *j* will be luminous, since like phases meet in that point. This alternation will be repeated a few times, till the luminous points become crowded and feeble. If the aperture is made narrower,

Fig. 319.



the intervals *hi*, *ij*, &c, will increase, as they obviously must, in order to preserve *ci—di* equal to a half wave, and *cj—dj* equal to a wave. Violet light produces the narrowest lines, red the widest, and white light the prismatic series, for the same reason as in Newton's rings. If A*c*, the left side of the screen, is entirely removed, so that light passes only one edge, *d*, the fringes will still exist, though somewhat modified.

774. The other cases of inflection, and the phenomena of *striation*, as well as the *supernumerary* rainbows, are fully accounted for on the principle of interference. The careful measurements, which have been made in nearly all these instances, have led, by so many independent methods, to the accurate determination of the length of a wave, and the consequent rate of vibration, in each color. The results are given in the following table.

Colors.	Length of a wave in decimals of an inch.	Number of vibrations per second.
Extreme red	.0000266	458,000,000,000,000
Red	.0000256	477,000,000,000,000
Orange............	.0000240	506,000,000,000,000
Yellow............	.0000227	535,000,000,000,000
Green.............	.0000211	577,000,000,000,000
Blue..............	.0000196	622,000,000,000,000
Indigo	.0000185	658,000,000,000,000
Violet	.0000174	699,000,000,000,000
Extreme violet......	.0000167	727,000,000,000,000
Mean	.0000225	541,000,000,000,000

775. *Change of the vibrations in polarized light.*—It has been remarked (Art. 745), that the vibrations of the ether, in the case of common light, must be supposed *transverse in all directions*. But, in accordance with the usual method of resolving motions, it is customary to regard the vibrations as transverse in *any one* assumed direction, and also in another at right angles to it. Thus, if a star in the zenith shines upon us, we may consider each atom of the ether, between it and the observer, to vibrate north and south, and also east and west, or in any other two horizontal directions at right angles with each other.

This being the nature of common light, it is easy to state what is meant by polarized light. It is that in which the vibrations are performed in only *one* of the transverse directions. For example, in the star-light just supposed, if all the easterly and westerly motion were destroyed, so that the ether retained its north and south motion, the light of that star would be called polarized. It is immaterial what particular transverse motion is cut off, provided all the motion at right angles to it is retained.

776. By recurring to Chap. VII., it will be seen that this effect may be produced on light in several ways.

1. *By reflection.*—The vibrations *in* the plane of incidence are in a greater or less degree weakened, while those *perpendicular to* this plane are unaffected. How much effect will be produced depends on the elasticity of the ether in the medium (as its refractive power does also), and on the angle of incidence.

It will now be readily understood how the analyzing plate (Fig. 312), proves the light to be polarized. Suppose the reflectors A and C are so perfect polarizers, that vibration in the plane of incidence is *entirely* destroyed. Along RA, the particles of ether vibrate across it both horizontally and vertically; and as the plane of incidence, RAC, is horizontal, the atoms along AC will vibrate only vertically. Now let C be placed so as to reflect horizontally; the light will not be weakened by this reflection, because there are no horizontal vibrations to be destroyed. But let C be turned, so as to reflect vertically, for instance, upward; now there can be no reflection, since all the vibrations left in AC are in the vertical plane, which is the plane of incidence; and they are destroyed. For the same reason that reflection at A extinguished all *horizontal* motions in the atoms of ether, the reflection at C extinguishes all *vertical* motions; hence there is *no* motion beyond C.

2. *By transmission through a bundle of plates.*—At each of the surfaces, some reflection occurs, and all vibrations in the plane of incidence at length disappear from the reflected ray, even though the laminæ are not *perfect* polarizers; while all vibrations perpendicular to this plane are preserved: hence, the reverse must be true of the transmitted ray; it will retain the vibrations, so far as they coincide with the plane of incidence, and lose them, so far as they are perpendicular to it. Thus, the two sets of rectangular vibrations are separated from each other; one exists in the reflected ray, the other in the transmitted ray. The two rays are polarized in planes at right angles to each other.

3. *By absorption.*—A tourmaline absorbs, or in some way extinguishes, the vibrations, so far as they are perpendicular to its crystalline axis, but leaves all motion, which is parallel to its axis, unimpaired. It is at once apparent, why a second tourmaline analyzes; for, if its axis is parallel to that of the first, the same vibrations which could pass the one, could pass the other also; but, if the two axes are at right angles, the same

system of vibrations which could pass the first, because parallel to its axis, will be absorbed by the second, because perpendicular to its axis.

4. *By double refraction.*—In doubly-refracting crystals, the ether possesses different degrees of elasticity in different directions; hence, so far as vibrations lie in *one* plane, they may be more retarded in their progress, and in a plane at right angles to that, they may be less retarded (Art. 767); thus, the two systems become separated, and emerge at different places. Each ray is of course polarized, having vibrations in only one direction; and each is polarized in a plane at right angles to that of the other.

777. Since the discovery was made, that the etherial atoms may by certain methods be thrown into circular movements, and by others, into vibrations in an ellipse with a fixed axis, the polarization already described has been called *plane polarization*, since the atoms vibrate in a plane. *Circular polarization* is that in which the atoms revolve in circles; and *elliptical polarization* denotes a state of vibration in ellipses, whose major axes lie in one plane.*

CHAPTER IX.

VISION.

778. As a preparation for studying the optical structure of the eye and the laws of vision, it will be useful first to learn in what way images of external objects are formed in a dark room, by light admitted through a hole in the window shutter.

779. *Image by light through an aperture.*—If light from an external object pass through a small opening of any form, in the wall of a dark room, it will form an ill-defined inverted image on the opposite wall. Imagine a minute square orifice, through which the light enters, and falls on a screen several feet distant. A pencil of light, in the form of a square pyramid, emanating from the highest point of the object, passes through the aperture, and forms a luminous square near the bottom of the screen. From an adjacent point another pencil, crossing the former at the aperture, forms another square, overlapping, and nearly coinciding with the former. Thus every point of the object is represented by its square on the screen; and as the pencils all cross at the aperture, the image formed is every way inverted. It is also indistinct, because the squares overlap, and the light of contiguous points is mingled together. If the orifice is smaller, the image is less luminous, but more distinct, because the pencils which form it overlap in a less degree. If the hole is circular, or triangular, or of irregular form, there is no change in the image, which is now composed of small circles, or triangles, or irregular figures, whose shape is completely lost by overlapping.

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* For an explanation of the phenomena of color by polarized light, see Herschel on Light, in Encyc. Metrop., Lloyd's Lectures on the Wave Theory, Airy's Tracts, &c.

A room fitted for exhibiting such images, or those described in the next article, is called a *camera obscura.*

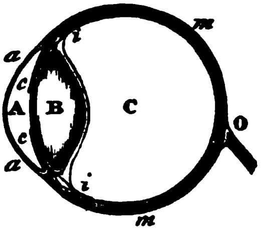
780. The image will become distinct and more luminous if the aperture be enlarged, and then covered by a convex lens of two or three inches in diameter. The image will be *distinct*, because the rays from each point of the object are converged to a point again, and *luminous*, in proportion as the lens has a larger area than the aperture before employed. A *scioptic ball* is a sphere containing a lens, and so fitted in a socket that it can be turned in any direction, and thus bring into the room the images of different parts of the landscape.

Instead of connecting the lens with the wall of a room, it is frequently attached to a portable box or case, within which the image is formed. The *Daguerreotype*, or *photograph*, is the image produced by the convex lens, and rendered permanent by the chemical action of light on a surface properly prepared. The lens for photographic purposes needs to be achromatic, and corrected, also, as far as possible, for spherical aberration.

The eye is a camera obscura, and the analogy existing between its principal parts and the contrivances employed to form a picture of external objects, as in the preceding experiments, will appear very striking on comparison.

781. *Description of the eye.*—The *eye* is an assemblage of lenses which concentrate the rays emanating from each point of external objects on a delicate tissue of nerves, called the *retina*, there forming an image or exact representation of every object, which is the thing immediately perceived or *felt* by the retina. Fig. 320 is a section of the human eye through its axis, in a horizontal plane. Its figure is, generally speaking, spherical, but in front considerably more prominent than the corresponding portion of a sphere. The eye consists of three principal chambers, filled with media of perfect transparency, whose refractive powers differ somewhat among themselves, but none of them is greatly different from pure water. The first of these media, A, occupying the anterior chamber, is called the *aqueous humor*, and consists, in fact, chiefly of pure water, holding in solution a little common salt and gelatine, with a trace of albumen. Its refractive index (Art. 691), is almost precisely that of water, viz., 1.337, that of water being 1.336. The cell in which the aqueous humor is contained, is bounded, on its anterior side, by a strong, transparent coating, called the *cornea*, which is not spherical, like the rest of the eye, but ellipsoidal, having, in fact, exactly the form which calculation gives for preventing spherical aberration. Near the posterior surface of the chamber A, but within the aqueous humor, is the iris, *cc*, an opaque colored screen composed of muscular fibers, by which the aperture through its center, called the *pupil*, is enlarged and contracted, according to the intensity of the light. In the human eye the pupil is circular, but in the eyes of some animals there is a special adaptation to the habits of the animal, the pupil being elongated horizontally for those that graze, and vertically for beasts and birds of prey. The changes in the size of the pupil are involuntary, and may be readily observed by any one, if while before a glass he alters considerably the intensity of the light.

FIG. 320.



782. The *crystalline lens*, B, forms the posterior boundary of the chamber A; it is a double convex lens, having a little greater refractive power than the aqueous, 1.384. From its edges toward the axis, it increases in density, which as effectually prevents spherical aberration as an increase of curvature near the axis. The posterior chamber, which occupies by far the greatest part of the eye, is filled with the *vitreous* humor, differing very little from the aqueous; both of them, in their consistency, much resemble the white of an egg. Back of the vitreous humor the *retina* lines the interior of the cavity up to the edges of the crystalline, *ii*. The retina is a *net-like* expansion of nerve, on which the images are depicted for the purpose of vision. Every fiber of the retina branches from the *optic nerve*, which enters at O, on the side of the axis of the eye toward the nose. The next coating is the *choroid membrane*, *mm*, which has a surface of jetty blackness, *pigmentum nigrum*, next the retina, whose office it is to absorb the light, and prevent internal reflections that would confuse the vision. The whole is inclosed by the *sclerotica*, a strong coating, which unites with the edge of the cornea in front, and constitutes what is called the white of the eye.

783. *Adaptations.*—By the prominence of the cornea rays of considerable obliquity are converged into the pupil, so that the eye, without being turned, has a range of vision more or less perfect, through an angle of about 150°.

When an image is formed by a lens, as the object recedes from the lens, the image approaches it, and *vice versa;* so that, in order to keep a distinct image on the screen, either the screen must be moved, or the convexity of the lens altered. In like manner, since we can see distinctly at various distances, there must be a power in the eye, either by change of curvature, or length of axis, or more probably by both combined, of adapting itself to the changes of distance. We are at least conscious of some muscular effort, whenever we change considerably the adjustment of the eye for different distances.

The eyes of animals are adapted, in respect to their refractive power, to the medium which surrounds them. The human eye being fitted for seeing in air, is unfit for distinct vision in water, since its refractive power is nearly the same as that of water, and therefore a pencil of parallel rays from water entering the eye, would scarcely be converged at all.

784. The image on the retina is a *real*, not an apparent image, and therefore is *inverted.* It should occasion no surprise that we see objects *erect* while the images are inverted; for the image is not the thing *seen*, but *that by means of which* we see; and we very naturally judge that part of a body to be highest which requires that we turn the eye upward to see it directly.

An object appears as *one*, though we see it by means of *two* images; but this is only one of many instances in which we have learned by experience to refer two or more sensations to one thing as the cause. Provided the images fall on parts of the retina, which in our ordinary vision *correspond* with each other, then by experience we refer both impressions to one object; but if we press one eye aside the image falls in a new place, and the object seems double.

785. *Long-sightedness and short-sightedness.*—The eyes of most persons are adapted to converge to a focus on the retina, those rays which are parallel or moderately divergent. Hence they can see with distinctness all objects which are not nearer than a certain distance, called the *limit of distinct vision.* This limit is different for different eyes, and for the same eyes at different periods of life. In early life it is generally not far from *five inches;* but increases as the eye gradually loses its convexity, and before the arrival of old age it is often at an infinite distance, so that even parallel rays are not converged to a focus on the retina. When the eye has become *long-sighted*, an object five or six inches from it gives an indistinct image; because the conical pencils are cut off by the retina before reaching their vertices, and therefore each point of the object is represented by a small circle instead of a point, and these circles overlap and cause a blur. *Convex* lenses are employed to supply the want of convexity in long-sighted eyes.

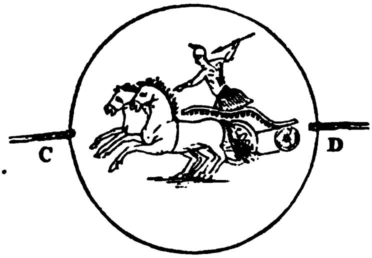
Sometimes the limit of distinct vision is much less than five inches; and when this is the case, there is usually also a limit, *beyond* which objects are not distinctly seen. This state of the eye is called *short-sightedness*, and results from too great convexity. If objects are at or beyond the common limit of distinct vision, the short-sighted person can not see distinctly, because the light is converged too quickly, and the rays of every pencil cross each other, and fall on the retina in a circle, instead of a point, producing the same kind of confusion as in long-sightedness. *Concave* lenses of the proper focal length are used to neutralize the excess of convexity in short-sighted eyes.

786. *Direct and indirect vision.*—To obtain a clear and satisfactory view of an object, the axes of the eyes are turned directly upon it, in which case each image is at the center of the retina. But when the light from an object is exceedingly faint, it is better seen by *indirect vision*, that is, by looking to a point a little on one side, and especially by changing the direction of the eyes from moment to moment, so that the image may fall in various places *near* the center of the retina. Many heavenly bodies are plainly discerned by indirect vision which are too faint to be seen by direct vision.

There is one spot on the retina which is entirely destitute of of sensibility, and is hence called the *punctum cæcum;* it is that at which the optic nerve enters the eye. An object whose image falls there is wholly invisible. If the right eye is shut, and the left is directed about 15° to the right of any small object, it entirely disappears, though, on turning the eye still further away, it is plainly seen.

787. *Continuance of impressions.*—The impression which a visible object makes upon the retina continues about one-eighth or one-ninth of a second; so that if the object is removed for that length of time, and then occupies its place again, the vision is uninterrupted. A coal of fire whirled round a center at the rate of eight or nine times per second, appears in all parts of the circumference at once. Two pictures, on opposite sides of a disk, are brought into view together, as parts of one and the same picture, by whirling the disk rapidly on one of its diameters. Such an instrument is called a *thaumatrope*. Fig. 321 represents the appearance produced when horses are painted on one side, and chariot and charioteer on the other. The *phantasmascope* is constructed on the same principle. Several pictures are painted in the sectors of a circular disk, representing the same object in a series of positions. These are viewed in a mirror through holes in the disk, as it revolves quickly in its own plane. Each glimpse, which is caught whenever a hole comes before the eye, presents the object in a new attitude; and all these views are in such rapid succession that they appear like one object going through the series of movements.

FIG. 321.



788. There are impressions on the retina of another kind, which are produced by intense lights; they continue longer, and are in respect to color unlike the objects which cause them. They are commonly called *accidental colors*. If a particular part of the retina is for some time affected by the image of a bright colored object, and then the eyes are shut, or turned upon a white surface, the *form* appears to remain, but the *color* is complementary to that of the object; and its continuance is for a few seconds, or several minutes, according to the vividness of the impression. This is the cause of the *green* appearance of the sky between clouds of brilliant *red* in the morning or evening.

789. *Estimate of distance and size of bodies.*—

1. If objects are near, we judge of relative distance by the *inclination of the optic axes* to each other. The greater that inclination is, or, which is the same thing, the greater the change of direction in an object, as it is viewed by one eye, and then by the other, the nearer it is. If objects are *very* near, we judge of their distance by the divergence of the rays which enter one eye alone.

2. If objects are known, we estimate their distance by the *visual angle* which they fill, having by experience learned to associate together their distance and their *apparent*, that is, their *angular* size.

3. Our judgment of distant objects is influenced by their *clearness* or *obscurity*. Mountains, and other features of a landscape, if seen for the first time when the air is remarkably pure, are estimated by us nearer than they really are; and the reverse, if the air is unusually hazy.

4. Our estimate of distance is more correct when *many objects intervene*. Hence it is that we are able to place that part

of the sky which is near the horizon, further from us than that which is over our heads. The apparent sky is not a hemisphere, but a flattened semi-ellipsoid.

790. Our judgments of distance and of magnitude are closely associated. If objects are known, we estimate their distance by their visual angle, as has been stated; but if unknown, we must first acquire our notion of their distance by some other means, and then their visual angle gives us a definite impression as to their size. And if our judgment of distance is erroneous, a corresponding error attaches to our estimate of their magnitude. An insect crawling slowly on the window, if by mistake it is supposed to be some rods beyond the window, will appear like a bird flying in the air. The moon near the horizon seems larger than above us, because we locate it at a greater distance.

791. *Binocular vision.—The stereoscope.*—If an object is placed near us, we obtain simultaneously *two views*, which are essentially different from each other—one with one eye, and one with the other. By the right eye, more of the right side, and less of the left side, is seen, than by the left eye. And we associate with these combined views the form and extent of a body, or group of bodies, particularly in respect to distance of parts from us. It is, then, by means of *vision with two eyes*, or *binocular vision*, that we are enabled to get accurate perceptions of prominence or depression of surface, reckoned in the visual direction. A picture offers no such advantage, since all its parts are on one surface, at a common distance from the eyes. But, if two perspective views of an object should be prepared, differing as those views do, which are seen by the two eyes, and if the right eye could then see only the right-hand view, and the left eye only the left-hand view, and if, furthermore, these two views could be made to appear on one and the same ground, the vision would then be the same as is obtained of the real object by both eyes. This is effected by the *stereoscope*. *Two* photographic views are taken, in directions which make a small angle with each other, and these views are seen at once by the two eyes respectively, through a pair of half-lenses, placed with their thin edges toward each other, so as to turn the visual pencils away from each other, as though they emanated from one object. An appearance of *relief* and *reality* is thus given to superficial pictures, precisely like that obtained from viewing the objects themselves.

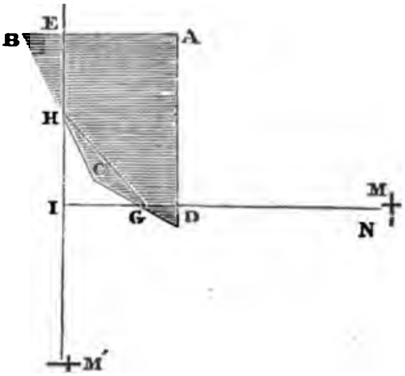
When the pictures change places the effect is exactly reversed, and the instrument, when thus used, has received the name, *pseudoscope*. By such an arrangement a *cameo* becomes an *intaglio*, and a mould for making a cast is changed into the cast itself.

CHAPTER X.

OPTICAL INSTRUMENTS.

792. *The camera lucida.*—This is a four-sided prism, so contrived as to form an apparent image at a surface on which that image may be copied, the surface and image being both visible at the same time. It has the form represented by the section in Fig. 322; A = 90°, C = 135°; B and D, of any convenient size, their sum of course = 135°. A pencil of light from the object M, falling perpendicularly on AD, proceeds on, and makes, with DC, an angle equal to the complement of D. After suffering total reflection at G, and again at H, its direction HE is perpendicular to ND. For, produce ND and EH, till they intersect in I; then, since C = 135°, CGH + CHG = 45°; but IGH = 2CGH, and IHG = 2CHG; ∴ IGH + IHG = 90°; ∴ I = 90°. Therefore HE emerges at right angles to AB, and is not refracted. Now, if the pupil of the eye be brought so near the edge B, that, while EH enters, there may also be a pencil from the surface at M′, then both the surface, M′, and the object, M, will be seen coinciding with each other, and the hand may therefore sketch M on the surface at M′. The reason for two reflections of the light is, that the inversion produced by one reflection may be restored by the second.

FIG. 322.



One of the most useful applications of the camera lucida, is in connection with the compound microscope, where it is employed in copying with exactness the forms of natural objects, which are of almost infinitesimal minuteness.

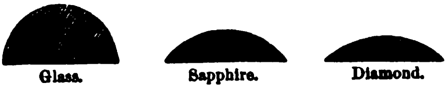
793. *The single microscope.*—The *microscope*, as the name signifies, is an instrument intended for *viewing minute* objects. The simplest kind is *a convex lens.* The nearer an object is brought to the eye, the larger is the angle which it fills, and therefore the more perfect is the view, provided the rays of each pencil are converged to a point on the retina. But, if the object is nearer than the limit of distinct vision, the eye is unable to produce sufficient convergency. If the letters of a book are brought close to the eye, they become blurred and wholly illegible. But let a pin-hole be pricked through paper, and interposed between the eye and the letters, and, though faint, they are *distinct* and *much enlarged.* The *distinctness* is owing to the fact that the outer and divergent rays are excluded, and the eye is able to converge the few central rays of each pencil to a focus. The letters appear *magnified,* because they are so near, and fill a large angle.

794. A convex lens, instead of excluding the outer rays, bends them toward the central ones, and enables the eye to converge the whole pencil to a focus on the retina, thus preserving all the light, while it enables the eye to see the object under a large visual angle. Since the lens may be close to the eye, the apparent enlargement of the object will be in proportion to its nearness to the lens. Taking *five* inches as the limit of distinct vision, if the principal focal distance is *one-fourth* of an inch, then we may consider the object 20 times nearer the eye than in viewing it without a lens, and therefore magnified 20 times in diameter, or 400 times in area. Now glass lenses have been made, whose focal length is not more than $\frac{1}{50}$ inch, and whose magnifying power, therefore, is $5 : \frac{1}{50}, = 250$ in diameter, or 62,500 in area.

795. When, however, an object is so near to the eye, a very minute space covers the whole field of vision, and it is only the minutest objects, or the smallest parts of a body, that are visible in such microscopes. The extent of parts seen by a microscope is called the *field of view.* A microscope of small focal distance has a proportionally small field of view. Moreover, when the object is so near to the lens, the outer rays are very diverging, and there is great spherical aberration. Only the central portion of the lens, therefore, can be used, and this is called its *aperture.* The aperture of a lens of small focal distance and high magnifying powers, must of necessity be small, and one of the principal difficulties in the use of such microscopes is the want of sufficient light. Hence microscopes of different focal distances are required for different purposes. Where we wish to view a large object at once, we must use a lens which has a large field of view, and of course but a comparatively small magnifying power. Such are the glasses used by watchmakers and other artists. Microscopes which magnify but little, yet afford a large field of view, are called *magnifiers*, or *magnifying glasses.* Such are the large lenses employed for viewing pictures. But for inspecting the minute parts of a small insect, we require a much higher power; and, the object being very small, a large field of view is not necessary. The only difficulty to be obviated is the want of light; and this evil is remedied, either by placing the object in the sun, or by condensing upon it a still stronger light, by means of apparatus specially adapted to that purpose.

796. Microscopes are sometimes made of the hardest precious gems, especially the *diamond* and the *sapphire*. The diamond seems to unite in itself almost every desirable quality for this purpose. This substance is distinguished for its high refractive power, its index of refraction being 2.439, while that of crown-glass is only 1.530 (Art. 692); hence a given refracting, and, of course, magnifying, power may be attained with a lens of less curvature, and consequently (Art. 709) subject to less *spherical aberration* than glass lenses of the same power. Indeed, it is estimated that the indistinctness arising from spherical aberration in a diamond lens is only $\frac{1}{10}$th as great as in a glass lens of equivalent power. The sapphire has analogous properties, as also the garnet. The comparative curvatures and thicknesses of three lenses of the same refracting power, made respectively of glass, sapphire, and diamond, are exhibited in the annexed diagrams. Since, moreover, a diamond lens admits of being made much thinner than a glass lens of the same power, the loss of light by absorption is far less, and the *brightness* of the image is proportionally augmented. Another valuable property of the diamond is its low dispersive power, on which account its *chromatic aberration* is small. But while the diamond possesses these favorable qualities, yet the expensiveness of the material, and the labor of grinding so hard a substance, prevent its frequent use.

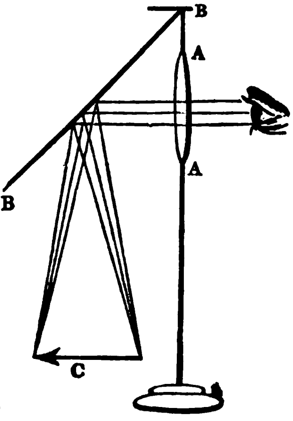
FIG. 823.



Glass. Sapphire. Diamond.

797. *The perspective glass.*—Fig. 824 represents an instrument of this kind. It is a single microscope, consisting of a large convex lens, AA, having behind it a plane mirror, BB, inclined at 45°. The picture, C, is placed horizontally below, and appears, therefore, vertical before the observer, and somewhat magnified. The distance from the picture to the mirror, and thence to the lens, must be less than the principal focal distance, so that the virtual image may be distinct. One reason that the pictures seen thus appear more real, is

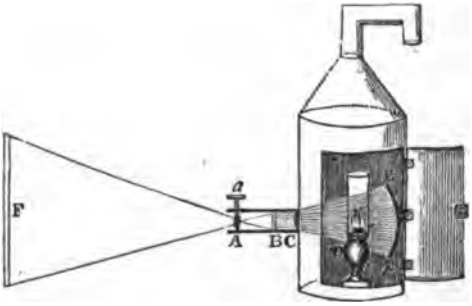
FIG. 824.



that other objects are excluded from view. The imagination is aided in a similar manner, whenever we view a picture through the hand, or a roll of paper.

798. *The magic-lantern.*—This is an instrument for producing large images of transparent paintings. It consists of a box, represented in Fig. 325, containing a lamp, and having openings so arranged as to permit the air to pass freely through it, without letting light escape. In front of the lamp is a tube containing a concentrating lens, C, the painting on glass, B, and the lens, A, for producing the image; back of the lamp may be a concave mirror for reflecting additional light on the lens C. The transparency B is a painting on glass, and the strong light which falls on it proceeds through the lens A, as from an original object brilliantly colored. It is a little further from A than its principal focus, and therefore the rays from any point are converged to the conjugate focus in a real image, F, on a distant screen.

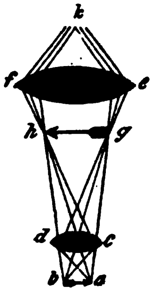
Fig. 325.



This image is of course inverted relatively to the object, and, therefore, if the picture B is inverted, F will be erect. The lens may be placed at various distances from B by the adjusting screw *a*, so as to give the greatest distinctness to the image at any given distance of the screen. According to Art. 706, the diam. of B : diam. of F : : AB : AF; and therefore, theoretically, the image may be as large as we please. But spherical aberration will increase rapidly, as the image is enlarged, and even if this evil could be remedied, the want of light would render the image too faint to be well seen; for the illumination is as much less than that of the painting as the area is greater. The magic-lantern is used for purposes of amusement, and also for illustrating some branches of science.

799. *The solar microscope.*—This does not differ in principle from the magic-lantern. For illumination, the solar or electric light is employed, and images are formed, not of artificial paintings, but of minute natural objects. The lens, which forms the image, is fixed in the end of a tube, and at the other end is a mirror which can be turned on a hinge to incline at any angle with the tube. This apparatus is attached to a window-shutter, the mirror on the outside, and the tube within. By adjusting screws the mirror is inclined so as to reflect the sunbeam along the tube, where it is concentrated by lenses upon the minute object lying just back of the image-lens. In this way the light may be made so intense that the images of minute objects will be plainly seen, when magnified millions of times in area. Spherical aberration is, however, inevitably considerable; and the instrument is of service, not for investigation and discovery, but for illustration.

FIG. 326.

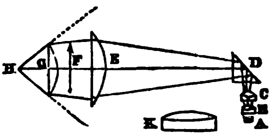


800. *The compound microscope.*—It is so called, because it consists of two parts, an object-glass, by which a real and magnified image is formed, and an eye-glass, by which that image is again magnified. Its general principle may be explained by Fig. 326, in which *ab* is the minute object, *cd* the object-glass, and *ef* the eye-glass. Let *ab* be a little beyond the principal focus of *cd*, and then the image *gh* will be real, on the opposite side of *cd*, and many times larger than *ab*. Now apply *ef* as a single microscope, for viewing *gh*, as though it were an object of comparatively large size. Therefore, *gh* must be slightly nearer than the principal focus of *ef*, so that the rays of each pencil shall be a little, and only a little, divergent; they will, therefore, come to the eye at *k*, from an *apparent* image on the same side as the *real* one, *gh*; and the extreme pencils, *ek*, *fk*, if produced backward, will include the image between them, *ekf* being the angle which it fills.

801. The *magnifying power* of the compound microscope is estimated by compounding two ratios; first, the distance of the image from the object-glass, to the distance of the object from the same; and secondly, the limit of distinct vision, to the distance of the image from the eye-glass. For, the image itself is enlarged in the first ratio (Art. 706); and the eye-glass enlarges that image in the second ratio (Art. 794). The advantage of this form over the single microscope, is not so much that a great magnifying power is obtained, as that a given magnifying power is accompanied by a larger field of view.

802. Great improvements have been made in the compound microscope, principally by combining lenses in such a manner as greatly to reduce the chromatic and spherical aberrations. The object-glass generally consists of one, two, or three achromatic pairs, each pair having the form represented in Fig. 300. The eye-piece usually contains two plano-convex lenses, a combination which is found to be the most favorable for diminishing the spherical aberration. For convenience, the direction of the rays is, in many instruments, changed from a vertical to a horizontal direction, by total reflection in a right-angled prism. In Fig. 327, A is the object; B and C, achromatic plano-convex lenses, the plano-concave part being of flint-glass, the double convex part of crown-glass, and the two parts fitted and cemented together (the form shown at K); D, the right-angled prism; E, the first eye-glass, though it strictly forms a part of the object-glass, since the image is beyond it at F, and it hastens the convergence of the rays to the foci in that image; G, the second eye-glass, converging the pencils to the eye at H, while the rays of each pencil diverge a little, as from the magnified image back of G. The image seen at H fills the angle made by the axes of the two extreme pencils.

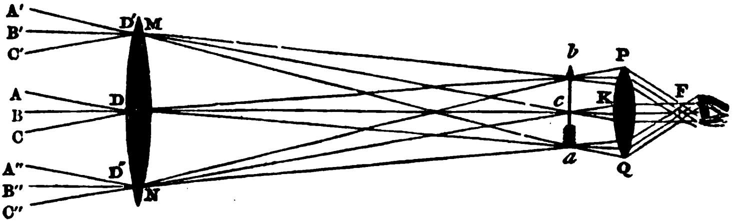
Fig. 327.



803. *Telescopes.*—The telescope aids in *viewing distant* bodies, as the name implies. An image of the distant body is first formed in the principal focus of a convex lens or a concave mirror; and then a microscope is employed to magnify that image as though it were a small body. The image is much more luminous than that formed in the eye, when looking at the heavenly body, because there is concentrated in the former the large beam of light which falls upon the lens or mirror, while the latter is formed by the slender pencil only, which enters the pupil of the eye. If the image in a telescope is formed by a lens, the instrument is called a *refracting telescope;* but if by a mirror, a *reflecting telescope.*

804. *Astronomical telescope.*—This is the most simple of the refracting telescopes, consisting of a lens to form an image of the heavenly body, and a single microscope for magnifying that image. The former is called the *object-glass*, the latter the *eye-glass*. The image is of course at the principal focus of the object-glass, and the eye-glass is placed at its own focal distance beyond the image, in order that the ray of each pencil may emerge parallel; therefore the two lenses are separated from each other by the sum of their focal distances. (See Fig. 328.)

FIG. 323.



MN, object-glass. PQ, eye-glass.
A′D′, AD, A″D″, parallel rays from the top of the object.
B′D′, BD, B″D″, " " " center do.
C′D′, CD, C″D″, " " " bottom do.
ba, inverted image formed in the focus of parallel rays.
*b*PF, a pencil of rays, proceeding from the top of the image to the eye-glass, and rendered parallel.
*c*KF, a similar pencil from the center.
*a*QF, do. from the bottom.
F, point where the different pencils cross the axis.
PFQ, the visual angle filled by the virtual image.

805. But an instrument constructed in this manner, without special care to remove the spherical and chromatic aberrations, and to secure glass free from inequalities of structure, would at the present day be regarded as worthless. It is obvious that the amount of light in the image depends on the area of the aperture of the object-glass; and many astronomical objects require all the light which can be commanded. It is desirable therefore to make the aperture as large as possible. But if it is large, the focal length must be proportionally increased, otherwise the surfaces of the lens become greater portions of the whole spherical surface, and the aberrations increase rapidly. Likewise, with every enlargement of the lens, the difficulty of obtaining glass of uniform density is greatly increased. The great length of instrument which a large aperture requires, is simply a matter of inconvenience; but the difficulty of making glass (especially flint-glass) which is free from inequalities of density, is so great, that no excellent instrument has yet been made, having an aperture more than *fifteen inches* in diameter.*

806. The object-glass of a telescope consists of a convex lens, united with a concave lens of higher dispersive power, so adapted to each other, in respect to curvatures, as to destroy as far as possible the chromatic aberration. (Art. 721.) The *eye-glass* usually employed is that *of Huygens*, composed of two plano-convex lenses, with the plane surfaces toward the eye, and so arranged that the image of the object-glass falls between them. The lenses E and F (Fig. 329) produce much less aberration, both spherical and chromatic, than the double convex lens, D, of the same refractive power. But if a micrometer is to be used for measuring angular diameters or distances, the *eye-piece of Ramsden* is used, so constructed that the image does not fall between its two lenses.

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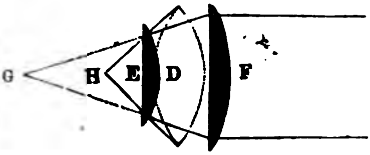
* The only refractors, whose aperture is so large as fifteen inches, are those in the observatories of Pulkova, Russia, and Harvard University, Cambridge, Mass. The object-glass alone for the last-named instrument cost 18,000 dollars.

FIG. 329.

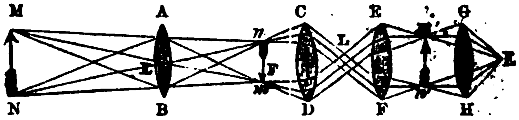


807. The *magnifying power* of the astronomical telescope is expressed by the ratio of the *focal distance of the object-glass to that of the eye-glass.* For (Fig. 328), the object, as seen by the naked eye, fills the angle ADC, between the axes of its extreme pencils. But, since the axes cross each other in straight lines at the optic center of the lens, ADC = *a*D*b*. Therefore, to an eye placed at the object-glass, the image, *ab*, appears just as large as the object; while, at the eye-glass, it appears as much larger in diameter, as the distance is less. Great *magnifying* power is of little value, unless it is accompanied by a sufficient *illuminating* power, obtained by giving a large size to the object-glass, and a good *defining* power, which it is most difficult to secure, since it depends on the uniformity of structure in the glass, and a careful removal of the spherical and chromatic aberrations.

808. The *equatorial mounting* of large telescopes is quite essential for accuracy of observation or measurement. When the magnifying power is great, the diurnal motion is very perceptible, and the body quickly leaves the field of view. To prevent this, the telescope is so mounted as to revolve on an axis parallel to the earth's axis, and then by means of a clock it has a motion communicated to it, by which it exactly keeps up with the apparent motion of a heavenly body. Another axis, at right angles with the former, allows the telescope to be directed to a point at any distance north or south of the celestial equator.

809. *Terrestrial telescope.*—In order to secure simplicity, and thus the highest excellence, in the astronomical telescope, the image is allowed to be *inverted,* which circumstance is of no importance in viewing heavenly bodies. But, for terrestrial objects, it would be a serious inconvenience; and, therefore, a *terrestrial telescope,* or *spy-glass,* has additional lenses for the purpose of forming a second image, inverted, compared with the first, and, therefore, erect, compared with the object. In Fig. 330, MN is the object; AB, the object-glass; *mn*, the first image; CD, the first eye-glass, which converges the pencils of parallel rays to L. Instead of placing the eye at L, the pencils are allowed to cross and fall on the second eye-glass, EF, by which the rays of each pencil are converged to a point in the second image, *m'n'*, which is viewed by the third eye-glass, GH. The second and third lenses are commonly of equal focal length, and add nothing to the magnifying power.

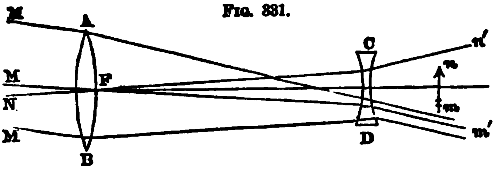
FIG. 330.



Such instruments are usually of a portable size, and hence the aberrations are corrected with comparative ease, by the methods already described. The spy-glass, for convenient transportation, is made of a series of tubes, which slide together in a very compact form.

810. *Galileo's telescope.*—This was the first form of telescope, having been invented by Galileo, as the name implies. It differs from the common astronomical telescope, in having for the eye-glass a *concave* instead of a convex lens, which receives the rays at such a distance from the focus to which they tend, as to render them parallel. Thus, the rays, M (Fig. 331), from the top of the object, are converged by the object-glass, AB, toward *m*, in the image; and the pencil from the bottom of the object, whose axis is NF, is converged toward *n*; but the concave lens CD is interposed at such a point as to render these converging rays parallel, and in this way they come to the eye situated behind the lens. But, though the *rays* converge, before they reach the concave lens, the *pencils* diverge, having crossed at F; therefore, in passing the concave lens, they are made to diverge more, and will enter the eye as if they had crossed at a much nearer point than F. The angle between these extreme pencils is the angle which the object appears to fill; and the magnifying power is in the ratio of this angle to the angle MFN = *m*F*n*; and that equals the ratio of the focal distance of AB to the focal distance of CD. The object appears *erect* in the

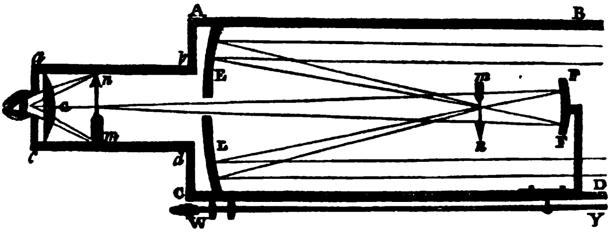
FIG. 331.



Galilean telescope, since the pencil, which comes from the top of the object, *appears* to come from the top of the virtual image; thus, the parts of the object and image are similarly situated. It is obvious that, since the pencils diverge, only the central ones, within the size of the pupil, can enter the eye. This circumstance exceedingly limits the field of view, and unfits the instrument for telescopic use. It is employed for opera-glasses, having a power usually of only *two* or *three* in diameter.

811. *Gregorian telescope.*—This is the most frequent form of reflecting telescope, and receives its name from the inventor, Dr. Gregory, of Scotland. The light from a heavenly body, entering the open tube (Fig. 332), is received on the large concave

FIG. 332.



speculum, EE, which forms an inverted image, *mn*, at the principal focus; the rays of each pencil crossing there next meet the small concave mirror F, which forms an erect image, *nm*, at the conjugate focus, beyond the speculum, the center of the latter being perforated to let the light pass through. The eye-glass, G, magnifies this image. WY is an adjusting screw for the small mirror. To avoid confusion, only the rays of the central pencil are drawn in the figure. Rays from the *top* of the object would enter the tube inclining slightly downward, and be reflected to *n*, and again to *n*. Rays from the *bottom* would ascend, and be reflected to *m* in each image.

812. *Herschelian telescope.*—Sir William Herschel modified the Gregorian, by dispensing with the small reflector, F, and inclining the speculum EE, so as to form the image near the edge of the tube at D, where the eye-glass is attached. Thus, the observer is situated with his back to the object. The speculum of Herschel's telescope was about four feet in diameter, and weighed more than 2000 pounds, and its focal length was forty feet. The Earl of Rosse has recently constructed a Herschelian telescope having an aperture of *six* feet, and a focal length of *fifty* feet.

$v = g t^2$

$\frac{1}{2} g t^2 = 16\frac{1}{2} \times 25 =$

$$\begin{array}{r} 16\frac{1}{2} \\ 25 \\ \hline 80 \\ \hline 408\frac{1}{2} \end{array}$$

$32\frac{1}{6} \times 5 = 160\frac{5}{6}$

$v = 32\frac{1}{6} \times 5 = 160\frac{5}{6}$

$\frac{6a + 9b}{a + b} = 14 = \frac{6a + 9b}{11} = 154 - 35 = 119$

$7a = 105$

$\frac{a + b}{a + b} = \frac{7}{2} = \frac{6(7+3)}{c(a+b)} = 7a + 7b = 21b$